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Research Article

Two Weighted Fuzzy Goal Programming Methods to Solve Multiobjective Goal Programming Problem

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We propose two new methods to find the solution of fuzzy goal programming (FGP) problem by weighting method. Here, the relative weights represent the relative importance of the objective functions. The proposed methods involve one additional goal constraint by introducing only underdeviation variables to the fuzzy operator λ (resp., 1-λ), which is more efficient than some well-known existing methods such as those proposed by Zimmermann, Hannan, Tiwari, and Mohamed. Mohamed proposed that every fuzzy linear program has an equivalent weighted linear goal program where the weights are restricted as the reciprocals of the admissible violation constants. But the above proposition of Mohamed is not always true. Furthermore, the proposed methods are easy to apply in real-life situations which give better solution in the sense that the objective values are sufficiently closer to their aspiration levels. Finally, for illustration, two real examples are used to demonstrate the correctness and usefulness of the proposed methods.

1. Introduction

In real life, the decision maker is always confronted with different conflicting objectives. So it is necessary to conduct trade-off analysis in multiobjective decision analysis (MODA). Therefore, the goal programming technique has been developed to consider such type of problem. In 1955, the roots of goal programming lie in the journal (Management Science) by Charnes et al. [1]. Goal programming (GP) has been widely implemented to different problems by the famous researchers [2–9].

Most of the methodologies for solving multiobjective linear or fractional goal programming problem [10–12] were computationally burdensome. In economical and physical problems of mathematical programming generally, and in the linear or fractional programming problems in particular, the coefficients in the problems are assumed to be exactly known. However, in practice, this assumption is seldom satisfied by great majority of real-life problems. Usually, the coefficients (some or all) are subjected to errors of measurement or vary with market conditions.

To overcome such a problem, the fuzzy set theory (FST) initially introduced by Zadeh [13] has been used to decision-making problems with imprecise data. Bellman and Zadeh [14] state that a fuzzy decision is defined as the fuzzy set of alternatives resulting from the intersection of the goals or objectives and constraints. The concept of fuzzy programming was first introduced by Tanaka et al. [15] in the framework of fuzzy decision of Bellman and Zadeh. Afterwards, fuzzy approach to linear programming (LP) with several objectives was studied by Zimmermann [16]. Luhandjula [17] used a linguistic variable approach in order to present a procedure for solving multipleobjective fractional programming problems (MOLFPP).

In 1980, Narasimhan [18] was the first to study the use of fuzzy set theory in Gp. Hannan [19] introduced interpolated membership functions (i.e., piecewise linear membership functions) into the fuzzy goal programming (FGP) model, then the FGP model could be solved using the linear programming method. Many real-world problems [20–22] are solved by fuzzy multiobjective linear or fractional goal programming technique.

In 1997, Mohamed discussed the relationship between goal programming and fuzzy programming where the highest degree of each of the membership goals is achieved by minimizing their underdeviation variables [23]. During the past, some pioneers [24, 25] proposed a novel approach to solve fuzzy multiobjective fractional goal programming (FMOFGP) problems. In 2007, Chang gives the idea of binary behavior of fuzzy programming [26].

In the recent past, several pioneer researchers projected some new approaches and works in the field of fuzzy multiobjective linear or fractional goal programming with consideration of both the under- and overdeviation variables to the membership goals [8, 27–39]. By using the existing methods, the obtained solutions are approximate not exact and also it is very difficult to apply the existing methods to find the better optimal solution of fuzzy goal programming (FGP) problems in the sense that there may exist a situation where a decision maker would like to make a decision on the FGP problem, which involves the achievement of fuzzy goals, in which some of them may meet the behavior of the problem and some are not. In such situations, the estimation of the relative weights attached to the goals plays an important role in multiobjective decision-making process. In order to reflect the relative importance of the fuzzy goals, various pioneer researchers proposed FGP approaches using different weights for the various goals [16, 18, 19, 40].

The main purpose of this paper is to point out the shortcomings of the existing FGP methods and to overcome these shortcomings; two new weighted fuzzy goal programming methods has been proposed for finding the correct efficient solutions, where weights are attached to the fuzzy operator in the constraint and only underdeviation variables are introduced in the goal constraint. Here, we notice that there are some fuzzy linear programs in the real-world decision-making environment, which have an equivalent weighted fuzzy linear goal program where weights are not restricted as the reciprocals of the admissible violation constants. Again it reveals that not every fuzzy linear program has an equivalent weighted fuzzy linear goal program if the weights are varied. In this paper, we have investigated fuzzy goal programming problems with different important levels to determine the desirable and realistic solutions for each goal. Our proposed methods can ensure the more important fuzzy goal, if the weights are varied that is, if the decision maker may change the relative importance of fuzzy goals. For illustration, two real examples adopted from [26, 29] are used to demonstrate the usefulness of the proposed methods. The obtained results are discussed and compared with the results of the existing methods.

This paper is organized as follows: following the introduction, in Section 2, formulation of multiobjective linear programming problem and multiobjective fractional programming problem is discussed in brief. In Section 3, fuzzy goal programming formulation has been described. In Section 4, the shortcomings of the existing methods are explained. In Section 5, construction of membership goals has been proposed for solving FGP problems. In Section 6, the existing and proposed weighted fuzzy goal programming methods have been presented. Numerical examples and their results compared with the existing methods are discussed in Section 7. In Section 8, advantages of the proposed methods over the existing methods are described. Section 9 deals with the concluding remarks.

2. Problem Formulation

The general format of the multiobjective linear programming problem (MOLPP) can be written as

$$\begin{gathered} \text{Optimize } Z_k(x) = c_k x, \quad k = 1, 2, \ldots, K, \\ \text{where } x \in X = \left\{ x \in R^n \mid Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b, x \geq 0, b^T \in R^m \right\}, \\ \text{where } c_k^T \in R^n. \end{gathered} \tag{2.1}$$

If the numerator and denominator in the objective function as well as the constraints are linear, then it is called a linear fractional programming problem (LFPP). The general format of the multiobjective fractional programming problem (MOFPP) can be written as

$$\begin{gathered} \text{Optimize } Z_k(x) = \frac{c_k x + \alpha_k}{d_k x + \beta_k}, \quad k = 1, 2, \ldots, K, \\ \text{where } x \in X = \left\{ x \in R^n \mid Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b, x \geq 0, b^T \in R^m \right\}, \\ \text{where } c_k^T, d_k^T \in R^n;\ \alpha_k, \beta_k \text{ are constants and } d_k x + \beta_k > 0. \end{gathered} \tag{2.2}$$

3. Fuzzy Goal Programming Formulation

3.1. Construction of Fuzzy Goals

In multiobjective fractional programming, if an imprecise aspiration level is introduced to each of the objectives then these fuzzy objectives are termed as fuzzy goals. Let g_k be the aspiration level assigned to the kth objective $Z_k(x)$. Then the fuzzy goals are

(i) $Z_k(x) \gtrsim g_k$ [for maximizing $Z_k(x)$] and

(ii) $Z_k(x) \lesssim g_k$ [for minimizing $Z_k(x)$];

where "$\gtrsim$" and "$\lesssim$" represent the fuzzified versions of "$\geq$" and "$\leq$". These are to be understood as "essentially greater than" and "essentially less than" in the sense of Zimmermann [16].

3.2. Construction of Fuzzy Multiobjective Goal Programming

Hence, the fuzzy multiobjective goal programming can be formulated as follows:

$$\begin{gathered}
\text{find } x, \\
\text{so as to satisfy } Z_k(x) \gtrsim g_k, \quad k = 1,2,\ldots,k_1, \\
Z_k(x) \lesssim g_k, \quad k = k_1+1,\ldots,K, \\
\text{subject to } Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b, \\
x \geq 0.
\end{gathered} \tag{3.1}$$

3.3. Construction of Membership Functions

Now the membership function μ_k for the kth fuzzy goal $Z_k(x) \gtrsim g_k$ can be expressed as follows:

$$\mu_k(Z_k(x)) = \left\{ \begin{array}{cc} 1 & \text{if } Z_k(x) \geq g_k \\ \dfrac{Z_k(x) - l_k}{g_k - l_k} & \text{if } l_k \leq Z_k(x) \leq g_k \\ 0 & \text{if } Z_k(x) \leq l_k \end{array} \right\}, \tag{3.2}$$

where l_k is the lower tolerance limit for the kth fuzzy goal and $(g_k - l_k)$ is the tolerance (p_k) which is subjectively chosen. Again the membership function μ_k for the kth fuzzy goal $Z_k(x) \lesssim g_k$ can be expressed as follows:

$$\mu_k(Z_k(x)) = \left\{ \begin{array}{cc} 1 & \text{if } Z_k(x) \leq g_k \\ \dfrac{u_k - Z_k(x)}{u_k - g_k} & \text{if } g_k \leq Z_k(x) \leq u_k \\ 0 & \text{if } Z_k(x) \geq u_k \end{array} \right\}, \tag{3.3}$$

where u_k is the upper tolerance limit for the kth fuzzy goal and $(u_k - g_k)$ is the tolerance which is subjectively chosen.

3.3.1. Construction of Existing Membership Goals

In fuzzy programming approaches, the highest possible value of membership function is 1. Thus, according to the idea of Mohamed [23], the linear membership functions in (3.2) and (3.3) can be expressed as the following functions (i.e., the achievement of the highest membership value):

$$\frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- - d_k^+ = 1 \quad \text{for} \ \gtrsim \text{type fuzzy goals,} \tag{3.4}$$

$$\frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- - d_k^+ = 1 \quad \text{for} \ \lesssim \text{type fuzzy goals,} \tag{3.5}$$

where x, d_k^-, d_k^+ (≥ 0); $d_k^- \times d_k^+ = 0$ and d_k^- and d_k^+ represent the underdeviation and over-deviation variable from the aspired levels.

4. Shortcomings of the Existing Methods

In this section, the shortcomings of some of the existing methods for solving FGP problems are mentioned.

(i) The well-known existing methods, namely, Zimmermann's method [16] and Hannan's method [19] do not always yield the value of fuzzy operator λ contained in $[0, 1]$ that is, yield $\lambda > 1$, for the fuzzy goal programming problems when the weights are taken as $w_k \leq 1$ and $\sum w_k = 1$, $k = 1, 2, \ldots, K$.

(ii) Mohamed suggested that every fuzzy linear program has an equivalent weighted linear goal program where the weights are restricted as the reciprocals of the admissible violation constants [23]. But this assertion of Mohamed is not always true.

(iii) Tiwari et al. [40] have proposed a weighted additive model that incorporates each goal's weight into the objective function, where weights (w_k) reveal the relative importance of the fuzzy goals. Here, weights are taken as $\sum w_k = 1$, $k = 1, 2, \ldots, K$. This model yields the value of fuzzy operator $\lambda(\lambda = \min(\mu_k(x)))$ contained in $[0, 1]$ always, but it may produce same feasible solutions when the weights are changed which does not reflect the relative importance of the fuzzy goals.

In this paper, two new methods of solving fuzzy goal programming problems have been proposed to get rid of these shortcomings.

Now, the construction of the membership goals had been followed by using Mohamed's FGP method where two deviation variables d_k^- and d_k^+ are introduced. But introduction of both deviation variables to the membership goals is unnecessary [41].

5. Construction of Proposed Membership Goals

In (3.4) or (3.5), if the overdeviation variables $d_k^+ > 0$ then the underdeviation variables d_k^- must be zero, since $d_k^- \times d_k^+ = 0$. Thus, $\mu_k(Z_k(x)) - d_k^+ = 1$ and it implies that any overdeviation from the fuzzy objective goals indicates that the membership value is greater than 1, which is not possible. So d_k^+ should be zero always. On the other hand, the Zimmerman's type membership function $\mu_k(Z_k(x))$ of the kth fuzzy goals $Z_k(x) \gtrsim g_k$ is given by (3.2). Now, we see that $(Z_k(x) - l_k)/(g_k - l_k) \leq 1$ always, when $l_k \leq Z_k(x) \leq g_k$. Since our aim is to achieve membership value of the fuzzy goals close to 1 as best as possible and $(Z_k(x) - l_k)/(g_k - l_k) \leq 1$ (similarly, $(u_k - Z_k(x))/(u_k - g_k) \leq 1$), that is, $\mu_k(Z_k(x)) \leq 1$, then only underdeviation variables need to be introduced in the kth membership goals [41]. The FGP methods where membership goals are based on (3.4) and (3.5) do not give completely correct solution always. From the above consideration, the proposed membership goals with the aspired level 1 can be represented as

$$\frac{Z_k(x) - l_k}{g_k - l_k}(\text{resp., } \lambda) + d_k^- = 1, \tag{5.1}$$

$$\frac{u_k - Z_k(x)}{u_k - g_k}(\text{resp., } \lambda) + d_k^- = 1. \tag{5.2}$$

Here, d_k^- represents the underdeviation variables, $k = 1, 2, \ldots, K$. $\mu_k(Z_k(x))$ represents the membership function for the objective $Z_k(x)$ of "$\geq$" type or "$\leq$" type. The objectives $Z_k(x)$ may be linear or fractional.

6. The Existing Weighted Fuzzy Goal Programming (FGP) Formulation

6.1. Hannan's Weighted FGP Formulation

Consider the following:

$$\begin{aligned}
\text{Minimize } & \sum w_k \left(d_k^- + d_k^+\right), \\
\text{Subject to } & \frac{Z_k(x) - l_k}{g_k - l_k} + d_k^- - d_k^+ = 1, \\
& \frac{u_k - Z_k(x)}{u_k - g_k} + d_k^- - d_k^+ = 1, \\
& Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b, \\
& \lambda + d_k^- - d_k^+ \leq 1, \\
& \lambda \geq 0,
\end{aligned} \tag{6.1}$$

where $x, d_k^-, d_k^+ \geq 0$; $d_k^- \times d_k^+ = 0$; $\sum w_k = 1$, $k = 1, 2, \ldots, K$.

6.1.1. Zimmermann's FGP Formulation

Consider the following:

$$\begin{aligned}
& \text{Max } \lambda, \\
& \text{Subject to } \lambda \leq \mu(Z_k(x)), \\
& Ax \begin{pmatrix} \geq \\ = \\ \leq \end{pmatrix} b, \\
& \lambda \geq 0,
\end{aligned} \tag{6.2}$$

where, $x \geq 0$.

6.2. Proposed Weighted Fuzzy Goal Programming Formulation

Now it is known that in the Zimmermann's weighted FGP method, there is no condition that $\lambda \leq 1$. In fact, λ can be more than unity when the weights $w_k < 1$. But the actual achieved level for each objective will never exceed unity. So the slack variables s_k are introduced to the kth constraint $w_k\lambda \leq \mu_k(Z_k(x))$ in the modified Zimmermann's weighted FGP method (WFGP). In the modified Zimmermann's weighted FGP method, the kth constraint $w_k\lambda \leq \mu_k(Z_k(x))$ is replaced by $w_k\lambda + s_k \leq \mu_k(Z_k(x))$ to keep $\lambda \leq 1$ when $w_k < 1$, $k = 1, 2, \ldots, K$. As λ is maximized, the slack variables s_k are minimized [42]. But it has been observed that after the introduction of the slack variables to the kth constraint $w_k\lambda \leq \mu_k(Z_k(x))$, still there is no guarantee that $\lambda \leq 1$ when $w_k < 1$, $k = 1, 2, \ldots, K$.

In 1987, Tiwari et al. [40] had proposed a weighted additive model, where $\lambda \in [0, 1]$ is satisfied always when $w_k < 1$, $k = 1, 2, \ldots, K$. Different weights in this weighted additive model are used for the various goals in order to reflect the relative importance of the fuzzy goals. But, this model may produce undesirable solutions when the weights are changed.

To overcome the drawbacks of the existing WFGP methods, we propose two new WFGP methods where the desired belongingness of fuzzy operator λ to $[0, 1]$ is fulfilled. These proposed methods allow the decision maker to determine clearly an acceptable solution for each fuzzy goal which is more realistic and also ensures the more important fuzzy goal even though the weights attached to the fuzzy operator may change.

6.2.1. Method 1

In this paper, we attempt to introduce a new weighted FGP method for fuzzy goal programming (FGP) problem by introducing only underdeviational variables d_k^- in the goal constraint for the fuzzy multiobjective goal programming problem with aspiration level one, $k = 1, 2, \ldots K$. Then this FGP method is used to achieve highest degree of membership for each of the goals by using max-min operator. The weights are also attached to the fuzzy operator λ in the constraints.

According to the idea of proposed membership goals based on (5.1) and (5.2), the proposed weighted FGP method 1 of fuzzy goal programming problem can be written as

$$\begin{gathered}\text{Find } x,\\ \text{Max } \lambda,\\ \text{Subject to } w_k\,\lambda \leq \mu_k(Z_k(x)),\\ \lambda + d_k^- = 1,\\ Ax\begin{pmatrix}\geq\\=\\\leq\end{pmatrix}b,\\ \lambda \geq 0,\end{gathered} \tag{6.3}$$

where $x, d_k^- \geq 0$; $k = 1, 2, \ldots K$.

Three different modes of weights are considered: $w_k = 1/p_k$; $\sum w_k = 1$; $w_k \leq 1$.

6.2.2. Method 2

Similarly, here we attempt to introduce a new weighted FGP method for fuzzy goal programming (FGP) problem by introducing only underdeviational variables d_k^- in the goal constraint for the fuzzy multiobjective goal programming problem with aspiration level one, $k = 1, 2, \ldots, K$. Then this FGP method is used to achieve the highest degree of membership for each of the goals by using min-max operator. The weights are also attached to fuzzy operator $(1 - \lambda)$. According to the idea of proposed membership goals based on (5.1) and (5.2), the proposed weighted FGP method 2 can be written as

$$
\begin{aligned}
&\text{Find } x,\\
&\text{Min}(1-\lambda),\\
&\text{Subject to } w_k\ (1-\lambda) \geq \left(1-\mu_k(Z_k(x))\right),\\
&(1-\lambda)+d_k^- = 1,\\
&Ax\begin{pmatrix}\geq\\=\\\leq\end{pmatrix}b,\\
&(1-\lambda)\geq 0,
\end{aligned}
\tag{6.4}
$$

where $x, d_k^- \geq 0$; $k = 1, 2, \ldots K$, $\lambda \geq 0$.

Three different modes of weights are considered: $w_k = 1/p_k$; $\sum w_k = 1$; $w_k \leq 1$.

The symbol d_k^- represents the underdeviation variables, p_k represents the tolerances, and w_k represents the weights; $k = 1, 2, \ldots, K$.

7. Illustrative Examples

The computational superiority and effectiveness of the proposed methods over existing methods are illustrated through two real examples by varying different weights.

One real example adopted from [29] is used to demonstrate the solution procedures of the fuzzy multiobjective fractional goal programming problem (FMOLFGPP) by the proposed FGP methods and other is adopted from [26] to illustrate the solution procedures of the fuzzy multiobjective linear goal programming problem (FMOLGPP) by the proposed FGP methods. The obtained results are compared with the solution of existing methods.

7.1. Example 1

This example adopted from Chang [29] is used to clarify the effectiveness of the proposed methods.

The fractional programming problem is represented as

$$\text{Max } Z(x) = \left(\frac{\text{the total user satisfaction}}{\text{total investment budget}}\right),$$
$$\text{Max } Z(x) = \frac{(\mathcal{A})}{(\mathcal{B})}, \tag{7.1}$$

$$\text{Subject to } 3x_1 + 2x_2 + 3x_3 + 3x_4 + 2x_5 + x_6 + 2x_7 + 3x_8 + 4x_9 + 3x_{10} + 2x_{11} + x_{12} \leq 15, \quad \text{(Manpower constraint)} \tag{7.2}$$

$$.1x_1 + .2x_2 + .2x_3 + .2x_4 + .2x_5 + .3x_6 + .2x_7 + .1x_8 + .2x_9 + .1x_{10} + .2x_{11} + .2x_{12} \leq 1.6, \quad \text{(Capital constraint)} \tag{7.3}$$

$$x_1+x_2+x_3+x_4+x_5+x_6+x_7+x_8+ x_9 + x_{10}+x_{11}+x_{12} \geq 6 (\text{At least six E-Learning Systems}) \tag{7.4}$$

where $\mathcal{A}$ denotes $2.16x_1 + 1.095x_2 + 1.4x_3 + 1.7x_4 + .69x_5 + .544x_6 + 1.3x_7 + .64x_8 + 1.7x_9 + 1.34x_{10} + .64x_{11} + 2.04x_{12}$, $\mathcal{B}$ denotes $.1x_1 + .2x_2 + .2x_3 + .2x_4 + .2x_5 + .3x_6 + .2x_7 + .1x_8 + .2x_9 + .1x_{10} + .2x_{11} + .2x_{12}$, and $x_k \geq 0$; $k = 1, 2, \ldots, 12$.

Now, we find the aspiration level for the objective $Z(x)$ of the above example, following the conventional technique [41]. In the solution process, we first maximize objective functions in the numerator (N) and also minimize objective functions in the denominator (D) with respect to the crisp constraints by using linear programming technique. Therefore, the aspiration level (g) for the fractional objective is $g = N^0(x_1, \ldots, x_{12})/D^0(x_1, \ldots, x_{12}) = N^0(2.8, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6.6)/D^0(4.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.5) = 26$, where $N^0(x_1, \ldots, x_{12}) = \max N(x_1, \ldots, x_{12})$ and $D^0(x_1, \ldots, x_{12}) = \min D(x_1, \ldots, x_{12})$.

Then the fuzzy goal of the problem becomes

$$\frac{(2.16x_1 + 1.095x_2 + \cdots + .64x_{11} + 2.04x_{12})}{(.1x_1 + .2x_2 + \cdots + .2x_{11} + .2x_{12})} \gtrsim 26. \tag{7.5}$$

Assume that the tolerance (p) of the fuzzy fractional objective goal is 9. The membership function of the problem is obtained as follows:

$$\mu(Z(x)) = \frac{((2.16x_1 + 1.095x_2 + \cdots + 2.04x_{12})/(.1x_1 + .2x_2 + \cdots + .2x_3)) - 17}{9}. \tag{7.6}$$

Table 1: Solution of fuzzy fractional goal programming problem by proposed method 1.

Weight	$g = 10, p = 9$	$g = 15, p = 5$	$g = 20, p = 5$
$w = 1/p$	ATUS = 102%,	Infeasibility present	Infeasibility present
$w = 1$	ATUS = 102%,	Infeasibility present	Infeasibility present
$w < 1$	ATUS = 102%,	$Z(x) = 12.99$, $w = .6$, ATUS = 99%, $Z(x) = 13.5$, $w = .7$, ATUS = 99%, $Z(x) = 13.99$, $w = .8$, ATUS = 87%, $Z(x) = 14.49$, $w = .9$, ATUS = 105%,	ATUS = 106%

Therefore, the proposed weighted FGP model of the above problem based on (6.3) is given by

$$\begin{aligned}
&\text{Maximize } \lambda, \\
&\text{Subject to } w\lambda \leq \frac{((2.16x_1 + 1.095x_2 + \cdots + 2.04x_{12})/(.1x_1 + .2x_2 + \cdots + .2x_{12})) - 17}{9}, \\
&\qquad\qquad \lambda + d^- = 1, \\
&\qquad\qquad \lambda \geq 0, \\
&\text{Equation (7.2)–(7.4)},
\end{aligned} \tag{7.7}$$

where $x_k \geq 0$, $k = 1, 2, \ldots, 12$; $d^- \geq 0$; $w = 1/p$ and $w \leq 1$.

We get infeasible solution by varying the weights.

Now assume that aspiration level $(g) = 19$, tolerance $(p) = 9$. We get infeasible solution by varying the weights.

In most of the real-life multiobjective decision situations, it was observed that the decision maker (DM) is often faced with the challenge of setting the exact aspiration levels to each objective due to inherent imprecise nature of model parameters involved with the practical problems. Setting the aim of achieving higher realistic value to the average total user satisfaction (ATUS) as best as possible, we first examine the different set of solutions of the above problem based on (7.7) by varying the weights, aspiration levels, and tolerances and then select the most suitable. The obtained results are tabulated in Table 1.

From Table 1, we see that the solution of the given fuzzy fractional goal programming problem is more realistic only when the aspiration level $g = 15$, tolerance $p = 5$ and weight $w = .7$ in the sense that the objective value is sufficiently close to the aspiration level with satisfactory realistic ATUS solution. Here, $\lambda = 1$.

Further, the above fuzzy fractional programming problem has been solved by using proposed FGP method 2 based on (6.4) under different weights. The results are shown in Table 2.

Table 2 shows that the proposed method 2 is not suitable to solve the above fuzzy fractional goal programming problems.

Now for comparison, the fuzzy fractional goal programming problem is solved by proposed methods 1 and 2, where the goal constraints are constructed by introducing both the under and overdeviation variables (based on the membership goals suggested by Mohamed). Also compare the results obtained from the well-known existing methods based on (6.1), (6.2), by varying the weights at different aspiration levels and tolerances. The comparison results are shown in the Table 3.

Table 2: Solution of proposed method 2.

Weight	$g = 10, p = 9$	$g = 15, p = 5$	$g = 20, p = 5$
$w = 1/p$	ATUS = 102%, $\lambda = 1$	Infeasibility present	Infeasibility present
$w = 1$	ATUS = 102%, $\lambda = 1$	Infeasibility present	ATUS = 106%, $\lambda = .408$
$w = .9$	ATUS = 102%, $\lambda = 1$	Infeasibility present	ATUS = 106%, $\lambda = .342$

Table 3: Comparison.

Weight w	Model class	Proposed methods 1 and 2 with $\lambda + d_k^- - d_k^+ = 1$	Zimmermann's method	Hannan's method
$w = 1/p, w \leq 1$	NLP	$\lambda > 1$	$\lambda > 1$	$\lambda > 1$

Table 4: Solution of the fuzzy fractional goal programming problem.

	Tiwari's weighted additive model		
Weight (w)	ATUS	Aspiration level, tolerance	λ
$w = 1$	106%	(10, 9) or (15, 5) or (20, 5), and so forth	.782
$w < 1$	106%	(10, 9), (15, 5)	.782

From Table 3, it is evident that the above fuzzy fractional programming problem cannot be solved by Zimmermann's method, Hannan's method. Also, the problem cannot be solved by the proposed methods, if the goal constraints are constructed by using both the under and overdeviation variables. Because the restriction $\lambda \in [0, 1]$ is not satisfied when $w_k \leq 1$, $\sum w_k = 1$, $k = 1, 2, \ldots K$.

Further, the solutions of the above fuzzy fractional goal programming (FFGP) problem by applying Tiwari's weighted additive model [40] have been summarized in Table 4.

From the Table 4, it has been seen that $\lambda \in [0, 1]$ is always satisfied. But the fuzzy fractional goal programming problem cannot be solved by Tiwari's weighted additive model because the value of average total user satisfaction (ATUS) is not realistic when the weights are changed.

Now, the solutions of the said fuzzy fractional goal programming (FFGP) problem obtained from the proposed methods 1 and 2 under different weights are shown in Table 5.

From Table 5, it is clear that the proposed method 1 yields better solution for the considered fuzzy fractional goal programming problem than the proposed method 2 in the sense that the ATUS solution is more realistic with $\lambda = 1$. Thus, the proposed method 2 fails to obtain the feasible solution for the fuzzy fractional goal programming problem, whereas the proposed method 1 gives efficient solution without any computational difficulties.

But it could be realized that the membership goals in fuzzy fractional goal programming problems are inherently nonlinear in nature and this may create computational difficulies in the solution process. To avoid such problems, the conventional linearization procedure [24, 25] is preferred.

The fuzzy fractional programming problem is now converted into fuzzy linear programming problem by first-order Taylor series and compared with the solutions of the existing methods.

Solving the fuzzy fractional goal programming problem by the proposed method 1, varying the aspiration levels, tolerances, and weights, we get the best solution as $x_1 = 2.2942$, $x_{10} = .9038$, and $x_{12} = 2.8019$, where aspiration level (g) = 15, tolerance (p) = 5, and weight (w) = .7.

Table 5: Comparison.

	Proposed method 1	Proposed method 2
Model class	NLP	NLP
ATUS	99%	Nil
Weight (w)	$w = .7$	$w \leq 1$ ($w = 1/p$)
Aspiration level	15	≥10
Tolerance	5	5

The fractional membership function corresponding to the objective function becomes

$$\begin{aligned}&\mu(Z(x)) = \frac{((2.16x_1 + 1.095x_2 + \cdots + 2.04x_{12})/(.1x_1 + .2x_2 + \cdots + .2x_3)) - 10}{5},\\ &\text{At the points } (x_1 = 2.2942, x_{10} = .9038, x_{12} = 2.8019),\\ &\mu(Z(2.2942, 0, 0, 0, 0, 0, 0, 0, 0, .9038, 0, 2.8019)) = .7,\end{aligned} \tag{7.8}$$

Then the fractional membership function is transformed into linear membership function at the best solution points ($x_1 = 2.2942$, $x_{10} = .9038$, $x_{12} = 2.8019$) by first-order Taylor series as follows:

$$\begin{aligned}\overline{\mu}(Z(x)) &= \mu(Z(2.2942, 0, 0, 0, 0, 0, 0, 0, 0, .9038, 0, 2.8019))\\ &\quad + (x_1 - 2.2942)\frac{\delta}{\delta x_1}(Z(2.2942, 0, 0, 0, 0, 0, 0, 0, 0, .9038, 0, 2.8019))\\ &\quad + (x_2 - 0)\frac{\delta}{\delta x_2}(Z(2.2942, 0, 0, 0, 0, 0, 0, 0, 0, .9038, 0, 2.8019)) + \cdots\\ &\quad + (x_{10} - .9038)\frac{\delta}{\delta x_{10}}(Z(2.2942, 0, 0, 0, 0, 0, 0, 0, 0, .9038, 0, 2.8019)) + \cdots\\ &\quad + (x_{12} - 2.8019)\frac{\delta}{\delta x_{10}}(Z(2.2942, 0, 0, 0, 0, 0, 0, 0, 0, .9038, 0, 2.8019)), \overline{\mu}(Z(x))\\ &= .7 + (x_1 - 2.2942).1841 - (x_2 - 0).3647 - (x_3 - 0).2954\\ &\quad - (x_4 - 0).2272 - (x_5 - 0).4567 - (x_6 - 0).7967 - (x_7 - 0).3180 - (x_8 - 0).1613\\ &\quad - (x_9 - 0).2272 - (x_{10} - .9038).0023 - (x_{11} - 0).4681 - (x_{12} - 2.8019).1500.\end{aligned} \tag{7.9}$$

Therefore, the proposed weighted FGP model of the fuzzy linear goal programming problem by proposed methods 1 and 2 can be written as

$$\begin{aligned}&\text{Max } \lambda && \text{Min } (1 - \lambda),\\ &\text{Subject to } w\lambda \leq \overline{\mu}(Z(x)) && \text{Subject to } w(1 - \lambda) \geq 1 - \overline{\mu}(Z(x)),\\ &\lambda + d^- = 1 && 1 - \lambda + d^- = 1,\\ &\lambda \geq 0 && 1 - \lambda \geq 0,\end{aligned} \tag{7.10}$$

where d^- represents the underdeviation variable, $d^- \geq 0$, $\lambda \geq 0$, the weights $w \geq 0$, $w < 1$, $w = 1/p$.

Table 6: Solution of the fuzzy linear goal programming problem by applying different methods.

Weight (w)	Proposed method 1	Proposed method 2	Zimmermann's method	Tiwari's method
.2	ATUS = 75%,	ATUS = 106%,		ATUS = 95%
.5	ATUS = 77%,	ATUS = 95%,		ATUS = 95%
.7	ATUS = 84%,	ATUS = 95%,		ATUS = 95%
.8	ATUS = 88%,	ATUS = 95%,		ATUS = 95%
.9	ATUS = 91%,	ATUS = 95%,		ATUS = 95%
1	ATUS = 95%,	ATUS = 95%,	$\lambda > 1$	ATUS = 95%
Model class	LP ($\lambda = 1$)	LP ($\lambda = 1$)	LP	LP ($\lambda = 1$)

The solutions are given in Table 6.

From Table 6, it has been shown that to avoid the drawbacks of the fuzzy linear fractional goal programming problem (FLGPP) by Zimmermann's method when the weights $w \leq 1$, the problem has been solved by Tiwari's weighted additive model, proposed linearized methods 1 and 2. Here, $\lambda = 1$.

If the weights are varied then same ATUS solution is obtained for the fuzzy linear goal programming problem when solved by Tiwari's weighted additive model, proposed linearized method 2. So the attachment of weights in these FGP methods is unnecessary.

Now, the comparison between the solutions of fuzzy fractional programming problem obtained from the proposed method 1, using linearization procedure and Chang's binary FGP method [29], Pal et al. Method, and using linearization procedure [24], has been made in the Table 7.

Based on the ATUS solution, it is clear from Table 7 that the proposed method 1, using linearization procedure, gives better and more realistic solution of the fuzzy fractional goal programming problem when $w = 1$.

Note 1. If we solve the fuzzy fractional programming problem using conventional linearization procedure [25] by determining the individual best solutions of the fractional objective function $Z(x)$ based on (7.1) by maximizing $Z(x)$ and correspondingly worst solutions by minimizing $Z(x)$ subject to the system constraints, then the individual best solutions are $Z^B(x_1,0,0,0,0,0,0,0,0,0,0,x_{12}) = 17$ at $(x_1 = 4.5, x_{12} = 1.5)$, and the individual worst solutions are $Z^W(0,0,0,0,0,x_6,0,0,0,0,0,0) = 1.81$ at $(x_6 = 15)$. Therefore, the fuzzy goal is $Z(x) \gtrsim 17$.

The fractional membership function corresponding to the objective function becomes

$$\mu(Z(x)) = \frac{Z(x) - 1.81}{17 - 1.81}. \tag{7.11}$$

At the pt $(x_1 = 4.5,\ x_{12} = 1.5)$, $\mu(Z(4.5,0,0,0,0,0,0,0,0,0,0,1.5)) = (((9.72+3.06)/(.45+.3)) - 1.81)/(17 - 1.81) = 1$.

Then, the fractional membership function is transformed into linear membership function at the individual best solution points by first-order Taylor series as follows:

$$\begin{aligned}\overline{\mu}(Z(x)) &= \mu(Z(4.5,0,0,0,0,0,0,0,0,0,0,1.5)) \\ &\quad + (x_1 - 4.5)\left(\frac{\delta}{\delta x_1}\right)(Z(4.5,0,0,0,0,0,0,0,0,0,0,1.5))\end{aligned}$$

Table 7: Comparison.

	Proposed method 1	Chang's binary method	Pal et al. method
Weight (w)	$w = 1$	$w = 1$	$w = 1$, $w = 1/p$
ATUS	95%	68.77%	93.74%
Model class	LP	LP	LP
Aspiration level	15	10	15
Tolerance	5	9	5
	$\lambda = 1$	$\mu = .9925$	$\mu \in [0, 1]$

$$
\begin{aligned}
&+ (x_2 - 0)\left(\frac{\delta}{\delta x_2}\right)(Z(4.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.5)) + \cdots \\
&+ (x_{12} - 1.5)\left(\frac{\delta}{\delta x_{12}}\right)(Z(4.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.5)). \\
\therefore \overline{\mu}(Z(x)) &= 1 + (x_1 - 4.5).0049 - (x_2 - 0).0977 \\
&- (x_3 - 0).070 - (x_4 - 0).0446 - (x_5 - 0).1332 \\
&- (x_6 - 0).2429 - (x_7 - 0).0797 - (x_8 - 0).0407 - (x_9 - 0).0446 \\
&- (x_{10} - 0).0207 - (x_{11} - 0).1376 - (x_{12} - 1.5).0147.
\end{aligned}
\tag{7.12}
$$

Now, solving the fuzzy linear goal programming problem by the proposed method 1, we get the solution as $x_1 = 4.5$, $x_{12} = 1.5$. Where the weights are $w \geq 0$, $w \leq 1$, $w = 1/p$, $\lambda = 1$.

But this solution is not acceptable because the average total user satisfaction (ATUS) is 102%. So this procedure is not applicable to convert the fuzzy fractional programming problem based on (7.1) into fuzzy linear programming problem.

Further, to illustrate the usefulness of the proposed methods 1 and 2, another example of fuzzy linear goal programming problem has been considered.

7.2. Example 2

This example considered by Chang [26] is used to clarify the effectiveness of the weights in the fuzzy linear goal programming problem.

The fuzzy linear goal programming problem is represented as

$$
\begin{aligned}
&3x_1 + 1.5x_2 + 2x_3 + 2.5x_4 + x_5 + .5x_6 \gtrsim 9, \\
&x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \gtrsim 4, \\
&\text{Subject to } 3x_1 + 2x_2 + 3x_3 + 3x_4 + 2x_5 + x_6 \leq 10 \ (\text{Manpower constraint}), \\
&.1x_1 + .2x_2 + .2x_3 + .2x_4 + .2x_5 + .3x_6 \leq 1 \ (\text{Capital constraint}), \\
&x_1 + x_3 + x_4 = 3 \ (\text{Basic ring trunking network constraint}), \\
&\text{where } x_i \geq 0, \quad i = 1, 2, \ldots 6.
\end{aligned}
\tag{7.13}
$$

Assuming that the tolerance limits of the above two fuzzy objective goals are (1,1), respectively.

Table 8: Solution and comparison.

Weight (w)	Proposed FGP 1 with d_k^-	Proposed FGP 2 with d_k^-	Zimmermann's method
1/8, 1/3	$\lambda = 1, Z_1(x) = 9, Z_2(x) = 3$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$	$\lambda > 1$
1, 1	$\lambda = 1, Z_1(x) = 9, Z_2(x) = 4$	$\lambda = 1, Z_1(x) = 9, Z_2(x) = 4$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$
<1	$\lambda = 1, Z_1(x) = 9, Z_2(x) = 3$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$	$\lambda > 1$
.9, .1	$\lambda = 1, Z_1(x) = 8, Z_2(x) = 3$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$	$\lambda > 1$
.8, .2	$\lambda = 1, Z_1(x) = 7, Z_2(x) = 3$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$	$\lambda > 1$
.7, .3	$\lambda = 1, Z_1(x) = 6, Z_2(x) = 3$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$	$\lambda > 1$
.6, .4	$\lambda = 1, Z_1(x) = 9, Z_2(x) = 3$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$	$\lambda > 1$
.5, .5	$\lambda = 1, Z_1(x) = 9, Z_2(x) = 3$	$\lambda = 1, Z_1(x) = 9.5, Z_2(x) = 4$	$\lambda > 1$
Model class	LP	LP	LP

Table 9: Solution of Tiwari's weighted additive FGP method.

Weight (w)	Model class	Tiwari's weighted additive FGP method
$w_k \le 1, 1/p_k, \sum w_k = 1; k = 1, 2$	LP	$Z_1(x) = 9, Z_2(x) = 4, \lambda_1 = 1, \lambda_2 = 1$

Now, we solve the above fuzzy linear goal programming problem by proposed methods 1, 2 and also by comparison with the solution obtained by the Zimmerman's method based on (6.2). Table 8 summarises the results.

From Table 8, it is seen that the fuzzy linear goal programming problem (FLGPP) based on (7.13) when solved by Zimmermann's method and the proposed methods 2 yield same results, namely, $Z_1(x) = 9.5$, $Z_2(x) = 4$ where weights are less than one that is, $w_k < 1$, $k = 1, 2$. Thus, the assertion that every fuzzy linear program has an equivalent weighted linear goal program where the weights are restricted as the reciprocals of the admissible violation constants is not always true. On the other hand, both the proposed methods yield the same solutions, namely, $Z_1(x) = 9$, $Z_2(x) = 4$ where weights are equal to unity. So both the proposed methods are equivalent FGP when weights are equal to unity. Here, both goals are completely achieved.

Again, the fuzzy linear goal programming problem based on (7.13) has been solved by Tiwari's weighted additive FGP method and the solutions are summarised in the Table 9.

From Table 9, it has been seen that the fuzzy linear goal programming problem (FLGPP) based on (7.13) when solved by Tiwari's method yields same results, namely, $Z_1(x) = 9$, $Z_2(x) = 4$ when weights are varied that is, $w_k \le 1$, $1/p_k$, $\sum w_k = 1$; $k = 1, 2$. Here, both goals are fully achieved.

Comparing the solutions for the fuzzy linear goal programming problem by the proposed methods 1 and 2, and Tiwari's method, it has been seen that Tiwari's method and proposed method 2 produce same solutions whereas the proposed method 1 produces different solutions when the weights are varied which represents the relative importance of the objective functions.

Further, the above fuzzy linear goal programming problem has been solved by the well-known existing method based on (6.1) and proposed methods, where both the under- and overdeviation variables are introduced in the goal constraint. The results are shown in Table 10.

Table 10 clearly shows that the fuzzy linear goal programming problem (FLGPP) [26] gives infeasible solution, when solved by the proposed methods 1 and 2, where the goal constraints are constructed by using both the under and overdeviation variables. Also

Table 10: Comparison.

Weight (w)	Proposed method 1 with $\lambda + d_k^- - d_k^+ = 1$	Proposed method 2 with $\lambda + d_k^- - d_k^+ = 1$	Hannan's method
$1/8, 1/3$	$\lambda > 1$	Infeasibility present	
$w_k \le 1, k = 1,2$	Infeasibility present	Infeasibility present	
$\sum w_k = 1, k = 1,2$	Infeasibility present	Infeasibility present	Infeasibility present
Model class	NLP	NLP	NLP

Table 11: Comparison.

Weight (w)	Chang [26]	Proposed method 1	Proposed method 2
$1/8, 1/3$		$b_1 = 10, Z_1 = 9, Z_2 = 3$	$b_1 = 10, Z_1 = 9.5, Z_2 = 4$
		$b_1 = 11, Z_1 = 9, Z_2 = 3$	$b_1 = 11, Z_1 = 10.5, Z_2 = 4$
$1, 1$	$b_1 = 10, Z_1 = 8, Z_2 = 0$	$b_1 = 10, Z_1 = 9, Z_2 = 4$	$b_1 = 10, Z_1 = 9, Z_2 = 4$
	$b_1 = 11, Z_1 = 9, Z_2 = 4$	$b_1 = 11, Z_1 = 9, Z_2 = 4$	$b_1 = 11, Z_1 = 9, Z_2 = 4$
w < 1		$b_1 = 10, Z_1 = 9, Z_2 = 3$	$b_1 = 10, Z_1 = 9.5, Z_2 = 4$
		$b_1 = 11, Z_1 = 9, Z_2 = 3$	$b_1 = 11, Z_1 = 10.5, Z_2 = 4$
.6, .4		$b_1 = 10, Z_1 = 9, Z_2 = 3$	$b_1 = 10, Z_1 = 9.5, Z_2 = 4$
		$b_1 = 11, Z_1 = 9, Z_2 = 3$	$b_1 = 11, Z_1 = 10.5, Z_2 = 4$
Model class	LP	LP	LP
	$\mu = 1$	$\lambda = 1$	$\lambda = 1$

if Hannan's method is applied for the solution of the same (FLGP) problem, infeasibility occurs. The conclusion is that in the proposed methods 1 and 2, the goal constraint cannot be constructed by introducing both the under and overdeviation variables d_k^-, d_k^+. So only underdeviation variables d_k^- $(k = 1, 2, \ldots, K)$ are necessary to attach in the goal constraint of the proposed methods 1 and 2.

Again, based on this example, Table 11 shows the comparison between the solutions of the FLGP problem obtained by Chang [26] and also by proposed methods 1, 2.

Here, b_1 represents the resource of the first constraint of the considered fuzzy linear goal programming problem. In the method introduced by Chang, the goals are not completely achieved for $b_1 = 10$ but achieved fully when $b_1 = 11$ with weights $w_k = 1$, $k = 1, 2$. Table 11 shows that the solutions obtained by the proposed method 1 and 2, for $b_1 = 10$, (resp., for $b_1 = 11$), achieve the targets of the fuzzy goals completely only when the weights $w_k = 1$, $k = 1, 2$.

Now, we solve the considered fuzzy linear goal programming (FLGP) problem by two proposed methods, varying the resource of the first constraint. The results are given in Table 12.

From Table 12, it has been shown that feasible solutions are obtained when $b_1 \ge 9$ but both goals are completely achieved when the weights attached to the fuzzy operator in the goal constraint of the proposed methods 1 and 2 are unity and $b_1 \ge 10$. As the first constraint, that is, manpower constraint, is strictly less than, equal to 10, or then the resource b_1 must be 10.

Table 12: Solution of the FLGP problem by varying the resource of the first constraint.

b_1	Weight (w)	Proposed method 1	Proposed method 2
$b_1 = 8$	1, 1	No feasible solution	No feasible solution
	$w < 1$	No feasible solution	No feasible solution
$b_1 = 9$	1, 1	$Z_1 = 6, Z_2 = 3, \lambda = .66$	$Z_1 = 6, Z_2 = 3, \lambda = .66$
	$w < 1$	$Z_1 = 9, Z_2 = 3, \lambda = 1$	$Z_1 = 9, Z_2 = 3, \lambda = .33$
$b_1 = 10$	1, 1	$Z_1 = 9, Z_2 = 4, \lambda = 1$	$Z_1 = 9, Z_2 = 4, \lambda = 1$
	$w < 1$	$Z_1 = 9, Z_2 = 3, \lambda = 1$	$Z_1 = 9.5, Z_2 = 4, \lambda = 1$
$b_1 = 11$	1, 1	$Z_1 = 9, Z_2 = 4, \lambda = 1$	$Z_1 = 9, Z_2 = 4, \lambda = 1$
	$w < 1$	$Z_1 = 9, Z_2 = 3, \lambda = 1$	$Z_1 = 10.5, Z_2 = 4, \lambda = 1$

8. Advantages of the Proposed Methods over the Existing Methods

In this section, it is shown that by using the proposed methods the shortcomings, described in Section 4, are removed and also it is better to use the proposed methods for solving the FGP problems, occurring in real-life situations as compared to the existing methods.

(i) The advantage of the proposed methods to solve the fuzzy goal programming problems is that the condition $\lambda \in [0,1]$ is always satisfied when $w_k < 1$, $\sum w_k = 1$, $k = 1, 2, \ldots, K$, whereas the existing methods based on (6.1) and (6.2) failed to produce such results.

(ii) The advantage of the proposed methods over the existing method [23] is that there is no restriction on the weights attached to the fuzzy operator in the constraints. The assertion that every fuzzy linear program has an equivalent weighted linear goal program where the weights are restricted as the reciprocals of the admissible violation constants is not always true.

(iii) Instead of the Tiwari's weighted additive model and Mohamed's min-sum FGP method, the proposed methods allow the decision maker to determine the relative weights of the goals of the FGP problems according to the consideration of different types of weights, as the relative weights represent the relative importance of the fuzzy goals. Also, it is very easy to apply the proposed methods as compared to the existing methods for solving the FGP problems, occurring in real-life situations and the obtained result satisfies the fuzzy goals at best in the sense that the solutions are very close to the aspiration level.

(iv) It can be easily realized that the membership goals in (3.4), (3.5) and also in (5.1), (5.2) are inherently nonlinear in nature when the objectives are fractional and this may create computational difficulties in the solution process of existing methods. To avoid such problems, the conventional linearization procedure [24, 25] was preferred. The advantage of the proposed method 1 is that the solution of any fuzzy fractional goal programming problems (FFGPP) could be found efficiently without any computational difficulties. However, if the linearization procedure [25] is applied to covert the FFGPP to FLGPP, then varying the weights attached to the fuzzy operator in the goal constraint, the proposed method 1 gives better solution for the FLGPP in the sense that the solutions are more realistic and close to the aspiration level.

(v) The proposed method 1 can ensure that the more important goals can have higher achievement degrees even though a decision maker may change the weights.

(vi) Also the numbers of constraints, variables, and correspondingly the time required in the solution process of the problems by proposed methods are less than those in other methods.

9. Conclusions

In the decision-making problem, there may be situations where a decision maker has to content with a solution of the FGP problem where some of the fuzzy goals are achieved and some are not because these fuzzy goals are subject to the function of environment/resource constraints. Since the relative weights represent the relative importance of the objective functions, then the proposed max-min FGP method 1 is very effective and more realistic than the proposed min max FGP method 2 at finding the optimal solution or near optimal solution of the fuzzy goal programming problems and helps to achieve the goals completely. Further it is to be noted that there are some fuzzy linear programs in real-world decision-making environment which have an equivalent weighted fuzzy linear goal program where the weights are not restricted. Again, it has been shown that the proposed max-min FGP method 1 gives feasible solution for both fuzzy fractional and linear goal programming problems, whereas the proposed min max FGP method 2 gives feasible solution for only fuzzy linear goal programming problems.

Since different weights lead to different efficient points, which can be obtained by using an interaction with decision making, there left bright prospect for future research work on the proposed weighted fuzzy goal programming methods.

In this paper, the software LINGO (version 11) has been used to solve the problems.

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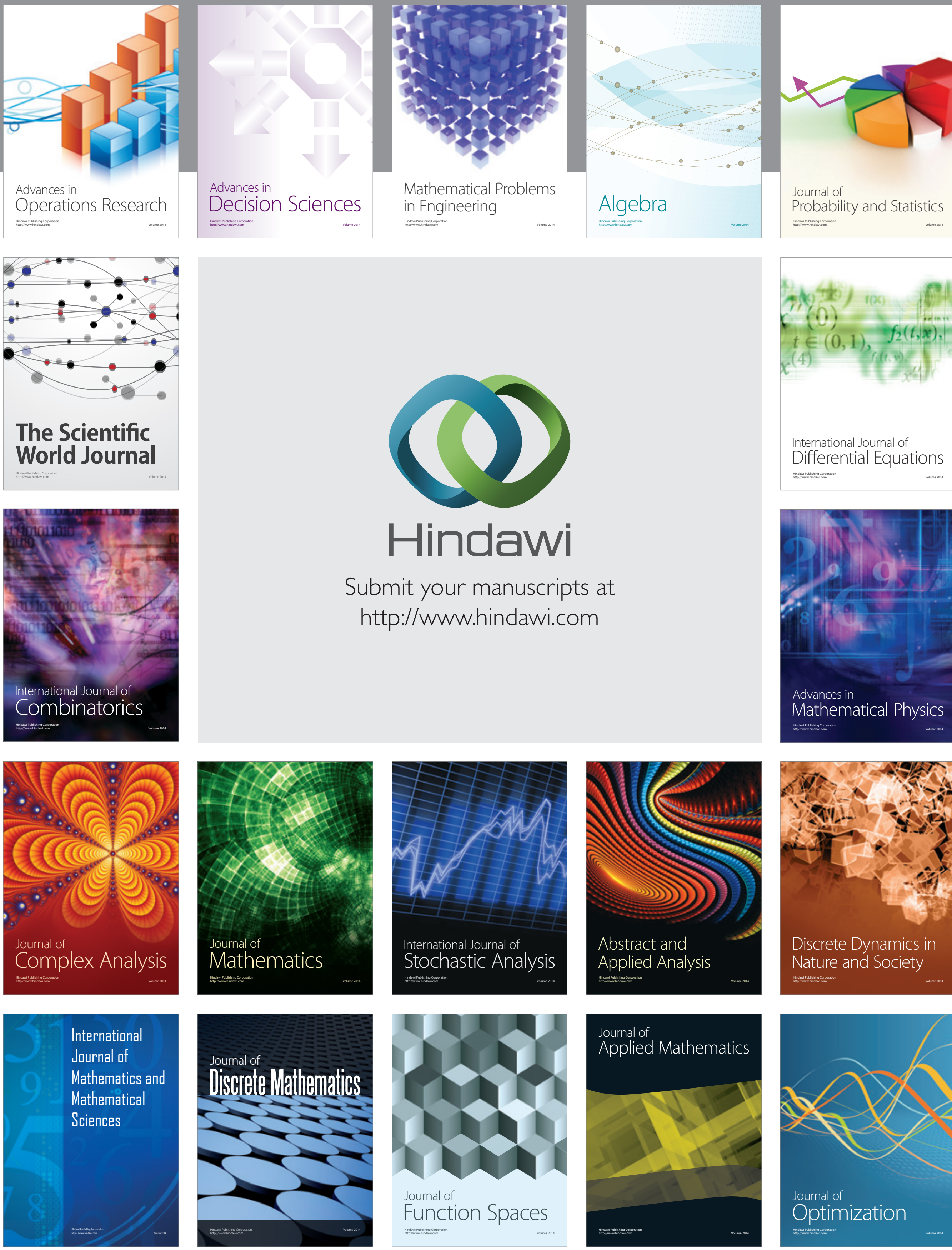
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