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Research Article

On Generalisation of Polynomials in Complex Plane

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The generalised Bell and Laguerre polynomials of fractional-order in complex z-plane are defined. Some properties are studied. Moreover, we proved that these polynomials are univalent solutions for second order differential equations. Also, the Laguerre-type of some special functions are introduced.

1. Introduction and Preliminaries

Special functions play important roles in applied mathematics. It has been seen that these functions have appeared in different frameworks, such as the mathematical physics [1], the combinatorial analysis [2], and the statistics [3]. Indeed, the explicit relationships between special functions and generalised hypergeometric functions have been obtained and mentioned in [4, 5]. Some extension of these polynomials already appeared in literature (see [6, 7]), and generalisation by using different type of calculus such as q-deform calculus [8, 9] and fractional calculus [10] has been studied.

Definition 1.1. The Bell polynomials take the form [11]

$$B_n(y) = e^{-y} D^n e^y = \sum_{d=1}^{n} B_{n,d}, \quad n \in \mathbb{N}_0, \tag{1.1}$$

where

$$B_{n,d} = \sum_{|k|=d,\|k\|=n} \frac{n!}{k!} \left(\frac{y_1}{1!}\right)^{k_1} \cdots \left(\frac{y_n}{n!}\right)^{k_n}, \tag{1.2}$$

such that $|k| = k_1 + \cdots + k_n$, $k_1 \geq 0, \ldots, k_n \geq 0$, $k! = k_1! \cdots k_n!$, and $B_0 = 1$.

Definition 1.2. The Laguerre polynomials take the form

$$L_n(x) = e^x D^n e^{-x} x^n, \quad L_0 = 1, \; n \in \mathbb{N}_0. \tag{1.3}$$

Recall that Bell polynomials and Laguerre polynomials are classical mathematical tools for representing the nth derivative of a composite functions. Moreover, the multidimensional polynomials of higher order are already defined, which are suitable to represent the derivative of a composite function of several variables (see [6]).

In this paper, we introduce definitions for these polynomials of arbitrary order (fractional order) in complex plane.

In [12] the definitions for fractional operators (derivative and integral) in the complex z-plane $\mathbb{C}$ are given as follows.

Definition 1.3. The fractional derivative of order α is defined, for a function $f(z)$, by

$$D_z^\alpha f(z) := \frac{1}{\Gamma(1-\alpha)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\alpha} d\zeta, \quad 0 \leq \alpha < 1, \tag{1.4}$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane $\mathbb{C}$ containing the origin and the multiplicity of $(z-\zeta)^{-\alpha}$ is removed by requiring $\log(z-\zeta)$ to be real when $(z-\zeta) > 0$. For $\alpha \in [n-1, n)$ and $n = 1, 2, \ldots$,

$$D_z^\alpha f(z) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dz^n} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\alpha} d\zeta. \tag{1.5}$$

Definition 1.4. The fractional integral of order α is defined, for a function $f(z)$, by

$$I_z^\alpha f(z) := \frac{1}{\Gamma(\alpha)} \int_0^z f(\zeta)(z-\zeta)^{\alpha-1} d\zeta, \quad \alpha \geq 0, \tag{1.6}$$

where the function $f(z)$ is analytic in simply-connected region of the complex z-plane ($\mathbb{C}$) containing the origin and the multiplicity of $(z-\zeta)^{\alpha-1}$ is removed by requiring $\log(z-\zeta)$ to be real when $(z-\zeta) > 0$. Further details in fractional calculus can be found in [13].

Remark 1.5. From Definition 1.3, we have $D_z^0 f(z) = f(0)$, $\lim_{\alpha\to 0} I_z^\alpha f(z) = f(z)$, and $\lim_{\alpha\to 0} D_z^{1-\alpha} f(z) = f'(z)$. Moreover,

$$D_z^\alpha\{z^\mu\} = \frac{\Gamma(\mu+1)}{\Gamma(\mu-\alpha+1)}\{z^{\mu-\alpha}\}, \quad \mu > -1,\ 0 \le \alpha < 1,$$
$$I_z^\alpha\{z^\mu\} = \frac{\Gamma(\mu+1)}{\Gamma(\mu+\alpha+1)}\{z^{\mu+\alpha}\}, \quad \mu > -1,\ \alpha \ge 0,\ z \ne 0. \tag{1.7}$$

Lemma 1.6 (see [14]). *For $\alpha \in [0,1)$ and f is a continuous function, then*

$$DI_z^\alpha f(z) = \frac{(z)^{\alpha-1}}{\Gamma(\alpha)} f(0) + I_z^\alpha D f(z); \quad D = \frac{d}{dz}. \tag{1.8}$$

By using the operators (1.5) *and* (1.6), *we define generalised polynomials in complex z-plane.*

Definition 1.7. Let $\alpha \in [n-1, n)$ and $n = 1, 2, \ldots$. The generalised Bell polynomials of order α and $-\alpha$ are

$$B_\alpha(z) = e^{-z} D_z^\alpha e^z, \tag{1.9}$$

$$B_{-\alpha}(z) = e^{-z} I_z^\alpha e^z, \tag{1.10}$$

respectively.

Definition 1.8. Let $\alpha \in [n-1, n)$ and $n = 1, 2, \ldots$. The generalised Laguerre polynomials of order α and $-\alpha$ are

$$L_\alpha(z) = e^z D_z^\alpha e^{-z} z^m\ , \tag{1.11}$$

$$L_{-\alpha}(z) = e^z I_z^\alpha e^{-z} z^m, \quad m \in \mathbb{N}_0, \tag{1.12}$$

respectively.

Our plan is as follows. In Section 2, we study the recurrence relations of the polynomials (1.9)–(1.12), the other three sections, we introduce the Laguerre-type of some special functions.

2. Recurrence Relations

In this section, we introduce some recurrence relations for the generalised Bell polynomials and Laguerre polynomials.

Theorem 2.1. *Let $\alpha \in [0,1)$. Then the generalised Bell polynomials of order α and $-\alpha$ satisfy*

$$\begin{aligned}
&(1)\quad DB_{\alpha}(z) = B_{\alpha+1}(z) - B_{\alpha}(z),\\
&(2)\quad DB_{-\alpha}(z) = B_{1-\alpha}(z) - B_{-\alpha}(z) = \frac{z^{\alpha-1}e^{-z}}{\Gamma(\alpha)},
\end{aligned} \tag{2.1}$$

where $D := d/dz$.

Proof. Let $\alpha \in [0,1)$, then we have

$$\begin{aligned}
(1)\quad DB_{\alpha}(z) &= D\left[e^{-z}D_z^{\alpha}e^{z}\right]\\
&= e^{-z}D[D_z^{\alpha}e^{z}] - e^{-z}D_z^{\alpha}e^{z}\\
&= e^{-z}D_z^{\alpha+1}e^{z} - e^{-z}D_z^{\alpha}e^{z}\\
&= B_{\alpha+1}(z) - B_{\alpha}(z),\\
(2)\quad DB_{-\alpha}(z) &= D\left[e^{-z}I_z^{\alpha}e^{z}\right]\\
&= e^{-z}D[I_z^{\alpha}e^{z}] - e^{-z}I_z^{\alpha}e^{z}\\
&= e^{-z}I_z^{\alpha-1}e^{z} - e^{-z}I_z^{\alpha}e^{z}\\
&= B_{1-\alpha}(z) - B_{-\alpha}(z).
\end{aligned} \tag{2.2}$$

On the other hand and in virtue of Lemma 1.6, we have

$$\begin{aligned}
DB_{-\alpha}(z) &= D\left[e^{-z}I_z^{\alpha}e^{z}\right]\\
&= e^{-z}D[I_z^{\alpha}e^{z}] - e^{-z}I_z^{\alpha}e^{z}\\
&= e^{-z}\left[\frac{(z)^{\alpha-1}}{\Gamma(\alpha)} + I_z^{\alpha}e^{z}\right] - e^{-z}I_z^{\alpha}e^{z}\\
&= \frac{z^{\alpha-1}e^{-z}}{\Gamma(\alpha)}.
\end{aligned} \tag{2.3}$$

□

Theorem 2.2. *Let $\alpha \in [0,1)$. Then the generalised Laguerre polynomials of order α and $-\alpha$ satisfy*

$$\begin{aligned}
&(1)\quad DL_{\alpha}(z) = L_{\alpha+1}(z) + L_{\alpha}(z),\\
&(2)\quad DL_{-\alpha}(z) = L_{1-\alpha}(z) + L_{\alpha}(z),\\
&(3)\quad DL_{-\alpha}(z) = \frac{m}{\Gamma(\alpha)}z^{m+\alpha-1}, \quad z \neq 0,
\end{aligned} \tag{2.4}$$

where $D := d/dz$.

Proof. Let $\alpha \in [0,1)$, then we have

$$\begin{aligned}
(1)\quad DL_{\alpha}(z) &= D\left[e^{z}D_{z}^{\alpha}e^{-z}z^{m}\right] \\
&= e^{z}D\left[D_{z}^{\alpha}e^{-z}z^{m}\right] + e^{z}D_{z}^{\alpha}e^{-z}z^{m} \\
&= e^{z}\left[D_{z}^{\alpha+1}e^{-z}z^{m}\right] + L_{\alpha}(z) \\
&= L_{\alpha+1}(z) + L_{\alpha}(z), \\
(2)\quad DL_{-\alpha}(z) &= D\left[e^{z}I_{z}^{\alpha}e^{-z}z^{m}\right] \\
&= e^{z}D\left[I_{z}^{\alpha}e^{-z}z^{m}\right] + e^{z}I_{z}^{\alpha}e^{-z}z^{m} \\
&= e^{z}\left[I_{z}^{\alpha-1}e^{-z}z^{m}\right] + L_{\alpha}(z) \\
&= L_{1-\alpha}(z) + L_{\alpha}(z).
\end{aligned} \tag{2.5}$$

For $z \neq 0$ and in view of Lemma 1.6, we have

$$\begin{aligned}
(3)\quad DL_{-\alpha}(z) &= D\left[e^{z}I_{z}^{\alpha}e^{-z}z^{m}\right] \\
&= e^{z}D\left[I_{z}^{\alpha}e^{-z}z^{m}\right] + e^{z}I_{z}^{\alpha}e^{-z}z^{m} \\
&= e^{z}\left[I_{z}^{\alpha}De^{-z}z^{m}\right] + e^{z}I_{z}^{\alpha}e^{-z}z^{m} \\
&= \frac{m}{\Gamma(\alpha)}z^{m+\alpha-1}.
\end{aligned} \tag{2.6}$$

□

In addition, we have the following results.

Theorem 2.3. *Let $\alpha \in [0,1)$. Then the generalised Bell polynomials $B_{\alpha}(z)$ are univalent solutions for the ordinary differential equation*

$$D^{2}B_{\alpha}(z) + 2DB_{\alpha}(z) + B_{\alpha}(z) = \rho_{\alpha}(z), \quad z \neq 0, \tag{2.7}$$

where $\rho_{\alpha}(z) := (\alpha e^{-z}z^{\alpha-1}[(\alpha+1)z^{-1} - 1] + e^{-z}z^{-\alpha} + z^{-\alpha})/\Gamma(1-\alpha)$.

Proof. Differentiating $DB_{\alpha}(z)$ in Theorem 2.1 (part 1), using the fact that $B_{\alpha+1}(z) = DB_{\alpha}(z) + B_{\alpha}(z)$ and using the properties in Lemma 1.6, into it, we obtain the result. Now for $z_1 \neq 0, z_2 \neq 0$ such that $z_1 \neq z_2$ and by applying Remark 1.5 on (1.9), we can verify that $B_{\alpha}(z)$ are univalent functions. □

Theorem 2.4. *Let $\alpha \in [0,1)$. Then the generalised Laguerre polynomials $L_{\alpha}(z)$ are univalent solutions for the ordinary differential equation*

$$D^{2}L_{\alpha}(z) + 2DL_{\alpha}(z) + L_{\alpha}(z) = \theta_{\alpha}(z), \quad z \neq 0, \tag{2.8}$$

where $\theta_{\alpha}(z) := e^{z}[(z^{-\alpha}/\Gamma(1-\alpha))(me^{-z}z^{m-1} - e^{-z}z^{m})]''$.

Proof. Differentiating $DL_\alpha(z)$ in Theorem 2.2 (part 1), using the fact that $L_{\alpha+1}(z) = DL_\alpha(z) - L_\alpha(z)$ and again using Lemma 1.6, into it, we obtain the result. Now for $z_1 \neq 0, z_2 \neq 0$ such that $z_1 \neq z_2$ and by applying Remark 1.5 on (1.11), we obtain that $L_\alpha(z)$ are univalent functions. □

3. Laguerre-Type Mittag-Leffler Function

In this section, the fractional Laguerre-type derivatives $\mathfrak{D}_L$ introduced and in connection with a fractional differential isomorphism denoted by the symbol $\mathfrak{D}_z^{-\beta}$, acting onto the space $\mathcal{A}$ of analytic functions of the z variable given as follows:

$$D := \frac{d}{dz} \longrightarrow \mathfrak{D}_L, \quad z \longrightarrow \mathfrak{D}_z^{-1}, \tag{3.1}$$

where

$$\mathfrak{D}_z^{-1} f(z) = \int_0^z f(\zeta) d\zeta. \tag{3.2}$$

In general,

$$\mathfrak{D}_z^{-\beta} f(z) := \frac{1}{\Gamma(\beta)} \int_0^z f(\zeta)(z-\zeta)^{\beta-1} d\zeta, \quad \beta \geq 0, \tag{3.3}$$

so that

$$\begin{aligned} F_0\left(z^\beta\right) &:= \mathfrak{D}_z^{-\beta}(1) = \int_0^z (z-\zeta)^{\beta-1} d\zeta = \frac{z^\beta}{\Gamma(\beta+1)}, \\ F_n\left(z^\beta\right) &= \mathfrak{D}_z^{-\beta}(z^n) = \frac{1}{\Gamma(\beta)} \int_0^z \zeta^n (z-\zeta)^{\beta-1} d\zeta = \frac{\Gamma(n+1)}{\Gamma(n+\beta+1)} z^{n+\beta}. \end{aligned} \tag{3.4}$$

According to this isomorphism, the Mittag-Leffler operator $E_\lambda(z)$ (see [15])

$$E_\lambda(z) := \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\lambda n+1)}, \quad \lambda > 0, \tag{3.5}$$

is transformed into the first Laguerre-type $E_\lambda^1(z)$

$$F_0(E_\lambda(z)) = \sum_{n=0}^{\infty} \frac{F_0(z^n)}{\Gamma(\lambda n+1)} = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n+1)\Gamma(\lambda n+1)} := E_\lambda^1(z). \tag{3.6}$$

This result can be generalised by considering the k Laguerre-type Mittag-Leffler

$$F_0^k(E_\lambda(z)) = \sum_{n=0}^{\infty} \frac{F_0(z^n)}{[\Gamma(\lambda n+1)]^k} = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n+1)[\Gamma(\lambda n+1)]^k} := E_\lambda^k(z). \tag{3.7}$$

Thus

$$\mathfrak{D}_L^k E_\lambda^k(az) = aE_\lambda^k(az), \quad a \in \mathbb{C}. \tag{3.8}$$

Note that when $\lambda = 1$ this reduces to exponential function (see [4]).

4. Laguerre-Type Hypergeometric Function

We use the same method of the previous section to obtain the Laguerre-type hypergeometric function

$${}_qF_p(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_p;z) = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n\cdots(\alpha_q)_n}{(\beta_1)_n\cdots(\beta_p)_n}\frac{z^n}{n!}, \tag{4.1}$$

where $(a)_n$ is the Pochhammer symbol defined by

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} 1, & n = 0, \\ a(a+1)\cdots(a+n-1), & n = \{1,2,\ldots\}. \end{cases} \tag{4.2}$$

According to the previous definition of Laguerre fractional derivative, the hypergeometric function ${}_qF_p(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_p;z)$ is transformed into the first Laguerre-type ${}_qF_p^1(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_p;z)$

$$\begin{aligned} \mathbf{F}_0\left({}_qF_p(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_p;z)\right) &= \sum_{n=0}^{\infty} \frac{(\alpha_1)_n\cdots(\alpha_q)_n}{(\beta_1)_n\cdots(\beta_p)_n}\frac{\mathbf{F}_0(z^n)}{\Gamma(n+1)} \\ &= \sum_{n=0}^{\infty} \frac{(\alpha_1)_n\cdots(\alpha_q)_n}{(\beta_1)_n\cdots(\beta_p)_n}\frac{z^n}{[\Gamma(n+1)]^2} \\ &= {}_qF_p^1(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_p;z). \end{aligned} \tag{4.3}$$

For k order we have

$$\begin{aligned} \mathbf{F}_0^k\left({}_qF_p(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_p;z)\right) &= \sum_{n=0}^{\infty} \left[\frac{(\alpha_1)_n\cdots(\alpha_q)_n}{(\beta_1)_n\cdots(\beta_p)_n}\right]^k \frac{\mathbf{F}_0(z^n)}{[\Gamma(n+1)]^k} \\ &= \sum_{n=0}^{\infty} \left[\frac{(\alpha_1)_n\cdots(\alpha_q)_n}{(\beta_1)_n\cdots(\beta_p)_n}\right]^k \frac{z^n}{[\Gamma(n+1)]^{k+1}} \\ &= {}_qF_p^k(\alpha_1,\ldots,\alpha_q;\beta_1,\ldots,\beta_p;z) \end{aligned} \tag{4.4}$$

the Laguerre-type hypergeometric function.

5. Laguerre-Type Fox-Wright Function

Lastly, we introduce the Laguerre-type Fox-Wright function by using the similar approach in Section 3. For complex parameters

$$\alpha_1,\ldots,\alpha_q \quad \left(\frac{\alpha_j}{A_j}\neq 0,\ -1,-2,\ldots;\ j=1,\ldots,q\right),$$
$$\beta_1,\ldots,\beta_p \quad \left(\frac{\beta_j}{B_j}\neq 0,\ -1,-2,\ldots;\ j=1,\ldots,p\right), \tag{5.1}$$

We have the Fox-Wright generalisation ${}_q\Psi_p[z]$ of the hypergeometric ${}_qF_p$ function by (see [16–18])

$$\begin{aligned}
{}_q\Psi_p\left[\begin{matrix}(\alpha_1,A_1),\ldots,(\alpha_q,A_q);\\(\beta_1,B_1),\ldots,(\beta_p,B_p);\end{matrix}\ z\right] &= {}_q\Psi_p\left[(\alpha_j,A_j)_{1,q};(\beta_j,B_j)_{1,p};z\right]\\
&:= \sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1+nA_1)\cdots\Gamma(\alpha_q+nA_q)}{\Gamma(\beta_1+nB_1)\cdots\Gamma(\beta_p+nB_p)}\frac{z^n}{n!}\\
&= \sum_{n=0}^{\infty}\frac{\prod_{j=1}^{q}\Gamma(\alpha_j+nA_j)}{\prod_{j=1}^{p}\Gamma(\beta_j+nB_j)}\frac{z^n}{n!},
\end{aligned} \tag{5.2}$$

where $A_j>0$ for all $j=1,\ldots,q$, $B_j>0$ for all $j=1,\ldots,p$, and $1+\sum_{j=1}^{p}B_j-\sum_{j=1}^{q}A_j\geq 0$ for suitable values $|z|$. The Laguerre-type ${}_q\Psi_p[(\alpha_j,A_j)_{1,q};(\beta_j,B_j)_{1,p};z]$ is

$$\begin{aligned}
\mathbf{F}_0\left({}_q\Psi_p\left[(\alpha_j,A_j)_{1,q};(\beta_j,B_j)_{1,p};z\right]\right) &= \sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1+nA_1)\cdots\Gamma(\alpha_q+nA_q)}{\Gamma(\beta_1+nB_1)\cdots\Gamma(\beta_p+nB_p)}\frac{\mathbf{F}_0(z^n)}{\Gamma(n+1)}\\
&= \sum_{n=0}^{\infty}\frac{\Gamma(\alpha_1+nA_1)\cdots\Gamma(\alpha_q+nA_q)}{\Gamma(\beta_1+nB_1)\cdots\Gamma(\beta_p+nB_p)}\frac{z^n}{[\Gamma(n+1)]^2}\\
&= {}_q\Psi_p^1\left[(\alpha_j,A_j)_{1,q};(\beta_j,B_j)_{1,p};z\right].
\end{aligned} \tag{5.3}$$

For k order we have

$$\begin{aligned}
\mathbf{F}_0^k\left({}_q\Psi_p\left[(\alpha_j,A_j)_{1,q};(\beta_j,B_j)_{1,p};z\right]\right) &= \sum_{n=0}^{\infty}\left[\frac{\Gamma(\alpha_1+nA_1)\cdots\Gamma(\alpha_q+nA_q)}{\Gamma(\beta_1+nB_1)\cdots\Gamma(\beta_p+nB_p)}\right]^k\frac{\mathbf{F}_0(z^n)}{[\Gamma(n+1)]^k}\\
&= \sum_{n=0}^{\infty}\left[\frac{\Gamma(\alpha_1+nA_1)\cdots\Gamma(\alpha_q+nA_q)}{\Gamma(\beta_1+nB_1)\cdots\Gamma(\beta_p+nB_p)}\right]^k\frac{z^n}{[\Gamma(n+1)]^{k+1}}\\
&= {}_q\Psi_p^k\left[(\alpha_j,A_j)_{1,q};(\beta_j,B_j)_{1,p};z\right],
\end{aligned} \tag{5.4}$$

the Laguerre-type Fox-Wright function.

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