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A GENERALIZATION OF AUXILIARY PROBLEM PRINCIPLE WITH APPLICATIONS TO VARIATIONAL INEQUALITIES

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We announce the approximation-solvability of the following class of nonlinear variational inequality (NVI) problems based on a new generalized auxiliary problem principle:

Find an element $x^* \in K$ such that

$$\langle T(x^*), x - x^*\rangle + f(x) - f(x^*) \geq 0 \text{ for all } x \in K,$$

where $T\colon K \to H$ is a mapping from a nonempty closed convex subset K of a real Hilbert space H into H, and $f\colon K \to R$ is a continuous convex functional on K.

The generalized auxiliary problem principle is described as follows: for a given iterate $x^* \in K$ and, for constants $\rho > 0$ and $\sigma > 0$, compute x^{k+1} such that

$$\langle \rho T(y^k) + h'(x^{k+1}) - h'(h^k), x - x^{k+1}\rangle + \rho[f(x) - f(x^{k+1})] \geq 0$$

for all $x \in K$ and for $k \geq 0$, where

$$\langle \sigma T(x^k) + h'(y^k) - h'(x^k), x - y^k\rangle + \sigma[f(x) - f(y^k)] \geq 0 \text{ for all } x \in K,$$

where $h\colon K \to R$ is twice Frechet-differential functional on K.

Theorem: *Let H be a real Hilbert space and $T\colon K \to H$ a γ-μ-partially relaxed monotone mapping from a nonempty closed convex subset K of H into H. Let $h\colon K \to R$ be twice continuous Frechet-differentiable on K with the following assumptions*:

$$\langle h''(x) - h''(y), (x-y)^2\rangle \geq 0$$

and

$$\| h''(x) \| \geq b.$$

Then for any fixed solution $x^* \in K$ of the NVI problem, the sequence $\{x^k\}$ is bounded and converges to x^* for

$$0 < \sigma < 2b/\gamma.$$

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