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Research Article

Random Three-Step Iteration Scheme and Common Random Fixed Point of Three Operators

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We construct random iterative processes with errors for three asymptotically nonexpansive random operators and study necessary conditions for the convergence of these processes. The results presented in this paper extend and improve the recent ones announced by I. Beg and M. Abbas (2006), and many others.

1. Introduction

Probabilistic functional analysis has come out as one of the momentous mathematical disciplines in view of its requirements in dealing with probabilistic models in applied problems. The study of random fixed points forms a central topic in this area. Random fixed point theorems for random contraction mappings on separable complete metric spaces were first proven by Špaček [1]. Subsequently, Bharucha-Reid [2] has given sufficient conditions for a stochastic analog of Schauder's fixed point theorem for a random operator. The study of random fixed point theorems was initiated by Špaček [1] and Hanš [3, 4]. In an attempt to construct iterations for finding fixed points of random operators defined on linear spaces, random Ishikawa scheme was introduced in [5]. This iteration and also some other random iterations based on the same ideas have been applied for finding solutions of random operator equations and fixed points of random operators (see [5]).

Recently, Beg [6], Choudhury [7], Duan and Li [8], Li and Duan [9], Itoh [10], and many others have studied the fixed point of random operators. Beg and Abbas [11] studied the different random iterative algorithms for weakly contractive and asymptotically nonexpansive random operators on arbitrary Banach spaces. They also established the convergence of an implicit random iterative process to a common random fixed point for a finite family of asymptotically quasi-nonexpansive operators.

More recently, Plubtieng et al. [12] studied weak and strong convergence theorems established for a modified Noor iterative scheme with errors for three asymptotically nonexpansive mappings in Banach spaces.

In this paper, we study the convergence of three-step random iterative processes with errors for three asymptotically nonexpansive random operators in Banach spaces. Our results extend and improve the corresponding ones announced by Beg and Abbas [11], and many others.

2. Preliminaries

Let (Ω,Σ) be a measurable space with Σ a sigma-algebra of subsets of Ω and let C be a nonempty subset of a Banach space X. A mapping $\xi:\Omega\to X$ is *measurable* if $\xi^{-1}(U)\in\Sigma$ for each open subset U of X. The mapping $T:\Omega\times C\to C$ is a *random map* if for each fixed $x\in C$, the mapping $T(\cdot,x):\Omega\to C$ is measurable, and it is *continuous* if for each $\omega\in\Omega$, the mapping $T(\omega,\cdot):C\to X$ is continuous. A measurable mapping $\xi:\Omega\to X$ is the *random fixed point* of the random map $T:\Omega\times C\to X$ if $T(\omega,\xi(\omega))=\xi(\omega)$, for each $\omega\in\Omega$. We denote by $\mathrm{RF}(T)$ the set of all random fixed points of a random map T and by $T^n(\omega,x)$ the nth iterate $T(\omega,T(\omega,T(,\ldots,T(\omega,x))))$ of T. The letter I denotes the random mapping $I:\Omega\times C\to C$ defined by $I(\omega,x)=x$ and $T^0=I$.

Definition 2.1. Let C be a nonempty subset of a separable Banach space X and let $T:\Omega\times C\to C$ be a random map. The map T is said to be

(a) a *nonexpansive random operator* if arbitrary $x,y\in C$, one has

$$\left|\left|T(\omega,x)-T(\omega,y)\right|\right|\le\|x-y\|, \tag{2.1}$$

for each $\omega\in\Omega$;

(b) an *asymptotically nonexpansive random operator* if there exists a sequence of measurable mappings $r_n:\Omega\to[0,\infty)$ with $\lim_{n\to\infty}r_n(\omega)=0$, for each $\omega\in\Omega$, such that for arbitrary $x,y\in C$,

$$\left|\left|T^n(\omega,x)-T^n(\omega,y)\right|\right|\le\left(1+r_n(\omega)\right)\|x-y\|,\quad\text{for each }\omega\in\Omega; \tag{2.2}$$

(c) a *uniformly L-Lipschitzian random operator* if arbitrary $x,y\in C$, one has

$$\left|\left|T^n(\omega,x)-T^n(\omega,y)\right|\right|\le L\|x-y\|, \tag{2.3}$$

where $n=1,2,\ldots,$ and L is a positive constant;

(d) a *semicompact random operator* if for a sequence of measurable mappings $\{\xi_n\}$ from Ω to C, with $\lim_{n\to\infty}\|\xi_n(\omega)-T(\omega,\xi_n(\omega))\|=0$, for every $\omega\in\Omega$, one has a subsequence $\{\xi_{n_k}\}$ of $\{\xi_n\}$ and a measurable mapping $\xi:\Omega\to C$ such that $\{\xi_{n_k}\}$ converges pointwisely to ξ as $k\to\infty$.

Definition 2.2 (three-step random iterative process, cf. [11]). Let $T:\Omega\times C\to C$ is a random operator, where C is a nonempty convex subset of a separable Banach space X. Let $\xi_0:\Omega\to C$ be a measurable mapping from Ω to C. Define sequence of functions $\{\zeta_n\}$, $\{\eta_n\}$, and $\{\xi_n\}$, as given below:

$$\begin{aligned}\zeta_n(\omega)&=\alpha_n''T^n(\omega,\xi_n(\omega))+\beta_n''\xi_n(\omega),\\ \eta_n(\omega)&=\alpha_n'T^n(\omega,\zeta_n(\omega))+\beta_n'\xi_n(\omega),\\ \xi_{n+1}(\omega)&=\alpha_nT^n(\omega,\eta_n(\omega))+\beta_n\xi_n(\omega)\quad\text{for each }\omega\in\Omega,\end{aligned}\tag{2.4}$$

$n=0,1,2,\ldots$, where $\{\alpha_n\}$, $\{\alpha_n'\}$, $\{\alpha_n''\}$, $\{\beta_n\}$, $\{\beta_n'\}$, and $\{\beta_n''\}$ are sequences of real numbers in $[0,1]$. Obviously $\{\zeta_n\}$, $\{\eta_n\}$, and $\{\xi_n\}$ are sequences of measurable functions from Ω to C.

Definition 2.3. Let $T_1,T_2,T_3:\Omega\times C\to C$ be three random operators, where C is a nonempty convex subset of a separable Banach space X. Let $\xi_0:\Omega\to C$ be a measurable mapping from Ω to C, let $\{f_n\}$, $\{f_n'\}$, $\{f_n''\}$ be bounded sequences of measurable functions from Ω to C. Define sequences of functions $\{\zeta_n\}$, $\{\eta_n\}$, and $\{\xi_n\}$, as given below:

$$\begin{aligned}\zeta_n(\omega)&=\alpha_n''T_3^n(\omega,\xi_n(\omega))+\beta_n''\xi_n(\omega)+\gamma_n''f_n''(\omega),\\ \eta_n(\omega)&=\alpha_n'T_2^n(\omega,\zeta_n(\omega))+\beta_n'\xi_n(\omega)+\gamma_n'f_n'(\omega),\\ \xi_{n+1}(\omega)&=\alpha_nT_1^n(\omega,\eta_n(\omega))+\beta_n\xi_n(\omega)+\gamma_nf_n(\omega)\quad\text{for each }\omega\in\Omega,\end{aligned}\tag{2.5}$$

$n=0,1,2,\ldots$, where $\{\alpha_n\}$, $\{\alpha_n'\}$, $\{\alpha_n''\}$, $\{\beta_n\}$, $\{\beta_n'\}$, $\{\beta_n''\}$, $\{\gamma_n\}$, $\{\gamma_n'\}$, and $\{\gamma_n''\}$ are sequences of real numbers in $[0,1]$ with $\alpha_n+\beta_n+\gamma_n=\alpha_n'+\beta_n'+\gamma_n'\alpha_n''+\beta_n''+\gamma_n''=1$.

Remark 2.4. If we take $T_1=T_2=T_3\equiv T$, and $\gamma_n=\gamma_n'=\gamma_n''\equiv 0$, then (2.5) reduces to (2.4).

The purpose of this paper is to establish several convergence results of the three-step random iterative process with errors given in (2.5) for three asymptotically nonexpansive random operators.

In the sequel, we will need the following lemma.

LEMMA 2.5 [13, Lemma 1.3]. *Let X be a uniformly convex Banach space with $x_n,y_n\in X$, real numbers $a\ge 0$, $\alpha,\beta\in(0,1)$, and let $\{\alpha_n\}$ be a real sequence of numbers which satisfies*

(i) $0<\alpha\le\alpha_n\le\beta<1$, *for all $n\ge n_0$ and for some $n_0\in\mathbb{N}$;*
(ii) $\limsup_{n\to\infty}\|x_n\|\le a$ *and* $\limsup_{n\to\infty}\|y_n\|\le a$;
(iii) $\lim_{n\to\infty}\|\alpha_nx_n+(1-\alpha_n)y_n\|=a$.
Then $\lim_{n\to\infty}\|x_n-y_n\|=0$.

3. Main results

In this section, we investigate the convergence of three-step random iterative process with errors for three asymptotically nonexpansive random operators to obtain the random solution of the common random fixed point. This iterative process includes three-step random iterative process for a random operator T as special case. Note that the proof given below is different form the method of the proof proved by Beg and Abbas [11]. In order to prove our main results, we need the following two lemmas.

LEMMA 3.1. *Let X be a uniformly convex separable Banach space, and let C be a nonempty closed and convex subset of X. Let T_1, T_2, T_3 be asymptotically nonexpansive random operators from $\Omega \times C$ to C with sequence of measurable mappings $r_{i_n}(\omega) : \Omega \to [0,\infty)$ satisfying $\sum_{n=1}^{\infty} r_{i_n}(\omega) < \infty$, for each $\omega \in \Omega$ and for all $i = 1,2,3$, and $F = \bigcap_{i=1}^{3} \mathrm{RF}(T_i) \neq \emptyset$. Let $\{\xi_n(\omega)\}$ be the sequence as defined by (2.5) with $\sum_{n=1}^{\infty} \gamma_n < \infty$, $\sum_{n=1}^{\infty} \gamma_n' < \infty$, and $\sum_{n=1}^{\infty} \gamma_n'' < \infty$. Then $\lim_{n\to\infty} \|\xi_n(\omega) - \xi(\omega)\|$ exists for all $\xi(\omega) \in F$ and for each $\omega \in \Omega$.*

Proof. Let $\xi : \Omega \to C$ be the random common fixed point of $\{T_1, T_2, T_3\}$. Since $\{f_n\}$, $\{f_n'\}$, and $\{f_n''\}$ are bounded sequences of measurable functions from Ω to C, we can put

$$M(\omega) = \sup_{n\geq 1} ||f_n(\omega) - \xi(\omega)|| \vee \sup_{n\geq 1} ||f_n'(\omega) - \xi(\omega)|| \vee \sup_{n\geq 1} ||f_n''(\omega) - \xi(\omega)||. \tag{3.1}$$

Then $M(\omega)$ is a finite number for each $\omega \in \Omega$. For each $n \geq 1$, let $r_n(\omega) = \max\{r_{i_n}(\omega) \mid i = 1,2,3\}$. Thus, we have $r_n(\omega) \geq 0$, $\lim_{n\to 0} r_{i_n}(\omega) = 0$, and

$$\begin{aligned}
||\xi_{n+1}(\omega) - \xi(\omega)|| &= ||\alpha_n T_1^n(\omega, \eta_n(\omega)) + \beta_n \xi_n(\omega) + \gamma_n f_n(\omega) - \xi(\omega)|| \\
&= \alpha_n ||T_1^n(\omega, \eta_n(\omega)) - \xi(\omega)|| + \beta_n ||\xi_n(\omega) - \xi(\omega)|| + \gamma_n ||f_n(\omega) - \xi(\omega)|| \\
&\leq \alpha_n (1 + r_n(\omega)) ||\eta_n(\omega) - \xi(\omega)|| + \beta_n ||\xi_n(\omega) - \xi(\omega)|| + \gamma_n ||f_n(\omega) - \xi(\omega)||.
\end{aligned} \tag{3.2}$$

Similarly, we have

$$||\eta_n(\omega) - \xi(\omega)|| \leq \alpha_n' (1 + r_n(\omega)) ||\zeta_n(\omega) - \xi(\omega)|| + \beta_n' ||\xi_n(\omega) - \xi(\omega)|| + \gamma_n' ||f_n'(\omega) - \xi(\omega)||, \tag{3.3}$$

$$||\zeta_n(\omega) - \xi(\omega)|| \leq \alpha_n'' (1 + r_n(\omega)) ||\xi_n(\omega) - \xi(\omega)|| + \beta_n'' ||\xi_n(\omega) - \xi(\omega)|| + \gamma_n'' ||f_n''(\omega) - \xi(\omega)||. \tag{3.4}$$

Substituting (3.4) in (3.3), we get

$$\begin{aligned}
&\|\eta_n(\omega)-\xi(\omega)\| \\
&\quad\le \alpha_n'\alpha_n''(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\|+\alpha_n'\beta_n''(1+r_n(\omega))\|\xi_n(\omega)-\xi(\omega)\| \\
&\qquad+\alpha_n'\gamma_n''(1+r_n(\omega))\|f_n''(\omega)-\xi(\omega)\|+\beta_n'\|\xi_n(\omega)-\xi(\omega)\|+\gamma_n'\|f_n'(\omega)-\xi(\omega)\| \\
&\quad=(1-\beta_n'-\gamma_n')\alpha_n''(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\|+\beta_n'\|\xi_n(\omega)-\xi(\omega)\| \\
&\qquad+(1-\beta_n'-\gamma_n')\beta_n''(1+r_n(\omega))\|\xi_n(\omega)-\xi(\omega)\|+m_n(\omega) \\
&\quad\le (1-\beta_n')\alpha_n''(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\|+\beta_n'(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\| \\
&\qquad+(1-\beta_n')\beta_n''(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\|+m_n(\omega) \\
&\quad\le (1-\beta_n')(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\| \\
&\qquad+\beta_n'(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\|+m_n(\omega) \\
&\quad=(1+r_n(\omega))^2\|\xi_n(\omega)-\xi(\omega)\|+m_n(\omega),
\end{aligned} \tag{3.5}$$

where

$$m_n(\omega)=\alpha_n'\gamma_n''(1+r_n(\omega))\|f_n''(\omega)-\xi(\omega)\|+\gamma_n'\|f_n'(\omega)-\xi(\omega)\|. \tag{3.6}$$

Note that $\sum_{n=1}^{\infty} m_n(\omega)<\infty$. Substituting (3.5) in (3.2), we have

$$\begin{aligned}
\|\xi_{n+1}(\omega)-\xi(\omega)\| &\le \alpha_n(1+r_n(\omega))^3\|\xi_n(\omega)-\xi(\omega)\|+\alpha_n(1+r_n(\omega))m_n(\omega) \\
&\quad+\beta_n\|\xi_n(\omega)-\xi(\omega)\|+\gamma_n\|f_n(\omega)-\xi(\omega)\| \\
&\le (\alpha_n+\beta_n)(1+r_n(\omega))^3\|\xi_n(\omega)-\xi(\omega)\|+b_n(\omega) \\
&=(1+r_n(\omega))^3\|\xi_n(\omega)-\xi(\omega)\|+b_n(\omega),
\end{aligned} \tag{3.7}$$

where

$$b_n(\omega)=\alpha_n(1+r_n(\omega))m_n(\omega)+\gamma_n\|f_n(\omega)-\xi(\omega)\|. \tag{3.8}$$

Since

$$\sum_{n=1}^{\infty} r_n(\omega)<\infty, \qquad \sum_{n=1}^{\infty} b_n(\omega)<\infty, \tag{3.9}$$

it follows from [10, Lemma 2] that $\lim_{n\to\infty}\|\xi_{n+1}(\omega)-\xi(\omega)\|$ exists for all $\omega\in\Omega$. □

LEMMA 3.2. *Let X be a uniformly convex separable Banach space, and let C be a nonempty closed and convex subset of X. Let T_1, T_2, T_3 be asymptotically nonexpansive random operators from Ω to C with sequence of measurable mappings $r_{i_n}(\omega) : \Omega \to [0,\infty)$ satisfying $\sum_{n=1}^{\infty} r_{i_n}(\omega) < \infty$, for each $\omega \in \Omega$ and for all $i = 1,2,3$, and $F = \bigcap_{i=1}^{3} \mathrm{RF}(T_i) \neq \emptyset$. Let $\{\xi_n(\omega)\}$ be the sequence defined as in (2.5) with the following restrictions:*

(1) $0 < \alpha \leq \alpha_n, \alpha'_n, \alpha''_n \leq 1-\alpha$, *for some* $\alpha \in (0,1)$, *for all* $n \geq n_0$, $\exists n_0 \in \mathbb{N}$,

(2) $\sum_{n=1}^{\infty} \gamma_n < \infty$, $\sum_{n=1}^{\infty} \gamma'_n < \infty$, *and* $\sum_{n=1}^{\infty} \gamma''_n < \infty$.

Then

$$\begin{aligned}\lim_{n\to\infty} \|T_1^n(\omega,\eta_n(\omega)) - \xi_n(\omega)\| &= \lim_{n\to\infty} \|T_2^n(\omega,\zeta_n(\omega)) - \xi_n(\omega)\| \\ &= \lim_{n\to\infty} \|T_3^n(\omega,\xi_n(\omega)) - \xi_n(\omega)\| = 0,\end{aligned} \tag{3.10}$$

for all $\omega \in \Omega$.

Proof. Let $\xi(\omega) \in F$. It follows from Lemma 3.1 that $\lim_{n\to\infty} \|\xi_{n+1}(\omega) - \xi(\omega)\|$ exists, for all $\omega \in \Omega$. Let $\lim_{n\to\infty} \|\xi_n(\omega) - \xi(\omega)\| = a$ for some $a \geq 0$. For each $n \geq 1$, let $r_n(\omega) = \max\{r_{i_n}(\omega) \mid i = 1,2,3\}$. Taking the upper limit in inequality (3.5), we obtain that

$$\limsup_{n\to\infty} \|\eta_n(\omega) - \xi(\omega)\| \leq \limsup_{n\to\infty} \|\xi_n(\omega) - \xi(\omega)\| = a. \tag{3.11}$$

So

$$\limsup_{n\to\infty} \|T_1^n(\omega,\eta_n(\omega)) - \xi(\omega)\| \leq \limsup_{n\to\infty} (1 + r_n(\omega)) \|\eta_n(\omega) - \xi(\omega)\| \leq a. \tag{3.12}$$

Next, consider

$$\begin{aligned}&\limsup_{n\to\infty} \|T_1^n(\omega,\eta_n(\omega)) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_n(\omega))\| \\ &\quad\leq \limsup_{n\to\infty} \|T_1^n(\omega,\eta_n(\omega)) - \xi(\omega)\| + \|\gamma_n(f_n(\omega) - \xi_n(\omega))\|.\end{aligned} \tag{3.13}$$

It follows from (3.12) that

$$\limsup_{n\to\infty} \|T_1^n(\omega,\eta_n(\omega)) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_n(\omega))\| \leq a. \tag{3.14}$$

By the triangle inequality,

$$\limsup_{n\to\infty} \|\xi_n(\omega) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_n(\omega))\| \leq a. \tag{3.15}$$

Moreover, we note that

$$\begin{aligned}
a &= \lim_{n\to\infty} ||\xi_{n+1}(\omega) - \xi(\omega)|| \\
&= \lim_{n\to\infty} ||\alpha_n T_1^n(\omega,\eta_n(\omega)) + \beta_n\xi_n(\omega) + \gamma_n(f_n(\omega) - (1-\alpha_n)\xi(\omega) - \alpha_n\xi(\omega))|| \\
&= \lim_{n\to\infty} ||\alpha_n T_1^n(\omega,\eta_n(\omega)) - \alpha_n\xi(\omega) + \alpha_n\gamma_n f_n(\omega) - \alpha_n\gamma_n\xi_n(\omega) \\
&\qquad + (1-\alpha_n)\xi_n(\omega) - (1-\alpha_n)\xi(\omega) - \gamma_n\xi_n(\omega) + \gamma_n f_n(\omega) - \alpha_n\gamma_n f_n(\omega) + \alpha_n\gamma_n\xi_n(\omega)|| \\
&= \lim_{n\to\infty} ||\alpha_n(T_1^n(\omega,\eta_n(\omega)) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_n(\omega))) \\
&\qquad + (1-\alpha_n)(\xi_n(\omega) - \xi(\omega) + \gamma_n(f_n(\omega) - \xi_n(\omega)))||.
\end{aligned} \tag{3.16}$$

It follows by (3.14), (3.15), and Lemma 3.2 that $\lim_{n\to\infty}\|T_1^n(\omega,\eta_n(\omega)) - \xi_n(\omega)\| = 0$. Next, we prove that $\lim_{n\to\infty}\|T_2^n(\omega,\zeta_n(\omega)) - \xi_n(\omega)\| = 0$. For each $n \geq 1$,

$$\begin{aligned}
||\xi_n(\omega) - \xi(\omega)|| &\leq ||T_1^n(\omega,\eta_n(\omega)) - \xi_n(\omega)|| + ||T_1^n(\omega,\eta_n(\omega)) - \xi(\omega)|| \\
&\leq ||T_1^n(\omega,\eta(\omega)) - \xi_n(\omega)|| + (1 + r_n(\omega))||\eta_n(\omega) - \xi_n(\omega)||.
\end{aligned} \tag{3.17}$$

Since $\lim_{n\to\infty}\|T_1^n(\omega,\eta_n(\omega)) - \xi_n(\omega)\| = 0 = \lim_{n\to\infty} r_n(\omega)$, it follows from (3.11) and (3.17) that

$$a = \lim_{n\to\infty} ||\xi_n(\omega) - \xi(\omega)|| \leq \liminf_{n\to\infty} ||\eta_n(\omega) - \xi_n(\omega)|| \leq \limsup_{n\to\infty} ||\eta_n(\omega) - \xi_n(\omega)|| \leq a. \tag{3.18}$$

Hence, $\lim_{n\to\infty}\|\eta_n(\omega) - \xi(\omega)\| = a$. Observe that $\zeta_n(\omega) - \xi(\omega)\| \leq (1 + r_n(\omega))\|\xi_n(\omega) - \xi(\omega)\| + \gamma_n''\|f_n''(\omega) - \xi(\omega)$. By boundedness of $\{f_n''(\omega)\}$ and $\lim_{n\to\infty} r_n(\omega) = 0 = \lim_{n\to\infty}\gamma_n''$, we have $\limsup_{n\to\infty}\|\zeta_n(\omega) - \xi(\omega)\| \leq \limsup_{n\to\infty}\|\xi_n(\omega) - \xi(\omega)\| \leq a$ and so $\limsup_{n\to\infty}\|T_2^n(\omega,\zeta_n(\omega)) - \xi(\omega)\| \leq \limsup_{n\to\infty}(1 + r_n(\omega))\|(\omega,\zeta_n(\omega)) - \xi(\omega)\| \leq a$. Next, we consider

$$\begin{aligned}
&||T_2^n(\omega,\zeta_n(\omega)) - \xi(\omega) + \gamma_n'(f_n'(\omega) - \xi_n(\omega))|| \\
&\qquad \leq ||T_2^n(\omega,\zeta_n(\omega)) - \xi(\omega)|| + \gamma_n'||(f_n'(\omega) - \xi_n(\omega))||.
\end{aligned} \tag{3.19}$$

Taking $\limsup_{n\to\infty}$ in both sides, we have $\limsup_{n\to\infty}\|T_2^n(\omega,\zeta_n(\omega)) - \xi(\omega) + \gamma_n'(f_n'(\omega) - \xi_n(\omega))\| \leq a$. By the triangle inequality, we see that $\limsup_{n\to\infty}\|\xi_n(\omega) - \xi(\omega) + \gamma_n'(f_n'(\omega) - \xi_n(\omega))\| \leq a$. Since $\lim_{n\to\infty}\|\eta_n(\omega) - \xi(\omega)\| = a$, we obtain

$$\begin{aligned}
a &= \lim_{n\to\infty} ||\xi_n(\omega) - \xi(\omega)|| = \lim_{n\to\infty} ||\alpha_n' T_2^n(\omega,\zeta_n(\omega)) + \beta_n'\xi_n(\omega) + \gamma_n' f_n'(\omega) - \xi(\omega)|| \\
&= \lim_{n\to\infty} ||\alpha_n'(T_2^n(\omega,\zeta_n(\omega)) - \xi(\omega) + \gamma_n'(f_n'(\omega) - \xi_n(\omega))) \\
&\qquad + (1-\alpha_n')(\xi_n(\omega) - \xi(\omega) + \gamma_n'(f_n'(\omega) - \xi_n(\omega)))||.
\end{aligned} \tag{3.20}$$

By Lemma 2.5, we obtain $\lim_{n\to\infty}\|T_2^n(\omega,\zeta_n(\omega))-\xi_n(\omega)\|=0$. Similarly, by using the same argument as in the proof above, we have $\lim_{n\to\infty}\|T_3^n(\omega,\xi_n(\omega))-\xi_n(\omega)\|=0$, for all $\omega\in\Omega$. This completes the proof. □

Theorem 3.3. *Let C be a nonempty closed and convex subset of a uniformly convex separable Banach space X. Let $T_1,T_2,T_3:\Omega\times C\to C$ be semicompact asymptotically nonexpansive random operators with sequence of measurable mappings $r_{i_n}(\omega):\Omega\to[0,\infty)$ satisfying $\sum_{n=1}^{\infty}r_{i_n}(\omega)<\infty$, for each $\omega\in\Omega$ and for each $i=1,2,3$ and $F=\bigcap_{i=1}^{3}\mathrm{RF}(T_i)\neq\varnothing$. Let ξ_0 be a measurable mapping from Ω to C. Define the sequence of functions $\{\xi_n\}$, $\{\eta_n\}$, and $\{\zeta_n\}$ by (2.5) with $\{\alpha_n\}$, $\{\alpha'_n\}$, $\{\alpha''_n\}$, $\{\beta_n\}$, $\{\beta'_n\}$, $\{\beta'_n\}$, $\{\gamma_n\}$, $\{\gamma'_n\}$, and $\{\gamma''_n\}$ satisfying*

(1) *$0<\alpha\le\alpha_n$, $\alpha'_n,\alpha''_n\le 1-\alpha$, for some $\alpha\in(0,1)$, for all $n\ge n_0$, $\exists n_0\in\mathbb{N}$,*

(2) *$\sum_{n=1}^{\infty}\gamma_n<\infty$, $\sum_{n=1}^{\infty}\gamma'_n<\infty$, and $\sum_{n=1}^{\infty}\gamma''_n<\infty$.*

Then sequences $\{\xi_n\}$, $\{\eta_n\}$, and $\{\zeta_n\}$ converge to a common random fixed point of F.

Proof. Let $\xi:\Omega\to C$ be the common random fixed point in F. By Lemma 3.2, we have

$$\begin{aligned}&\lim_{n\to\infty}||T_1^n(\omega,\zeta_n(\omega))-\xi_n(\omega)||\\&\quad=\lim_{n\to\infty}||T_2^n(\omega,\eta_n(\omega))-\xi_n(\omega)||=\lim_{n\to\infty}||T_3^n(\omega,\xi_n(\omega))-\xi_n(\omega)||=0\end{aligned}\tag{3.21}$$

for each $\omega\in\Omega$. This implies that $\|\xi_{n+1}(\omega)-\xi_n(\omega)\|\le\alpha_n\|T_1^n(\omega,\eta_n(\omega))-\xi_n(\omega)\|+\gamma_n\|f_n(\omega)-\xi_n(\omega)\|\to 0$, as $n\to\infty$, for each $\omega\in\Omega$. We note that

$$\begin{aligned}&||T_1^n(\omega,\xi_{n+1}(\omega))-\xi_{n+1}(\omega)||\\&\quad\le||T_1^n(\omega,\xi_{n+1}(\omega))-T_1^n(\omega,\xi_n(\omega))||+||T_1^n\xi_n(\omega)-\xi_n(\omega)||+||\xi_n(\omega)-\xi_{n+1}(\omega)||\\&\quad\le(1+\gamma_n)||\xi_{n+1}(\omega)-\xi_n(\omega)||+||T_1^n(\omega,\xi_n(\omega))-\xi_n(\omega)||\\&\qquad+||\xi_n(\omega)-\xi_{n+1}(\omega)||\longrightarrow 0,\quad\text{as } n\longrightarrow\infty,\end{aligned}\tag{3.22}$$

for each $\omega\in\Omega$. Using (3.22), we have

$$\begin{aligned}&||T_1(\omega,\xi_{n+1}(\omega))-\xi_{n+1}(\omega)||\\&\quad\le||T_1(\omega,\xi_{n+1}(\omega))-T_1^{n+1}(\omega,\xi_n(\omega))||+||T_1^{n+1}(\omega,\xi_{n+1}(\omega))-\xi_{n+1}(\omega)||\\&\quad\le(1+\gamma_1)||\xi_{n+1}(\omega)-T_1^n\xi_{n+1}(\omega)||\\&\qquad+||T_1^{n+1}(\omega,\xi_{n+1}(\omega))-\xi_{n+1}(\omega)||\longrightarrow 0,\quad\text{as } n\longrightarrow\infty,\end{aligned}\tag{3.23}$$

for each $\omega\in\Omega$. Thus, we have $\lim_{n\to\infty}\|T_1(\omega,\xi_n(\omega))-\xi_n(\omega)\|=0$ for each $\omega\in\Omega$. Similarly, we can show that

$$\lim_{n\to\infty}||T_2(\omega,\xi_n(\omega))-\xi_n(\omega)||\lim_{n\to\infty}||T_3(\omega,\xi_n(\omega))-\xi_n(\omega)||=0.\tag{3.24}$$

Since T_1 is a semicompact continuous random operator and $\lim_{n\to\infty}\|T_1(\omega,\xi_n(\omega))-\xi_n(\omega)\|=0$ for each $\omega\in\Omega$, there exist a subsequence $\{\xi_{n_k}\}$ of $\{\xi_n\}$ and a measurable mapping $\xi_0:\Omega\to C$ such that ξ_{n_k} converges pointwisely to ξ_0. The mapping $\xi_0:\Omega\to C$, being a pointwise limit of measurable mappings $\{\xi_{n_k}\}$, is measurable. Now,

$$\lim_{k\to\infty}\left\|\xi_{n_k}(\omega)-T_1(\omega,\xi_{n_k}(\omega))\right\|=\left\|\xi_0(\omega)-T_1(\omega,\xi_0(\omega))\right\|=0 \tag{3.25}$$

for each $\omega\in\Omega$. Hence, $\xi_0(\omega)$ is a random fixed point of T_1. Since $\lim_{n\to\infty}\|\xi_n(\omega)-\xi_0(\omega)\|$ exists, $\lim_{n\to\infty}\xi_n(\omega)=\xi_0(\omega)$ for each $\omega\in\Omega$. Similarly, we can show that $\xi_0(\omega)$ is also a random fixed point of T_2 and T_3. Observe that $\|\eta_n(\omega)-\xi_n(\omega)\|\le\alpha_n'\|T_2^n(\omega,\zeta_n(\omega))-\xi_n(\omega)\|+\gamma_n'\|f_n'(\omega)-\xi_n(\omega)\|\to 0$, and $\|\zeta_n(\omega)-\xi_n(\omega)\|\le\alpha_n''\|T_3^n(\omega,\xi_n(\omega))-\xi_n(\omega)\|+\gamma_n''\|f_n''(\omega)-\xi_n(\omega)\|\to 0$, as $n\to\infty$, for each $\omega\in\Omega$. Hence, $\lim_{n\to\infty}\eta_n(\omega)=\xi_0(\omega)$ and $\lim_{n\to\infty}\zeta_n(\omega)=\xi_0(\omega)$ for each $\omega\in\Omega$. Therefore $\{\xi_n\},\{\eta_n\}$, and $\{\zeta_n\}$ converge to a common random fixed point in F. □

If $T_1=T_2=T_3:=T$ and $\gamma_n=\gamma_n'=\gamma_n''\equiv 0$, then Theorem 3.3 reduces to the following known result.

Corollary 3.4 (see Beg and Abbas [11, Theorem 3.3]). *Let C be a nonempty closed bounded and convex subset of a uniformly convex separable Banach space X. Let $T:\Omega\times C\to C$ be completely continuous asymptotically nonexpansive random operator with sequence of measurable mappings $r_n(\omega):\Omega\to[0,\infty)$ satisfying $\sum_{n=1}^{\infty}r_n(\omega)<\infty$, for each $\omega\in\Omega$. Let ξ_0 be a measurable mapping from Ω to C. Define the sequence of functions $\{\xi_n\}$, $\{\eta_n\}$, and $\{\zeta_n\}$ by (2.4) with $\{\alpha_n\}$ and $\{\beta_n\}$ satisfying $0<\liminf_{n\to\infty}\alpha_n\le\limsup_{n\to\infty}\alpha_n<1$, and $0<\liminf_{n\to\infty}\beta_n\le\limsup_{n\to\infty}\beta_n<1$. Then sequences $\{\xi_n\}$, $\{\eta_n\}$, and $\{\zeta_n\}$ converge to a random fixed point of T.*

Proof. By Xu [14] and Ramírez [15], $F(T)\neq\emptyset$. Hence it follows from Theorem 3.3 that the sequences $\{\xi_n\}$, $\{\eta_n\}$, and $\{\zeta_n\}$ converge to a random fixed point of T. □

Remark 3.5. Theorem 3.3 is a generalized stochastic version of the result due to Plubtieng et al. [12].

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