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Research Article

Operations on Soft Sets Revisited

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The concept of soft sets introduced by Molodtsov is a general mathematical tool for dealing with uncertainty. Just as the conventional set-theoretic operations of intersection, union, complement, and difference, some corresponding operations on soft sets have been proposed. Unfortunately, such operations cannot keep all classical set-theoretic laws true for soft sets. In this paper, we redefine the intersection, complement, and difference of soft sets and investigate the algebraic properties of these operations along with a known union operation. We find that the new operation system on soft sets inherits all basic properties of operations on classical sets, which justifies our definitions.

1. Introduction

As a necessary supplement to some existing mathematical tools for handling uncertainty, Molodtsov [1] initiated the concept of soft sets via a set-valued mapping. A distinguishing feature of soft sets which is different from probability theory, fuzzy sets, and interval mathematics is that precise quantity such as probability and membership grade is not essential. This feature facilitates some applications because in most realistic settings the underlying probabilities and membership grades are not known with sufficient precision to justify the use of numerical valuations. Since its introduction, the concept of soft sets has gained considerable attention (see, e.g., [2–31]), including some successful applications in information processing [32–36], decision [37–41], demand analysis [42], clustering [43], and forecasting [44].

In [22], Maji et al. made a theoretical study of the soft set theory in more detail. Especially, they introduced the concepts of subset, intersection, union, and complement of soft sets and discussed their properties. These operations make it possible to construct new soft sets from given soft sets. Unfortunately, several basic properties presented in [22] are not true in general; these have been pointed out and improved by Yang [45], Ali et al. [46], and Sezgin and Atagun [47]. In particular, to keep some classical set-theoretic laws true for soft sets, Ali et al. defined some restricted operations on soft sets such as the restricted intersection, the restricted union, and the restricted difference and improved the notion of complement of a soft set. Based upon these newly defined operations, they proved that certain De Morgan's laws hold for soft sets. It is worth noting that the concept of complement [22, 46] (it should be stressed that there are two types of complements defined in [46]: one is defined with the NOT set of parameters and the other is defined without the NOT set of parameters), which is fundamental to De Morgan's laws is based on the so-called NOT set of a parameter set. It means that the logic conjunction NOT is a prerequisite for defining the complement of a soft set; this is considerably beyond the definition of soft sets. Moreover, the union of a soft set, and its complement is not exactly the whole universal soft set in general, which is considered less desirable.

The purpose of this paper is to develop the theory of soft sets by introducing new operations on soft sets that inherit all basic properties of classical set operations. To this end, we redefine the intersection, complement, and difference of soft sets and then examine the algebraic properties of these operations along with a known union operation. It turns out that all basic properties of operations on classical sets, including identity laws, domination laws, idempotent laws, commutative laws, associative laws, distributive laws, and De Morgan's laws, hold for soft sets with respect to the newly defined operations.

The remainder of this paper is structured as follows. In Section 2, we briefly recall the notion of soft sets. Section 3 is devoted to the definitions of new operations on soft sets. An example is also provided to illustrate the newly defined operations in this section. We address the basic properties of the operations on soft sets in Section 4 and conclude the paper in Section 5.

2. Soft Sets

For subsequent need, let us review the notion of soft sets. For a detailed introduction to the soft set theory, the reader may refer to [1, 22].

We begin with some notations. For classical set theory, the symbols $\emptyset$, A^c, $A \cup B$, $A \cap B$, $A \setminus B$ denote, respectively, the empty set, the complement of A with respect to some universal set U, the union of sets A and B, the intersection of sets A and B, and the difference of sets A and B whose elements belong to A but not to B. In what follows, we write $\mathscr{P}(U)$ for the power set of a universal set U and denote $\mathscr{P}(U) \setminus \{\emptyset\}$ by $\mathscr{P}^*(U)$.

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U. Usually, parameters are attributes, characteristics, or properties of objects in U. We now recall the notion of soft sets due to Molodtsov [1].

Definition 1 (see Molodtsov [1]). Let U be a universe and E the set of parameters. A *soft set* over U is a pair (F, A) consisting of a subset A of E and a mapping $F : A \to \mathscr{P}^*(U)$.

Note that the above definition is slightly different from the original one in [1] where F has $\mathscr{P}(U)$ as its codomain. In other words, we remove the parameters having the empty set as images under F. It seems rational since this means that if there exists a parameter $e \in A$ which is not the attribute, characteristic, or property of any object in U, then this parameter has no interest with respect to the knowledge stored in the soft set. As a result, a soft set of U in the sense of Definition 1 is a parameterized family of nonempty subsets of U.

Clearly, any soft set (F, A) over U gives a partial function $F' : E \to \mathscr{P}^*(U)$ defined by

$$F'(e) = \begin{cases} F(e), & \text{if } e \in A, \\ \text{undefined}, & \text{otherwise}, \end{cases} \tag{1}$$

for all $e \in E$. Conversely, any partial function f from E to $\mathscr{P}^*(U)$ gives rise to a soft set (F_f, A_f), where $A_f = \{e \in E \mid f(e) \text{ is defined}\}$ and F_f is the restriction of f on A_f.

To illustrate the above definition, Molodtsov considered several examples in [1], one of which we present blow.

Example 2. Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2, \ldots, h_5\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, \ldots, e_8\}$, where $e_1, e_2, \ldots, e_8$ stand for the attributes "expensive," "beautiful," "wooden," "cheap," "in the green surroundings," "modern," "in good repair," and "in bad repair," respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (F, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Alice, may be defined as:

$$\begin{gathered} A = \{e_2, e_3, e_4, e_5, e_7\}; \\ F(e_2) = \{h_2, h_3, h_5\}, \qquad F(e_3) = \{h_2, h_4\}, \\ F(e_4) = \{h_1\}, \qquad F(e_5) = U, \qquad F(e_7) = \{h_3, h_5\}. \end{gathered} \tag{2}$$

3. Operations on Soft Sets

In this section, we generalize the basic operations on classical sets to soft sets. The examination of more properties of these operations is deferred to the next section.

Let us start with the notions of empty and universal soft sets. Recall that in [22] a soft set (F, A) is called a null soft set if $F(e) = \emptyset$ for all $e \in A$. Because $\emptyset$ does not belong to the codomain of F in our framework, we redefine the concept of empty soft set as follows.

Definition 3. A soft set (F, A) over U is said to be *empty* whenever $A = \emptyset$. Symbolically, we write $(\emptyset, \emptyset)$ for the empty soft set over U.

Definition 4. A soft set (F, A) over U is called a *universal soft set* if $A = E$ and $F(e) = U$ for all $e \in A$. Symbolically, we write (U, E) for the universal soft set over U.

Let us now define the subsets of a soft set.

Definition 5. Let (F, A) and (G, B) be two soft sets over U. We say that (F, A) is a *subset* of (G, B), denoted $(F, A) \subseteq (G, B)$ if either $(F, A) = (\emptyset, \emptyset)$ or $A \subseteq B$ and $F(e) \subseteq G(e)$ for every $e \in A$. Two soft sets (F, A) and (G, B) are said to be *equal*, denoted $(F, A) = (G, B)$, if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

By definition, two soft sets (F, A) and (G, B) are equal if and only if $A = B$ and $F(e) = G(e)$ for all $e \in A$. In [22], a similar notion, called soft subset, was defined by requiring that $A \subseteq B$ and $F(e) = G(e)$ for every $e \in A$. By Definition 5, the empty soft set $(\emptyset, \emptyset)$ is a subset of any soft set. It also follows from Definition 5 that any soft set is a subset of the universal soft set (U, E). Formally, we have the following proposition.

Proposition 6. *For any soft set (F, A) over U,*

$$(\emptyset, \emptyset) \subseteq (F, A) \subseteq (U, E). \tag{3}$$

We are now in the position to introduce some operations on soft sets.

Definition 7. Let (F, A) and (G, B) be two soft sets over U. The *intersection* of (F, A) and (G, B), denoted by $(F, A) \cap (G, B)$, is defined as $(F \cap G, C)$, where

$$\begin{gathered} C = \{e \in A \cap B \mid F(e) \cap G(e) \neq \emptyset\}, \quad \forall e \in C, \\ (F \cap G)(e) = F(e) \cap G(e). \end{gathered} \tag{4}$$

In particular, if $A \cap B = \emptyset$ or $F(e) \cap G(e) = \emptyset$ for every $e \in A \cap B$, then we see that $(F, A) \cap (G, B) = (\emptyset, \emptyset)$.

The following definition of union of soft sets is the same as in [22].

Definition 8 (see [22], Definition 2.11). Let (F, A) and (G, B) be two soft sets over U. The *union* of (F, A) and (G, B), denoted by $(F, A) \cup (G, B)$, is defined as $(F \cup G, C)$, where

$$C = A \cup B, \quad \forall e \in C,$$
$$(F \cup G)(e) = \begin{cases} F(e), & \text{if } e \in A \setminus B \\ G(e), & \text{if } e \in B \setminus A \\ F(e) \cup G(e), & \text{otherwise.} \end{cases} \tag{5}$$

We now define the notion of complement in soft set theory. It is worth noting that this is rather different from those in the existing literature [22, 46], where the complement is usually based on the so-called NOT set of a parameter set and the union of a soft set, and its complement is not exactly the whole universal soft set in general.

Definition 9. Let (F, A) be a soft set over U. The *complement* of (F, A) with respect to the universal soft set (U, E), denoted by $(F, A)^c$, is defined as (F^c, C), where

$$\begin{aligned} C &= E \setminus \{e \in A \mid F(e) = U\} \\ &= \{e \in A \mid F(e) = U\}^c, \quad \forall e \in C, \\ F^c(e) &= \begin{cases} U \setminus F(e), & \text{if } e \in A \\ U, & \text{otherwise.} \end{cases} \end{aligned} \tag{6}$$

In certain settings, the difference of two soft sets (F, A) and (G, B) is useful.

Definition 10. Let (F, A) and (G, B) be two soft sets over U. The *difference* of (F, A) and (G, B), denoted by $(F, A) \setminus (G, B)$, is defined as $(F \setminus G, C)$, where

$$C = A \setminus \{e \in A \cap B \mid F(e) \subseteq G(e)\}, \quad \forall e \in C,$$
$$(F \setminus G)(e) = \begin{cases} F(e) \setminus G(e), & \text{if } e \in A \cap B \\ F(e), & \text{otherwise.} \end{cases} \tag{7}$$

By Definitions 9 and 10, we find that $(F, A)^c = (U, E) \setminus (F, A)$ holds for any soft set (F, A). That is, the complement of (F, A) with respect to the universal soft set (U, E) is exactly the difference of (U, E) and (F, A). In light of this, $(F, A) \setminus (G, B)$ is also called the relative complement of (G, B) in (F, A), while $(F, A)^c$ is also called the absolute complement of (F, A).

Let us illustrate the previous operations on soft sets by a simple example.

Example 11. Let us revisit Example 2. Recall that the soft set (F, A) describing the "attractiveness of the houses" in Alice's opinion was defined by

$$\begin{aligned} A &= \{e_2, e_3, e_4, e_5, e_7\}; \\ F(e_2) &= \{h_2, h_3, h_5\}, \quad F(e_3) = \{h_2, h_4\}, \\ F(e_4) &= \{h_1\}, \quad F(e_5) = U, \quad F(e_7) = \{h_3, h_5\}. \end{aligned} \tag{8}$$

In addition, we assume that the "attractiveness of the houses" in the opinion of another buyer, say Bob, is described by the soft set (G, B), where

$$\begin{aligned} B &= \{e_1, e_2, \ldots, e_7\}; \\ G(e_1) &= \{h_3, h_5\}, \quad G(e_2) = \{h_4\}, \\ G(e_3) &= \{h_2, h_4\}, \quad G(e_4) = \{h_1\}, \\ G(e_5) &= \{h_2, h_3, h_4, h_5\}, \quad G(e_6) = G(e_7) = \{h_3\}. \end{aligned} \tag{9}$$

Then by a direct computation, we can readily obtain $(F, A) \cap (G, B)$, $(F, A) \cup (G, B)$, $(F, A)^c$, and $(F, A) \setminus (G, B)$ as follows.

(i) $(F, A) \cap (G, B) = (F \cap G, \{e_3, e_4, e_5, e_7\})$, where $(F \cap G)(e_3) = \{h_2, h_4\}$, $(F \cap G)(e_4) = \{h_1\}$, $(F \cap G)(e_5) = \{h_2, h_3, h_4, h_5\}$, and $(F \cap G)(e_7) = \{h_3\}$. This means that both Alice and Bob think that h_2 and h_4 are wooden, h_1 is cheap, h_2, h_3, h_4, h_5 are in the green surroundings, and h_3 is in the good repair.

(ii) $(F, A) \cup (G, B) = (F \cup G, \{e_1, e_2, \ldots, e_7\})$, where $(F \cup G)(e_1) = \{h_3, h_5\}$, $(F \cup G)(e_2) = \{h_2, h_3, h_4, h_5\}$, $(F \cup G)(e_3) = \{h_2, h_4\}$, $(F \cup G)(e_4) = \{h_1\}$, $(F \cup G)(e_5) = U$, $(F \cup G)(e_6) = \{h_3\}$, and $(F \cup G)(e_7) = \{h_3, h_5\}$. This means that either Alice or Bob thinks that h_3 is expensive, either Alice or Bob thinks that h_5 is expensive, either Alice or Bob thinks that h_2 is beautiful, either Alice or Bob thinks that h_3 is beautiful, and so on.

(iii) $(F, A)^c = (F^c, \{e_1, e_2, e_3, e_4, e_6, e_7, e_8\})$, where $F^c(e_1) = U$, $F^c(e_2) = \{h_1, h_4\}$, $F^c(e_3) = \{h_1, h_3, h_5\}$, $F^c(e_4) = \{h_2, h_3, h_4, h_5\}$, $F^c(e_6) = U$, $F^c(e_7) = \{h_1, h_2, h_4\}$, and $F^c(e_8) = U$. This means that Alice thinks that none of these houses is expensive, neither h_1 nor h_4 is beautiful, h_1, h_3, h_5 are not wooden, and so on.

(iv) $(F, A) \setminus (G, B) = (F \setminus G, \{e_2, e_5, e_7\})$, where $(F \setminus G)(e_2) = \{h_2, h_3, h_5\}$, $(F \setminus G)(e_5) = \{h_1\}$, and $(F \setminus G)(e_7) = \{h_5\}$. This means that Alice thinks of h_2, h_3, and h_5 as beautiful, but Bob does not think that these are beautiful, and so on.

4. Algebraic Properties of Soft Set Operations

This section is devoted to some algebraic properties of soft set operations defined in the last section.

Let us begin with some properties involving intersections and unions. The first four laws are obvious. We omit their proofs here since the proofs follow directly from the definitions of intersection and union of soft sets.

Proposition 12 (Identity laws). *For any soft set* (F, A) *over* U*, we have that*

(1) $(F, A) \cap (U, E) = (F, A)$,

(2) $(F, A) \cup (\emptyset, \emptyset) = (F, A)$.

Proposition 13 (Domination laws). *For any soft set* (F, A) *over* U*, we have that*

(1) $(F, A) \cap (\emptyset, \emptyset) = (\emptyset, \emptyset)$,

(2) $(F, A) \cup (U, E) = (U, E)$.

Proposition 14 (Idempotent laws). *For any soft set* (F, A) *over* U*, we have that*

(1) $(F, A) \cap (F, A) = (F, A)$,

(2) $(F, A) \cup (F, A) = (F, A)$.

Proposition 15 (Commutative laws). *For any soft sets* (F, A) *and* (G, B) *over* U*, we have that*

(1) $(F, A) \cap (G, B) = (G, B) \cap (F, A)$,

(2) $(F, A) \cup (G, B) = (G, B) \cup (F, A)$.

Now we turn our attention to the associative laws.

Proposition 16 (Associative laws). *For any soft sets* (F, A), (G, B), *and* (H, C) *over* U*, we have that*

(1) $((F, A) \cap (G, B)) \cap (H, C) = (F, A) \cap ((G, B) \cap (H, C))$,

(2) $((F, A) \cup (G, B)) \cup (H, C) = (F, A) \cup ((G, B) \cup (H, C))$.

Proof. We only prove the first assertion since the second one is the same as Proposition 2.5(i) in [22]. For simplicity, we write (L, A'), (R, B'), and $(F \cap G, A_1)$ for $((F, A) \cap (G, B)) \cap (H, C)$, $(F, A) \cap ((G, B) \cap (H, C))$, and $(F, A) \cap (G, B)$, respectively. We thus get by definition that

$$\begin{aligned}
A' &= \{e \in A_1 \cap C \mid (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&= \{e \in A_1 \mid (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&\quad \cap \{e \in C \mid (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&= \{e \in A \cap B \mid (F \cap G)(e) \neq \emptyset, (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&\quad \cap \{e \in C \mid (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&= \{e \in A \cap B \mid (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&\quad \cap \{e \in C \mid (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&= \{e \in A \cap B \cap C \mid (F \cap G)(e) \cap H(e) \neq \emptyset\} \\
&= \{e \in A \cap B \cap C \mid F(e) \cap G(e) \cap H(e) \neq \emptyset\}.
\end{aligned} \tag{10}$$

By the same token, we have that $B' = \{e \in A \cap B \cap C \mid F(e) \cap G(e) \cap H(e) \neq \emptyset\}$, and thus $A' = B'$. Moreover, for any $e \in A'$, we have that

$$\begin{aligned}
L(e) &= (F \cap G)(e) \cap H(e) \\
&= F(e) \cap G(e) \cap H(e) \\
&= F(e) \cap (G(e) \cap H(e)) \\
&= F(e) \cap (G \cap H)(e) \\
&= R(e),
\end{aligned} \tag{11}$$

namely, $L(e) = R(e)$. Therefore, the assertion (1) holds. □

Proposition 17 (Distributive laws). *For any soft sets* (F, A), (G, B), *and* (H, C) *over* U*, we have that*

(1) $(F, A) \cap ((G, B) \cup (H, C)) = ((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C))$,

(2) $(F, A) \cup ((G, B) \cap (H, C)) = ((F, A) \cup (G, B)) \cap ((F, A) \cup (H, C))$.

Proof. We only verify the first assertion; the second one can be verified similarly. For simplicity, we write (L, A') and (R, B') for $(F, A) \cap ((G, B) \cup (H, C))$ and $((F, A) \cap (G, B)) \cup ((F, A) \cap (H, C))$, respectively. We thus see that

$$\begin{aligned}
A' &= \{e \in A \cap (B \cup C) \mid F(e) \cap (G \cup H)(e) \neq \emptyset\} \\
&= \{e \in (A \cap B) \cup (A \cap C) \mid F(e) \cap (G \cup H)(e) \neq \emptyset\} \\
&= \{e \in A \cap B \mid F(e) \cap (G \cup H)(e) \neq \emptyset\} \\
&\quad \cup \{e \in A \cap C \mid F(e) \cap (G \cup H)(e) \neq \emptyset\} \\
&= \{e \in A \cap B \cap C^c \mid F(e) \cap G(e) \neq \emptyset\} \\
&\quad \cup \{e \in A \cap B \cap C \mid F(e) \cap (G(e) \cup H(e)) \neq \emptyset\} \\
&\quad \cup \{e \in A \cap B^c \cap C \mid F(e) \cap H(e) \neq \emptyset\} \\
&= \{e \in A \cap B \cap C^c \mid F(e) \cap G(e) \neq \emptyset\} \\
&\quad \cup \{e \in A \cap B \cap C \mid F(e) \cap G(e) \neq \emptyset\} \\
&\quad \cup \{e \in A \cap B^c \cap C \mid F(e) \cap H(e) \neq \emptyset\} \\
&\quad \cup \{e \in A \cap B \cap C \mid F(e) \cap H(e) \neq \emptyset\} \\
&= \{e \in A \cap B \mid F(e) \cap G(e) \neq \emptyset\} \\
&\quad \cup \{e \in A \cap C \mid F(e) \cap H(e) \neq \emptyset\} \\
&= B',
\end{aligned} \tag{12}$$

namely, $A' = B'$. Furthermore, for any $e \in A'$, one can check that $L(e) = F(e) \cap (G \cup H)(e) = ((F \cap G) \cup (F \cap H))(e) = R(e)$ by a routine computation. We do not go into the details here. Hence, the assertion (1) holds. □

Like usual sets, soft sets are monotonic with respect to intersection and union.

Proposition 18. *Let (F_i, A_i) and (G_i, B_i), $i = 1, 2$, be soft sets over U. If $(F_i, A_i) \subseteq (G_i, B_i)$, $i = 1, 2$, then we have that*

(1) $(F_1, A_1) \cap (F_2, A_2) \subseteq (G_1, B_1) \cap (G_2, B_2)$,

(2) $(F_1, A_1) \cup (F_2, A_2) \subseteq (G_1, B_1) \cup (G_2, B_2)$.

Proof. It is clear by the definitions of intersection, union, and subset of soft sets. □

Recall that in classical set theory, we have that $X \subseteq Y$ if and only if $X \cap Y = X$, which is also equivalent to $X \cup Y = Y$. For soft sets, we have the following observation.

Proposition 19. *Let (F, A) and (G, B) be soft sets over U. Then the following are equivalent:*

(1) $(F, A) \subseteq (G, B)$,

(2) $(F, A) \cap (G, B) = (F, A)$,

(3) $(F, A) \cup (G, B) = (G, B)$.

Proof. Again, it is obvious by the definitions of intersection, union, and subset of soft sets. □

The following several properties are concerned with the complement of soft sets.

Proposition 20. *Let (F, A) and (G, B) be two soft sets over U. Then $(G, B) = (F, A)^c$ if and only if $(F, A) \cap (G, B) = (\emptyset, \emptyset)$ and $(F, A) \cup (G, B) = (U, E)$.*

Proof. If $(G, B) = (F, A)^c$, then we see by definition that $(F, A) \cap (F, A)^c = (\emptyset, \emptyset)$ and $(F, A) \cup (F, A)^c = (F, A) \cup (F^c, \{e \in A \mid F(e) = U\}^c) = (U, E)$. Whence, the necessity is true.

Conversely, assume that $(F, A) \cap (G, B) = (\emptyset, \emptyset)$ and $(F, A) \cup (G, B) = (U, E)$. The latter means that $A \cup B = E$. Moreover, we obtain that $F(e) = U$ for all $e \in A \setminus B$ and $G(e) = U$ for all $e \in B \setminus A$. For any $e \in A \cap B$, it follows from $(F, A) \cap (G, B) = (\emptyset, \emptyset)$ and $(F, A) \cup (G, B) = (U, E)$ that $F(e) \cup G(e) = U$ and $F(e) \cap G(e) = \emptyset$. As neither $F(e)$ nor $G(e)$ is empty, this forces that $B = \{e \in A \mid F(e) = U\}^c$. For any $e \in B$, if $e \in A$, then $G(e) = F(e)^c = F^c(e)$; if $e \in B \setminus A$, then $G(e) = U = F^c(e)$. This implies that $(F, A)^c = (F^c, \{e \in A \mid F(e) = U\}^c) = (G, B)$, finishing the proof. □

The following fact follows immediately from Proposition 20.

Corollary 21. *For any soft set (F, A) over U, we have that*

$$\left((F, A)^c\right)^c = (F, A). \tag{13}$$

Proof. Note that $(F, A)^c \cap (F, A) = (\emptyset, \emptyset)$ and $(F, A)^c \cup (F, A) = (U, E)$. It therefore follows from Proposition 20 that $(F, A) = ((F, A)^c)^c$, as desired. □

With the above corollary, we can prove the De Morgan's laws of soft sets.

Proposition 22 (De Morgan's laws). *For any soft sets (F, A) and (G, B) over U, we have that*

(1) $((F, A) \cap (G, B))^c = (F, A)^c \cup (G, B)^c$,

(2) $((F, A) \cup (G, B))^c = (F, A)^c \cap (G, B)^c$.

Proof. (1) For convenience, let $A_0 = \{e \in A \mid F(e) = U\}$, $B_0 = \{e \in B \mid G(e) = U\}$, $C_0 = \{e \in A \cap B \mid F(e) \cap G(e) = U\}$, and $C_1 = \{e \in A \cap B \mid F(e) \cap G(e) \neq \emptyset\}$. Then we have that

$$\begin{aligned}
&((F, A) \cap (G, B))^c \\
&\quad = \left(F \cap G, C_1\right)^c \\
&\quad = \left((F \cap G)^c, \{e \in C_1 \mid (F \cap G)(e) = U\}^c\right) \\
&\quad = \left((F \cap G)^c, \{e \in A \cap B \mid F(e) \cap G(e) = U\}^c\right) \\
&\quad = \left((F \cap G)^c, C_0^c\right).
\end{aligned} \tag{14}$$

On the other hand, we have that

$$\begin{aligned}
(F, A)^c \cup (G, B)^c &= \left(F^c, A_0^c\right) \cup \left(G^c, B_0^c\right) \\
&= \left(F^c \cup G^c, A_0^c \cup B_0^c\right) \\
&= \left(F^c \cup G^c, (A_0 \cap B_0)^c\right) \\
&= \left(F^c \cup G^c, C_0^c\right).
\end{aligned} \tag{15}$$

Therefore, to prove (1), it suffices to show that $(F \cap G)^c(e) = (F^c \cup G^c)(e)$ for all $e \in C_0^c$. In fact, since $C_0^c = (C_1 \setminus C_0) \cup C_1^c$ and $(C_1 \setminus C_0) \cap C_1^c = \emptyset$, we need only to consider two cases. The first case is that $e \in C_1 \setminus C_0$. In this case, $e \in A_0^c \cap B_0^c$, and thus we get that $(F \cap G)^c(e) = (F(e) \cap G(e))^c = F(e)^c \cup G(e)^c = (F^c \cup G^c)(e)$. The other case is that $e \in C_1^c$. In this case, we always have by definition that $(F \cap G)^c(e) = U = (F^c \cup G^c)(e)$. Consequently, $(F \cap G)^c(e) = (F^c \cup G^c)(e)$ for all $e \in C_0^c$, as desired.

(2) By Corollary 21 and the first assertion, we find that

$$\begin{aligned}
((F, A) \cup (G, B))^c &= \left(\left((F, A)^c\right)^c \cup \left((G, B)^c\right)^c\right)^c \\
&= \left(\left((F, A)^c \cap (G, B)^c\right)^c\right)^c \\
&= (F, A)^c \cap (G, B)^c.
\end{aligned} \tag{16}$$

Hence, the second assertion holds as well. This completes the proof of the proposition. □

Let us end this section with an observation on the difference of two soft sets.

Proposition 23. *For any soft sets (F, A) and (G, B) over U, we have that*

$$(F, A) \setminus (G, B) = (F, A) \cap (G, B)^c. \tag{17}$$

Proof. We set $B_0 = \{e \in B \mid G(e) = U\}$ and write $(F \setminus G, C)$ for $(F, A) \setminus (G, B)$. Then we see that $C = A \setminus \{e \in A \cap B \mid F(e) \subseteq$

$G(e)\}$ and $(G,B)^c = (G^c, B_0^c)$. As a result, $(F,A) \cap (G,B)^c = (F,A) \cap (G^c, B_0^c) = (F \cap G^c, B_1)$, where

$$\begin{aligned} B_1 &= \{e \in A \cap B_0^c \mid F(e) \cap G^c(e) \neq \emptyset\} \\ &= (A \setminus B) \cup \{e \in A \cap B \mid F(e) \not\subseteq G(e)\} \\ &= A \setminus \{e \in A \cap B \mid F(e) \subseteq G(e)\} \\ &= C, \end{aligned} \tag{18}$$

as desired. It remains to show that $(F \setminus G)(e) = (F \cap G^c)(e)$ for all $e \in C = B_1$. In fact, if $e \in C \setminus B$, then we have that $(F \setminus G)(e) = F(e) = F(e) \cap U = (F \cap G^c)(e)$; if $e \in C \cap B$, then $(F \setminus G)(e) = F(e) \setminus G(e) = F(e) \cap G^c(e) = (F \cap G^c)(e)$. We thus get that $(F \setminus G)(e) = (F \cap G^c)(e)$ for all $e \in C = B_1$. Consequently, $(F,A) \setminus (G,B) = (F,A) \cap (G,B)^c$, finishing the proof. □

5. Conclusion

In this paper, we have redefined the intersection, complement, and difference of soft sets. These operations, together with an existing union operation, form the fundamental operations for constructing new soft sets from given soft sets. By examining the algebraic properties of these operations, we find that all basic properties of operations on classical sets such as identity laws, domination laws, distributive laws, and De Morgan's laws hold for the newly defined operations on soft sets. From this point of view, the new operations on soft sets are reasonable. Motivated by the notion of Not set of a parameter set in [22], we will investigate the operations on soft sets by introducing more conjunctions including AND and OR into a parameter set. In addition, it is interesting to extend the notions of intersection, complement, difference of soft sets developed here to other soft structures such as fuzzy soft sets [29, 41], vague soft sets [28], and soft rough sets [42].

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References

[1] D. Molodtsov, "Soft set theory—first results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[2] H. Aktaş and N. Çağman, "Soft sets and soft groups," *Information Sciences*, vol. 177, no. 13, pp. 2726–2735, 2007.

[3] M. I. Ali, "A note on soft sets, rough soft sets and fuzzy soft sets," *Applied Soft Computing Journal*, vol. 11, no. 4, pp. 3329–3332, 2011.

[4] M. I. Ali, "Another view on reduction of parameters in soft sets," *Applied Soft Computing*, vol. 12, no. 6, pp. 1814–1821, 2012.

[5] M. I. Ali, M. Shabir, and M. Naz, "Algebraic structures of soft sets associated with new operations," *Computers & Mathematics with Applications*, vol. 61, no. 9, pp. 2647–2654, 2011.

[6] S. Alkhazaleh and A. R. Salleh, "Soft expert sets," *Advances in Decision Sciences*, vol. 2011, Article ID 757868, 12 pages, 2011.

[7] S. Alkhazaleh and A. R. Salleh, "Generalised interval-valued fuzzy soft set," *Journal of Applied Mathematics*, vol. 2012, Article ID 870504, 18 pages, 2012.

[8] K. V. Babitha and J. J. Sunil, "Transitive closures and ordering on soft sets," *Computers & Mathematics with Applications*, vol. 62, no. 5, pp. 2235–2239, 2011.

[9] T. Deng and X. Wang, "Parameter significance and reductions of soft sets," *International Journal of Computer Mathematics*, vol. 89, no. 15, pp. 1979–1995, 2012.

[10] F. Feng, Y. B. Jun, and X. Zhao, "Soft semirings," *Computers & Mathematics with Applications*, vol. 56, no. 10, pp. 2621–2628, 2008.

[11] X. Ge, Z. Li, and Y. Ge, "Topological spaces and soft sets," *Journal of Computational Analysis and Applications*, vol. 13, no. 5, pp. 881–885, 2011.

[12] A. A. Hazaymeh, I. B. Abdullah, Z. T. Balkhi, and R. I. Ibrahim, "Generalized fuzzy soft expert set," *Journal of Applied Mathematics*, vol. 2012, Article ID 328195, 22 pages, 2012.

[13] Y. Jiang, Y. Tang, Q. Chen, and Z. Cao, "Semantic operations of multiple soft sets under conflict," *Computers & Mathematics with Applications*, vol. 62, no. 4, pp. 1923–1939, 2011.

[14] Y. Jiang, Y. Tang, Q. Chen, J. Wang, and S. Tang, "Extending soft sets with description logics," *Computers & Mathematics with Applications*, vol. 59, no. 6, pp. 2087–2096, 2010.

[15] Y. B. Jun, "Soft BCK/BCI-algebras," *Computers & Mathematics with Applications*, vol. 56, no. 5, pp. 1408–1413, 2008.

[16] Y. B. Jun and S. S. Ahn, "Double-framed soft sets with applications in BCK/BCI-algebras," *Journal of Applied Mathematics*, vol. 2012, Article ID 178159, 15 pages, 2012.

[17] Y. B. Jun, K. J. Lee, and A. Khan, "Soft ordered semigroups," *Mathematical Logic Quarterly*, vol. 56, no. 1, pp. 42–50, 2010.

[18] Y. B. Jun, K. J. Lee, and C. H. Park, "Soft set theory applied to ideals in d-algebras," *Computers & Mathematics with Applications*, vol. 57, no. 3, pp. 367–378, 2009.

[19] Y. B. Jun and C. H. Park, "Applications of soft sets in ideal theory of BCK/BCI-algebras," *Information Sciences*, vol. 178, no. 11, pp. 2466–2475, 2008.

[20] Y. B. Jun, S. Z. Song, and K. S. So, "Soft set theory applied to p-ideals of BCI-algebras related to fuzzy points," *Neural Computing and Applications*, vol. 20, no. 8, pp. 1313–1320, 2011.

[21] Z. Kong, L. Gao, L. Wang, and S. Li, "The normal parameter reduction of soft sets and its algorithm," *Computers & Mathematics with Applications*, vol. 56, no. 12, pp. 3029–3037, 2008.

[22] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications*, vol. 45, no. 4-5, pp. 555–562, 2003.

[23] P. Majumdar and S. K. Samanta, "Generalised fuzzy soft sets," *Computers & Mathematics with Applications*, vol. 59, no. 4, pp. 1425–1432, 2010.

[24] W. K. Min, "Similarity in soft set theory," *Applied Mathematics Letters*, vol. 25, no. 3, pp. 310–314, 2012.

[25] J. H. Park, O. H. Kim, and Y. C. Kwun, "Some properties of equivalence soft set relations," *Computers & Mathematics with Applications*, vol. 63, no. 6, pp. 1079–1088, 2012.

[26] B. Tanay and M. B. Kandemir, "Topological structure of fuzzy soft sets," *Computers & Mathematics with Applications*, vol. 61, no. 10, pp. 2952–2957, 2011.

[27] Z. Xiao, K. Gong, S. Xia, and Y. Zou, "Exclusive disjunctive soft sets," *Computers & Mathematics with Applications*, vol. 59, no. 6, pp. 2128–2137, 2010.

[28] W. Xu, J. Ma, S. Wang, and G. Hao, "Vague soft sets and their properties," *Computers & Mathematics with Applications*, vol. 59, no. 2, pp. 787–794, 2010.

[29] X. Yang, T. Y. Lin, J. Yang, Y. Li, and D. Yu, "Combination of interval-valued fuzzy set and soft set," *Computers & Mathematics with Applications*, vol. 58, no. 3, pp. 521–527, 2009.

[30] Y. Yin, H. Li, and Y. B. Jun, "On algebraic structure of intuitionistic fuzzy soft sets," *Computers & Mathematics with Applications*, vol. 64, no. 9, pp. 2896–2911, 2012.

[31] P. Zhu and Q. Wen, "Probabilistic soft sets," in *Proceedings of the IEEE Conference on Granular Computing (GrC'10)*, pp. 635–638, IEEE Press, San Jose, Calif, USA, August 2010.

[32] N. Çağman, S. Enginoglu, and F. Citak, "Fuzzy soft set theory and its applications," *Iranian Journal of Fuzzy Systems*, vol. 8, no. 3, pp. 137–147, 2011.

[33] T. Herawan and M. M. Deris, "On multi-soft sets construction in information systems," in *Emerging Intelligent Computing Technology and Applications with Aspects of Artificial Intelligence: 5th International Conference on Intelligent Computing (ICIC'09)*, vol. 5755 of *Lecture Notes in Computer Science*, pp. 101–110, Springer, Ulsan, Republic of Korea, September 2009.

[34] T. Herawan, A. N. M. Rose, and M. M. Deris, "Soft set theoretic approach for dimensionality reduction," in *Proceedings of the Database Theory and Application: International Conference (DTA'09)*, pp. 171–178, Springer, Jeju Island, Republic of Korea, December 2009.

[35] M. M. Mushrif, S. Sengupta, and A. K. Ray, "Texture classification using a novel, soft-set theory based classification algorithm," in *Computer Vision—ACCV 2006*, vol. 3851 of *Lecture Notes in Computer Science*, pp. 246–254, 2006.

[36] Y. Zou and Z. Xiao, "Data analysis approaches of soft sets under incomplete information," *Knowledge-Based Systems*, vol. 21, no. 8, pp. 941–945, 2008.

[37] T. M. Basu, N. K. Mahapatra, and S. K. Mondal, "A balanced solution of a fuzzy soft set based decision making problem in medical science," *Applied Soft Computing*, vol. 12, no. 10, pp. 3260–3275, 2012.

[38] F. Feng, Y. B. Jun, X. Liu, and L. Li, "An adjustable approach to fuzzy soft set based decision making," *Journal of Computational and Applied Mathematics*, vol. 234, no. 1, pp. 10–20, 2010.

[39] F. Feng, Y. Li, and N. Çağman, "Generalized *uni-int* decision making schemes based on choice value soft sets," *European Journal of Operational Research*, vol. 220, no. 1, pp. 162–170, 2012.

[40] P. K. Maji, A. R. Roy, and R. Biswas, "An application of soft sets in a decision making problem," *Computers & Mathematics with Applications*, vol. 44, no. 8-9, pp. 1077–1083, 2002.

[41] A. R. Roy and P. K. Maji, "A fuzzy soft set theoretic approach to decision making problems," *Journal of Computational and Applied Mathematics*, vol. 203, no. 2, pp. 412–418, 2007.

[42] F. Feng and X. Liu, "Soft rough sets with applications to demand analysis," in *Proceedings of the International Workshop on Intelligent Systems and Applications (ISA'09)*, pp. 1–4, IEEE, Wuhan, China, May 2009.

[43] H. Qin, X. Ma, J. M. Zain, and T. Herawan, "A novel soft set approach in selecting clustering attribute," *Knowledge-Based Systems*, vol. 36, pp. 139–145, 2012.

[44] Z. Xiao, K. Gong, and Y. Zou, "A combined forecasting approach based on fuzzy soft sets," *Journal of Computational and Applied Mathematics*, vol. 228, no. 1, pp. 326–333, 2009.

[45] C.-F. Yang, "A note on: "Soft set theory" [Computers & Mathematics with Applications 45 (2003), no. 4-5, 555–562]," *Computers & Mathematics with Applications*, vol. 56, no. 7, pp. 1899–1900, 2008.

[46] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, "On some new operations in soft set theory," *Computers & Mathematics with Applications*, vol. 57, no. 9, pp. 1547–1553, 2009.

[47] A. Sezgin and A. O. Atagun, "On operations of soft sets," *Computers & Mathematics with Applications*, vol. 61, no. 5, pp. 1457–1467, 2011.