MATHEMATICAL MODELLING OF INFORMATION AGE CONFLICT

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Previous mathematical modelling of conflict has been based on Lanchester's equations, which relate to the grinding attrition of "industrial-age" warfare. Large blocks of force interact in order to force defeat by a process of wearing away the other. This is no longer so relevant as a way of conceptualising warfare, and we generalise the approach so that it is more appropriate to the "information age" into which we are now moving. It turns out that the solution to this problem is the development of a theory of what we call "scale-free systems." We first develop this theory, and then indicate how it can be applied.

1. Introduction

Previous mathematical modelling of conflict has been based on Lanchester's equations. A comprehensive review is given in Bowen and McNaught [4], with significant contributions by Taylor [9]. These equations relate to the grinding attrition of "industrial-age" warfare. Large blocks of force interact in order to force defeat by a process of wearing away the other. This is no longer so relevant as a way of conceptualising warfare, and we now need to generalise the approach so that it is more relevant to the "information age" into which we are now moving. One of the key ideas in this transition is the emergent behaviour of self-organising future forces in conflict (Alberts and Hayes [2]). Can we develop a mathematical treatment which shows the way forward?

In this paper, we show that the answer is based on the development of a new perspective on a certain class of systems which we term "scale-free." This is a more general definition than that considered by Barabasi in his development of scale-free networks (Albert and Barabási [1]). We firstly develop the theory of such scale-free systems, and then show how it can be applied to our problem.

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2. Scale-free systems

In order to introduce the idea, we firstly consider the classical Lanchester equations in a scale-free form. These equations represent a conflict of attrition between two warring parties.

This represents a two-sided conflict between Red and Blue in which each side uses "aimed fire" to attack the other side (Taylor [9]). Specifically we assume that

(i) each unit on each side is within weapon range of all units on the other side;
(ii) units on each side are identical but the units on one side may have a different kill rate to the units on the opposing side;
(iii) each firing unit is sufficiently well-aware of the location and condition of all enemy units so that when a target is killed, fire may be immediately shifted to a new target;
(iv) this new target is randomly chosen from the surviving targets.

At time t, and considering the small increment of time between t and $t+\delta t$, the number of Blue casualties $\delta b(t)$ is given by the number of remaining Red units times the number of targets they kill, that is, $r(t)k_r\delta t$, where k_r is the effectiveness of a single Red unit engagement, and $r(t)\delta t$ is a measure of the number of such engagements. From this we have the relationship $db/dt = k_r r(t)$. By symmetry we also have a similar relationship for the attrition of Red units, namely, $dr/dt = k_b b(t)$.

Hence, dividing one equation by the other we have $db/dr = k_r r/k_b b$ or $k_b b\,db = k_r r\,dr$. Integrating both sides of this equation, and using the initial values r_0 and b_0 for the number of Red and Blue units at the start of the conflict, we have the following relationship (from which the squared-law name derives):

$$k_b\left(b_0^2 - b^2\right) = k_r\left(r_0^2 - r^2\right). \tag{2.1}$$

Now define the nondimensional variables $x = 1 - r/r_0$ and $y = b/b_0$. We also define the nondimensional "Lanchester number:" $L = k_r r_0^2/k_b b_0^2$. We term this a "similarity parameter," by which we mean that battles with the same value of L will evolve in a similar way.

From the expression (2.1) above we have

$$\frac{k_b}{k_r} b_0^2\left\{1 - \left(\frac{b}{b_0}\right)^2\right\} = r_0^2\left\{1 - \left(\frac{r}{r_0}\right)^2\right\}, \tag{2.2}$$

thus

$$\frac{k_b}{k_r}\frac{b_0^2}{r_0^2}\left(1 - y^2\right) = \left\{\left(1 - \frac{r}{r_0}\right)^2 + 2\left(\frac{r}{r_0}\right)\left(1 - \frac{r}{r_0}\right)\right\}, \tag{2.3}$$

thus

$$\begin{gathered} L^{-1}\left(1 - y^2\right) = x^2 + 2(1-x)x = x(2-x), \\ 1 - y^2 = Lx(2-x), \end{gathered} \tag{2.4}$$

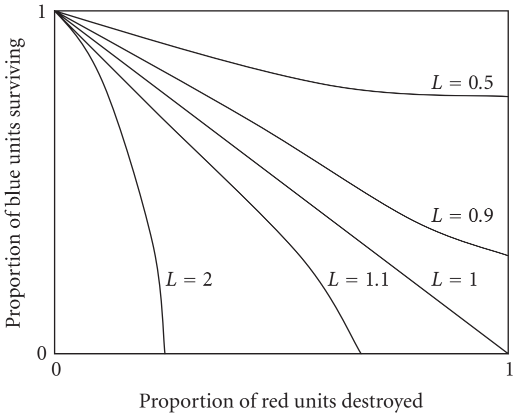

Figure 2.1. The relationship between the dimensionless variables y, x, and L for the Lanchester square law.

from which it follows that

$$y = \left\{1 - 2Lx\left(1 - \frac{x}{2}\right)\right\}^{1/2}. \tag{2.5}$$

If we plot this relationship for different values of the Lanchester number L, we get Figure 2.1.

Each of the lines in Figure 2.1 corresponds to an essentially different evolution of the battle over time, and corresponds to a different value of the similarity parameter L.

A first-order expansion of the expression at (2.5) is of the form

$$y \cong 1 - Lx\left(1 - \frac{x}{2}\right) \cong 1 - Lx. \tag{2.6}$$

The relationship $y = 1 - Lx$ corresponds to Lanchester's linear law (Taylor [9]). Thus we can see that when x, the proportion of red units destroyed, is small, the linear and square law relationships evolve in the same way, and then diverge as the nonlinear terms (such as x^2) grow larger.

Figure 2.1 is a normalised plot. This means that it remains valid for a large number of variations of the initial conditions b_0 and r_0, and the kill parameters k_b and k_r. Various combinations of these simply relate to particular values of the Lancester number L which is a dimensionless invariant describing a particular flow or evolution of the battle.

In terms of a general approach, we can express these laws in the form

$$y = \Phi(x, L), \tag{2.7}$$

where Φ is a normalised functional relationship of the two nondimensional parameters x and L.

This application of nondimensionality thus raises the question of whether it is possible to obtain the key nondimensional parameters of a system by a systematic means.

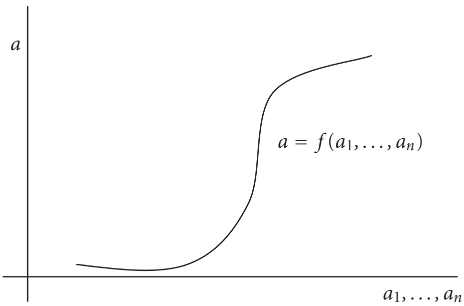

Figure 2.2. The system S.

2.1. The general approach. More generally, then, we assume that we have a representation of the system in the form

$$a = f(a_1, a_2, \ldots, a_n), \tag{2.8}$$

where a is a key output of the system, $a_1, a_2, \ldots, a_n$ are the key input variables which affect the behaviour of the system, and f represents some functional relationship between the two. This represents what we mean by a *metamodel* of the system, in the sense that the output of the system is mathematically related in some way to the input variables. There are two basic ways of trying to determine the nature of the functional relationship f. The first is to run the system model a sufficient number of times in order to produce a map of the response surface. Examples of such an approach are discussed, for example, in Law and Kelton [6]. This is essentially a black-box approach to the problem. We try to fit inputs to outputs with no understanding of how the system itself "works." Examples of this approach are the use of multiple regression analysis and neural networks.

However, in some cases, it is possible to exploit certain symmetries of the system in order to take a rather different approach. Although this is in some ways more challenging, it has the advantage that it can lead to a more fundamental insight into the behaviour of the system. One symmetry which can give such insights into system behaviour is invariance under changes to the scale at which we choose to represent the system. We call these changes in the measurement *scale changes*. Thus the symmetry we aim to exploit is one of invariance under changes in the scale, that is, *scale invariance*.

Principle of scale invariance. We elevate this to a principle that for the systems of interest (i.e., all scale-free systems), we require that observers using different scales should observe the same system behaviour. Thus the metamodel we produce should be scale-invariant. Examples of such systems are those based on fractal effects (since by definition, fractals are scale-invariant).

Consider now a system S for which we can measure the inputs and output, and these form a relationship as shown in Figure 2.2.

The system S is thus represented by the relationship $a = f(a_1, \ldots, a_n)$, where the function f represents the system response. If we now change the scales which we use to measure the system inputs and output, (e.g., shrinking or expanding the size of the system

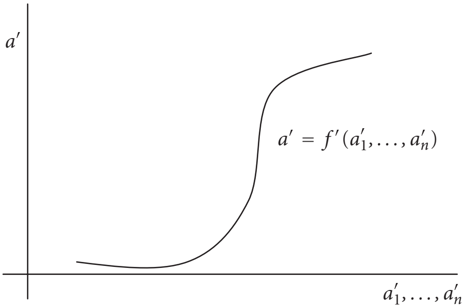

Figure 2.3. The system S'.

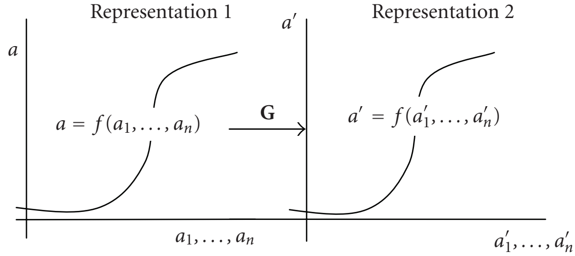

Figure 2.4. The effect of the gauge group **G**.

by changing our length scale) we have another representation of the system, shown at Figure 2.3.

In the new representation at Figure 2.3, the variable scales have changed. In general we also assume that the system response function f' differs from f.

We say that the system is invariant under this scale change if the system response is the same in both cases, in other words, $f = f'$. If this is true for all such changes of scale, we call the system *scale-free*.

To go from the frame of reference for the system as shown in Figure 2.2 to the frame of reference as shown in Figure 2.3 we change the scales we use. This corresponds to mapping from one representation to the other via a mapping **G** as shown in Figure 2.4. This mapping represents the rescaling required of the variables $a, a_1, \ldots, a_n$. Such rescalings in fact form a group, which we call a *gauge group*.

To understand what we mean by this more precisely, we have to define what is called a fundamental gauge class for the system (Sedov [8], Barenblatt [3]). We define a fundamental gauge class to mean the set of dimensions which we will use to define the dimensionality of each of our system variables a and $a_1, a_2, \ldots, a_n$. For example, this might be the dimensions {force, length, time} as used in the Lanchester example. It may need to be extended to include dimensions such as heat and temperature.

As shown in Figure 2.4, the effect of the gauge group **G** is thus to "scale" the input and output variables $a, a_1, \ldots, a_n$. However the system response f remains the same. This system is scale-free since its representation does not depend on particular fixed gauges or scales for the input and output values.

In terms of our fundamental gauge class, we can then define the dimensionality of the output variable a and the input variables $a_1, a_2, \ldots, a_n$. From this, some of the input variables will have "independent dimensions." This means that their dimensions cannot be defined in terms of products of powers of the dimensions of the other variables. Call these variables $a_1, \ldots, a_k$. Consider now the remaining "dependent dimension" variables. There are $n - k$ of these. Relabel them as $b_1, \ldots, b_{n-k}$. For simplicity we continue the explanation assuming that $n - k = 2$; that is, we have just two dependent dimension variables: b_1 and b_2. However, everything we say can be easily generalised to the case of more than two such variables.

Our general metamodel relation is then of the form

$$a = f(a_1, a_2, \ldots, a_k, b_1, b_2). \tag{2.9}$$

We assume our metamodel is scale-free, as discussed above. If we change the gauge of each of our independent dimension variables $a_1, \ldots, a_k$, their values will change (i.e., increase or decrease dependent on the nature of the gauge change). Since these are the variables with independent dimensions, these changes are independent of each other. Thus we can represent such a gauge change as the mapping **G**:

$$a_1' = A_1 a_1, \qquad a_2' = A_2 a_2, \qquad a_k' = A_k a_k, \tag{2.10}$$

where $A_1, A_2, \ldots, A_k$ can be varied independently.

The variables b_1 and b_2 have dependent dimensions, thus if we consider b_1; it is possible to find products of powers of the dimensions of $a_1, \ldots, a_k$ which equal the dimension of b_1. The same applies to b_2. Thus we can write (using square brackets to denote "dimension of")

$$\begin{aligned} [b_1] &= [a_1]^{p_1} \cdots [a_k]^{r_1}, \\ [b_2] &= [a_1]^{p_2} \cdots [a_k]^{r_2}. \end{aligned} \tag{2.11}$$

Since b_1 and b_2 do not contribute additional dimensionality to the metamodel in terms of independent dimensions, it also follows that

$$[a] = [a_1]^{p} \cdots [a_k]^{r}. \tag{2.12}$$

Under the gauge transformation **G** we then have

$$b_1' = A_1^{p_1} \cdots A_k^{r_1} b_1, \qquad b_2' = A_1^{p_2} \cdots A_k^{r_2} b_2, \qquad a' = A_1^{p} \cdots A_k^{r} a. \tag{2.13}$$

The transformation group **G** generated by $\{A_1, \ldots, A_k\}$ is a continuous gauge group.

It follows from (2.11) and (2.12) that the expressions

$$\Pi = \frac{a}{a_1^p \cdots a_k^r},$$
$$\Pi_1 = \frac{b_1}{a_1^{p_1} \cdots a_k^{r_1}}, \tag{2.14}$$
$$\Pi_2 = \frac{b_2}{a_1^{p_2} \cdots a_k^{r_2}}$$

are dimensionless, and hence gauge invariant—this means that they remain invariant under the action of the gauge group **G**.

Consider now the expression

$$\begin{aligned}\Pi = \frac{a}{a_1^p \cdots a_k^r} &= \frac{f(a_1,\ldots,a_k,b_1,b_2)}{a_1^p \cdots a_k^r} \\ &= \frac{f(a_1,\ldots,a_k,\Pi_1 a_1^{p_1} \cdots a_k^{r_1},\Pi_2 a_1^{p_2} \cdots a_k^{r_2})}{a_1^p \cdots a_k^r}.\end{aligned} \tag{2.15}$$

Thus we can write

$$\Pi = F(a_1,a_2,\ldots,a_k,\Pi_1,\Pi_2), \tag{2.16}$$

where F represents some functional relationship.

If we now consider the parameter a_1, then since a_1 has independent dimensions, we can change the value of a_1 through a change of gauge without changing the values of the other independent dimension variables $a_2,\ldots,a_k$. Since Π, Π_1, and Π_2 are dimensionless, they are also invariant under this change of gauge. Thus we can find a group parameter A such that

$$\Pi = F(Aa_1,a_2,\ldots,a_k,\Pi_1,\Pi_2) = F(a_1,a_2,\ldots,a_k,\Pi_1,\Pi_2). \tag{2.17}$$

Since A is a free parameter, it can take any value. The only way the expression at (2.17) can then hold is that Π is independent of the variable a_1. The same argument can also be applied individually to the variables $a_2,\ldots,a_k$. Thus Π must be independent of the variables $a_1,\ldots,a_k$. It follows from (2.17) that Π is a function only of Π_1 and Π_2. If we denote this function as Φ, then the expression for our gauge invariant metamodel must be of the form

$$\Pi = \Phi(\Pi_1,\Pi_2). \tag{2.18}$$

Now recall that $\Pi = a/a_1^p \cdots a_k^r = f(a_1,\ldots,a_k,b_1,b_2)/a_1^p \cdots a_k^r$.

Thus any metamodel function f which defines a scale-free relationship of the form

$$a = f(a_1,a_2,\ldots,a_n), \tag{2.19}$$

where there are n input variables, $k = n - 2$ of which have independent dimensions, and two (b_1 and b_2) have dependent dimensions, can be written as a gauge invariant function of two nondimensional variables in the following special form

$$a = f(a_1, a_2, \ldots, a_n) = f(a_1, a_2, \ldots, a_k, b_1, b_2) = a_1^p \cdots a_k^r \Phi(\Pi_1, \Pi_2). \tag{2.20}$$

The argument clearly generalises to the case where there are $m = n - k$ input variables with dependent dimensions, $b_1, \ldots, b_m$, and the metamodel in this case must have the gauge invariant form

$$a = a_1^p \cdots a_k^r \Phi(\Pi_1, \Pi_2, \ldots, \Pi_m). \tag{2.21}$$

2.2. Power laws. One signature of complex emergent systems is that of power law relationships between input and output variables, as discussed by Moffat [7]. We can see from the expression at (2.21) that such power laws (i.e., relations of the form $y = x^\alpha$) are to be expected for the type of scale-free systems we have considered, and they emerge naturally from our approach to metamodelling.

3. The renormalisation group

To keep things simple, we assume again that we have just two similarity parameters Π_1 and Π_2 corresponding to two input variables b_1 and b_2 which have dependent dimensions. Our metamodel is thus of the form

$$a = f(a_1, a_2, \ldots, a_k, b_1, b_2). \tag{3.1}$$

We can then turn this into a scale-free metamodel of the form

$$\Pi = \Phi(\Pi_1, \Pi_2). \tag{3.2}$$

The metamodel is then invariant under the continuous gauge group **G** of changes to the scale of the parameters with independent dimensions $\{a_1, \ldots, a_k\}$.

We now consider what happens if we require our metamodel to be invariant under a renormalisation group. Firstly consider the limit of $\Phi(\Pi_1, \Pi_2)$ as Π_2 tends to 0. If this limit is a constant, then for small values of Π_2 we can ignore this variable and just write our metamodel in the form

$$\Pi = \Phi(\Pi_1) \tag{3.3}$$

which is much simpler. This is equivalent to requiring that our metamodel is (in the limit) invariant under the group transformation $a_i \to a_i$, $b_1 \to b_1$, $b_2 \to Bb_2$ with $0 < B < 1$ since repeated application of this group transformation will produce $B^n b_2$ which tends to 0 as $n \to \infty$.

What happens if $\Phi(\Pi_1, \Pi_2)$ does not tend to a constant as Π_2 tends to 0? What does the gauge invariant function $\Phi(\Pi_1, \Pi_2)$ look like then?

As before, we let b_2, and hence Π_2, become small, and this is equivalent to continually multiplying it by a factor B with $0 < B < 1$. For each iteration of the factor B, we have a change of scale in the parameter b_2 and we renormalise the variables b_1 and a in order to ensure that our metamodel at (3.1) remains invariant under this change in the scale of one of the inputs with *dependent* dimensions.

This is achieved using the following renormalisation-group transformations (from Barenblatt [3]):

$$\begin{aligned} b_2' &= Bb_2, \\ b_1' &= B^{\alpha_1} b_1, \\ a_i' &= a_i, \quad \text{for } 1 \le i \le k, \\ a' &= B^{\alpha_2} a. \end{aligned} \tag{3.4}$$

If we substitute these into our metamodel and let $\mathbf{R}$ denote one iteration of this renormalisation process, then we have

$$Ra = a' = a'^{p}_{1} \cdots a'^{r}_{k} f\left(\frac{b_1'}{a'^{p_1}_{1} \cdots a'^{r_1}_{k}}, \frac{b_2'}{a'^{p_2}_{1} \cdots a'^{r_2}_{k}} \right). \tag{3.5}$$

Thus

$$B^{\alpha_2} a = a_1^{p} \cdots a_k^{r} f\left(\frac{B^{\alpha_1} b_1}{a_1^{p_1} \cdots a_k^{r_1}}, \frac{Bb_2}{a_1^{p_2} \cdots a_k^{r_2}} \right). \tag{3.6}$$

From this we can see that the form of the metamodel

$$\Pi = \Phi(\Pi_1, \Pi_2) = f\left(\frac{b_1}{a_1^{p_1} \cdots a_k^{r_1}}, \frac{b_2}{a_1^{p_2} \cdots a_k^{r_2}} \right) \tag{3.7}$$

is transformed by the renormalisation-group action into

$$\Pi = B^{-\alpha_2} \Phi(B^{\alpha_1} \Pi_1, B\Pi_2). \tag{3.8}$$

This has the same from as the "static-scaling" renormalisation group discussed in Moffat [7]. There we express this in the form

$$R_{b,\phi} u(x,t) = b^{-\alpha} u(b^{\phi} x, bt). \tag{3.9}$$

If we make the substitutions

$$b = B, \qquad \alpha_2 = \alpha, \qquad \alpha_1 = \phi, \qquad u = \Phi, \qquad x = \Pi_1, \qquad t = \Pi_2, \tag{3.10}$$

then these two expressions are the same.

Moffat [7] also shows that the limiting invariant form of the renormalised function $u(x,t)$ is

$$u^*(x,t) = t^{\alpha} \tilde{u}^* \left(\frac{x}{t^{\phi}} \right). \tag{3.11}$$

Making the substitutions above, this tells that the limiting form of $\Phi(\Pi_1, \Pi_2)$ is

$$\Phi^*(\Pi_1, \Pi_2) = \Pi_2^{\alpha_2} \tilde{\Phi}^* \left(\frac{\Pi_1}{\Pi_2^{\alpha_1}} \right). \tag{3.12}$$

This expression is then invariant under the action of the renormalisation-group.

We assume that our scale-free system is renormalisation-group invariant and thus has a form as shown at (3.12).

4. Application to Lanchester's equations

Our application of the above methods is to a generalisation of Lanchester's equations of conflict (described in the introduction), to take account of the transition from industrial-age conflict (mass on mass) as exemplified in the original equations, to the more dispersed and interactive conflict of the information age. We call this extension the fractal attrition equation.

The two key inputs to our metamodel, as we assume, are the unit effectiveness k and the time interval δt. Note that the variable k has dimensions $FT^{-1}F^{-1} = T^{-1}$ (with F the dimension of force, and T the dimension of time); thus $k\delta t$ is dimensionless. The key output is the attrition to the opposing force per time interval. We thus assume a metamodel of the form $E(\delta b/\delta t) \propto kr(t)\Phi(\Pi)$, with the dimensionless variable $\Pi = k\delta t$. E is a statistical expectation over some time period T which we normally assume to be significantly greater than δt. The next question is the precise functional form of the system response function Φ. This is governed by whether the system is fractal or not.

4.1. Investigation of the fractal nature of force laydown. We carried out an investigation of this using a combination of results from the HiLOCA closed form simulation model (an agent-based model of land conflict), and from field trials of tactical forces equipped with GPS receivers in order to record their positions during the battle. These trials took place at the suffield trials site in Canada, using UK forces, and go under the name of BATUS (British Army Training Unit (Suffield)) trials.

In order to determine the best estimate for a fractal dimension associated with the positions of deployed units, it is necessary to freeze the action, place a grid of cells across the battlefield, and analyse a graph of the log of the number of occupied cells versus $\log(1/d)$, where d is the cell size. This then yields a measure of the fractal dimension, if the distribution of units is fractal. Figure 4.1 shows the graph for the whole range of d values for the Blue forces near the start of a HiLOCA run. This shows a graph which is broadly linear but has a hook shape at small d (high $1/d$) and blocking effects at large d. It is therefore necessary to determine a range of d values that generate a good linear representation (if possible), omitting these end effects. This range is expressed as a percentage of the full range of d values. Figure 4.2 shows the same data with a range filter of 5–50% applied and a best linear fit shown. The gradient of this line is the estimate of the fractal dimension of the data (1.4 in this case).

Our research also showed that the fractal dimension of combined Red and Blue force dispositions varied over time and followed the advance, defence, and attack phases of the

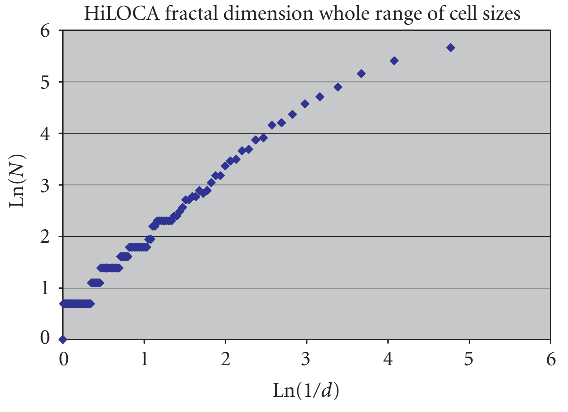

Figure 4.1. Computed fractal dimension of HiLOCA Blue forces.

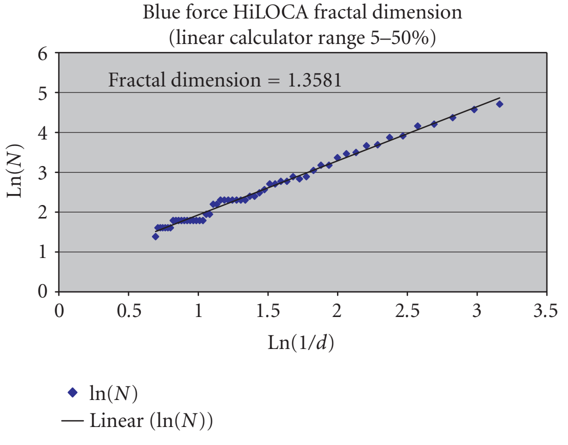

Figure 4.2. Range filter applied to the HiLOCA Blue force data.

simulation, as shown in Figure 4.3, indicating that we should expect the fractal dimension to adjust as circumstances change.

4.2. BATUS trials data fractal analysis. BATUS exercise data taken at the trials range in Canada has been analysed in a similar way to the HiLOCA data. Figure 4.4 shows the whole range of d values for the Blue forces in a particular BATUS trial, taken at a particular point during the trial, and based on GPS readings of the unit locations. Figure 4.5 shows the same data with a range filter applied. The straight line obtained indicates a fractal dimension of 0.8 in this case.

In Figure 4.6 we show how the fractal dimension varies over time for one of the BATUS exercises, computed in the way indicated above, at time steps of 5 minutes over the

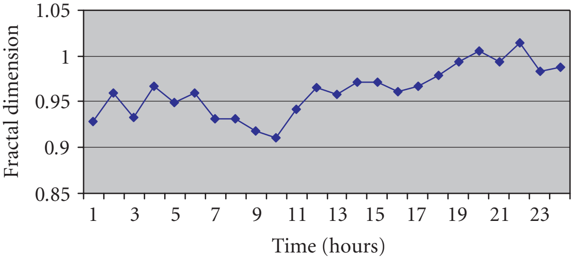

Figure 4.3. The fractal dimension of combined red and blue simulated force over time.

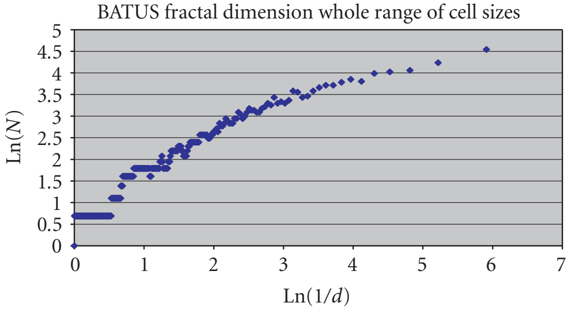

Figure 4.4. Calculation of the fractal dimension for a snapshot of the battlefield during a particular BATUS trial, based on GPS trials data.

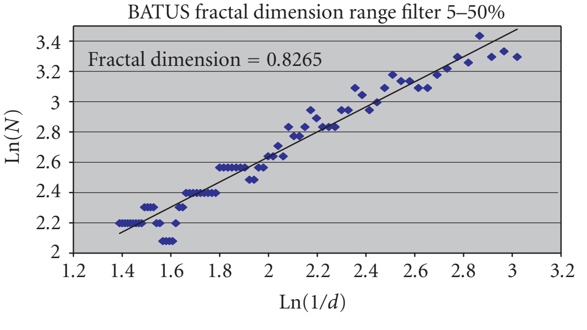

Figure 4.5. Calculation of the fractal dimension for BATUS data with the range filter applied.

duration of the data set. Only noncasualty vehicles are included. The graph shows the fractal dimension over time computed using two different fractal calculators. The linear calculator assumes a linear reduction in cell size during the calculation. The "Grainger logarithmic" approach assumes halving of the cell size at each step of the calculation. We have found that when these both give consistent plots, it is a useful indicator of a stable

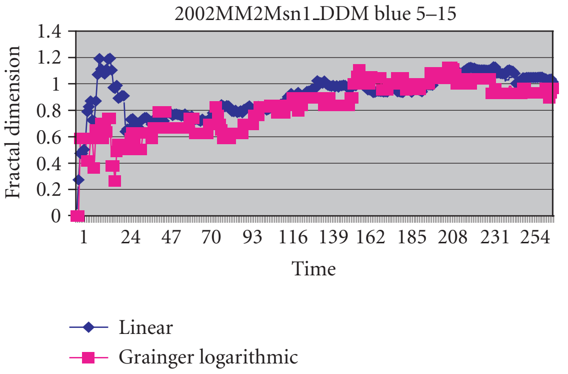

Figure 4.6. An example of the variation in fractal dimension over time for the BATUS trials data.

fractal clustering. The pattern of fractal dimension is to show a general increase, followed by a levelling off. There is some instability towards the end, possibly due to reduction in vehicles caused by casualties. A number of other BATUS data sets have been analysed in the same way and show similar characteristics.

5. Resultant theory

This evidence that our system is fractal in nature leads us to hypothesise that the function Φ is homogeneous, and so has the form $\Phi(x) = x^{\xi}$ with ξ an "anomalous dimension," as discussed by Barenblatt [3].

Thus we have the relationship

$$E_{0<t<T}\left(\frac{\delta b}{\delta t}\right) \propto k^{\xi+1}\delta t^{\xi} r(t). \tag{5.1}$$

In Lauren [5], a number of experiments were carried out using the cellular automata-based combat model ISAAC. For a number of different cases, the mean attrition rate for Blue was plotted against the Red kill probability. The data suggest that the exponent of k is of the form $D/2$ with D the Red force fractal dimension. Hence we assume that

$$\xi + 1 = \frac{D}{2}, \tag{5.2}$$

where D is the fractal dimension of the Red force clustering.

Thus

$$\xi = -\left(1 - \frac{D}{2}\right). \tag{5.3}$$

5.1. Correspondence requirement. When there is no dispersed fractal clustering, and Red agents form an uncorrelated set spread over the battlespace, with fractal dimension

$D=2$, we require that this relationship reverts to the square law Lanchester equation:

$$E_{0<t<T}\left(\frac{\delta b}{\delta t}\right) \propto kr(t). \tag{5.4}$$

This occurs when $\xi = 0$, which gives the correct exponent for k and also eliminates the term in δt as required.

5.2. General form. We thus assume that the fractal attrition relationship takes the general form

$$E_{0<t<T}\left(\frac{\delta b}{\delta t}\right) = E_{0<t<T}\left(-ck^{D/2}\delta t^{-(1-D/2)}r(t)\right), \tag{5.5}$$

where c is the constant of proportionality and D is a function of time t in general. There is a corresponding relationship for $\delta r/\delta t$ in terms of the Blue fractal dimension.

This then is the extension of Lanchester equations into the information age which we propose.

6. Conclusions

We have introduced the idea of a "scale-free" system in the context of defence modelling. We have demonstrated this approach by applying it to an important problem concerned with the mathematical modelling of information age conflict. This generalises the "industrial-age" Lanchester equations to make them more appropriate to the "information age." The resultant mathematical relationship gives us a new paradigm for consideration of information age conflict, and also acts as a useful adjunct to agent-based simulation modelling.

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