Research Article

Delay-Dependent Guaranteed Cost H_{∞} **Control of an Interval System with Interval Time-Varying Delay**

Min Xiao and Zhongke Shi

College of Automation, Northwestern Polytechnical University, Xi'an 710072, China

Correspondence should be addressed to Min Xiao, mx_nwpu@hotmail.com

Received 25 January 2009; Revised 30 June 2009; Accepted 16 August 2009

Recommended by Alexander I. Domoshnitsky

This paper concerns the problem of the delay-dependent robust stability and guaranteed cost H_{∞} control for an interval system with time-varying delay. The interval system with matrix factorization is provided and leads to less conservative conclusions than solving a square root. The time-varying delay is assumed to belong to an interval and the derivative of the interval time-varying delay is not a restriction , which allows a fast time-varying delay; also its applicability is broad. Based on the Lyapunov-Ktasovskii approach, a delay-dependent criterion for the existence of a state feedback controller, which guarantees the closed-loop system stability, the upper bound of cost function, and disturbance attenuation lever for all admissible uncertainties as well as out perturbation, is proposed in terms of linear matrix inequalities (LMIs). The criterion is derived by free weighting matrices that can reduce the conservatism. The effectiveness has been verified in a number example and the compute results are presented to validate the proposed design method.

Copyright © 2009 M. Xiao and Z. Shi. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction

Recently, stability analysis, and control synthesis of the uncertainty interval systems and the time-delay system have been discussed extensively [1–7]. Literature [4, 5] has proposed a design method for a specific structure of single-input interval system, it has further been developed in [6], it has proposed a solution technique of an interval system stability and control synthesis by using a Riccati equation, but for the parameter matrix, it has constraint conditions of full column rank when the control input matrix is the interval matrix and matrix factorization requires the solving a square root. Time-delay is generally a source of instability in practical engineering systems; considerable attention has been paid to the problem of stability analysis and controller synthesis for time-delay systems. The guaranteed cost-control approach aims at stabilizing the systems while maintaining an adequate level

of performance represented by the quadratic cost [8–10]. Fragility is a common dynamic problem and is caused by many factors; reduction in size and cost of digital control hardware results in limitations in available computer memory and word length capabilities of the digital processor [11–15]. Literature [7] studies the guaranteed cost control of the interval system but does not consider the time delay and the fragility or the results presented by the proposed M_{-} matrix conditions. The existing approaches are all limited and conservative, and the proposed Riccati equation algorithm cannot be guaranteed to be convergent [8]. Very little open literature covering the research guaranteed cost control of an interval system with time delay has been published and the fragility problem has not been considered.

The H_{∞} control is an effective way for dealing with the disturbance uncertainty [16]. Since the delay-dependent results are less conservative than the delay-independent ones, especially when the delay time is small, it is necessary to discuss the delay-dependent guaranteed cost H_{∞} control for interval time-varying delay systems. Recently, a special type of time delay in practical engineering systems, that is, interval time-varying delay, was identified and investigated [17–20]. A typical example of systems with interval time-varying delay is networked control systems (NCSs). Employing the Lyapunov-Ktasovskii approach, literature [19–21] requires both the upper bound of the time-varying delay and additional information on the derivative of the time varying delay, while literature [17, 18, 22, 23] has no restriction on the derivative of the time varying delay, which allows a fast time-varying delay. Moreover, the general model transformation and bounding technique may be the sources of conservatism results; to further improve the performance of delay-dependent stability criteria, much effort has been devoted recently to the development of the free weighting matrices method [24], in which neither the bounding technique nor model transformation is employed.

In this paper, we present a new method of dealing with the problem of the delaydependent stability and guaranteed cost H_{∞} control of interval system with interval timevarying delay based on the LMIs [25]. This method employs Lyapunov-Krasovskii functional and free weighting matrices approaches, which are used to reduce the conservative result, guaranteeing that the closed-loop system gives a better dynamic performance.

2. Problem Formulation

Consider uncertainty interval system with time-varying delay

$$\dot{x}(t) = Ax(t) + A_d x(t - h(t)) + Bu(t) + B_1 w(t),$$

$$Z(t) = Cx(t) + Du(t),$$

$$x(t) = \varphi(t), \quad t \in [-h_M, 0],$$
(2.1)

where $x(t) \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, and $w \in \mathbb{R}^w$ denote the state vector, the control vector, and the disturbance input, respectively, and $z(t) \in \mathbb{R}^z$ is the controlled output. The time-varying delay h(t) is a time-varying continuous function satisfying

$$0 \le h_m \le h(t) \le h_M,\tag{2.2}$$

where h_m , h_M are known constants. Moreover, $h_M > 0$ and the initial condition $\varphi(t)$ denotes a continuous vector-valued initial function of $t \in [-h_M, 0]$. *A* is the state matrix. *B* is the input matrix, B_1 is the disturbance matrix, and *C* and *D* are appropriate dimension constant matrices.

Interval matrix $A, A_d \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times p}$ vary with the parameters and can be shown as [3, 10]

$$A \in [A^{m}, A^{M}] = \left\{ [a_{ij}], a_{ij}^{m} < a_{ij} < a_{ij}^{M}, i, j = 1 \cdots n \right\},$$

$$B \in [B^{m}, B^{M}] = \left\{ [b_{ij}] : b_{ij}^{m} < b_{ij} < b_{ij}^{M}, i = 1 \cdots n, j = 1 \cdots p \right\},$$

(2.3)

where

$$A^{m} = \begin{bmatrix} a_{ij}^{m} \end{bmatrix}_{n \times n'} \qquad A^{M} = \begin{bmatrix} a_{ij}^{M} \end{bmatrix}_{n \times n} \text{ satisfying } a_{ij}^{m} < a_{ij}^{M},$$

$$B^{m} = \begin{bmatrix} b_{ij}^{m} \end{bmatrix}_{n \times p'}, \qquad B^{M} = \begin{bmatrix} b_{ij}^{M} \end{bmatrix}_{n \times p} \text{ satisfying } b_{ij}^{m} < b_{ij}^{M}.$$
(2.4)

Let

$$A_{0} := \frac{A^{M} + A^{m}}{2}, \qquad \overline{A}_{ij} := \frac{A^{M} - A^{m}}{2} : [\overline{a}_{ij}],$$

$$B_{0} := \frac{B^{M} + B^{m}}{2}, \qquad \overline{B}_{ij} := \frac{B^{M} - B^{m}}{2} : [\overline{b}_{ij}],$$
(2.5)

where A^m , A^M , B^m , and B^M are known real matrices; \overline{A}_{ij} denotes that the *i*, *j*th component is \overline{a}_{ij} with all other entries being zeros and can be factorized into $\overline{a}_{ij}e_i \times e_j^T$. A and B can be described as an equivalent form:

$$A = A_{0} + \Delta A = A_{0} + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{ij} \overline{A}_{ij} = A_{0} + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_{ij} \overline{a}_{ij} e_{i} \times e_{j}^{T} = A_{0} + D_{1}F_{1}E_{1},$$

$$A_{d} = A_{d0} + \Delta A_{d} = A_{d0} + \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \overline{A}_{dij} = A_{d0} + \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \overline{a}_{dij} e_{i} \times e_{j}^{T} = A_{d0} + D_{d}F_{d}E_{d}, \quad (2.6)$$

$$B = B_{0} + \Delta B = B_{0} + \sum_{i=1}^{n} \sum_{j=1}^{p} \beta_{ij} \overline{B}_{ij} = B_{0} + \sum_{i=1}^{n} \sum_{j=1}^{p} \beta_{ij} \overline{b}_{ij} e_{i} \times e_{j}^{T} = B_{0} + D_{2}F_{2}E_{2},$$

where $|\lambda_{ij}| \leq 1$, $i, j = 1 \cdots n$, $|\alpha_{ij}| \leq 1$, $i, j = 1 \cdots n$, $|\beta_{ij}| \leq 1$, $i = 1 \cdots n$, $j = 1 \cdots p$, and uncertainty matrices satisfy

$$F_1^T F_1 \le I_{n^2}, \quad F_d^T(t) F_d(t) \le I_{n^2}, \quad F_2^T F_2 \le I_{(n \times p) \times (n \times p)}.$$
(2.7)

The actual control input implemented is assumed to be $u = (K_{p \times n} + \Delta K)x = Kx$, where D_k , E_k are known constant dimension matrices and $F_k(t)$ satisfies

$$F_k^{\mathrm{T}} F_k \le \mu I_{(p \times n) \times (p \times n)},\tag{2.8}$$

where $\mu > 0$ is controller gain perturbation uncertainty bound.

The cost function associated with this system is

$$J = \int_{0}^{\infty} \left[x^{T}(t) R_{1} x(t) + u^{T}(t) R_{2} u(t) \right] dt,$$
(2.9)

where $R_1 > 0$, $R_2 > 0$ are given weighting matrices.

Substituting (2.1) into (2.10) the resulting closed-loop system is

$$\dot{x}(t) = [(A_0 + D_1F_1(t)E_1) + (B_0 + D_2F_2(t)E_2)(K + D_kF_k(t)E_k)]x(t) + (A_{d0} + D_dF_d(t)E_d)x(t - h(t)) + B_1w(t).$$
(2.10)

Our controllers design objective is described as follows.

The closed loop system (2.10) is asymptotically stable with disturbance attenuation γ , nonfragility μ , if the following is fulfilled for all time-varying delay and admissible uncertainties satisfying (2.7) and (2.8).

- (1) The closed close system (2.10) is asymptotically stable.
- (2) The closed loop system (2.10) guarantees, under aeroinitial conditions,

$$\|z(t)\|_{2} < \gamma \|w(t)\|_{2} \tag{2.11}$$

for all nonzero $w(t) \in L_2[0, \infty]$.

(3) The closed-loop cost function satisfies $J \leq J^*$.

Therefore, the objective of this paper is to design a nonfragile guaranteed cost H_{∞} controller in the presence of time-varying delay, time-varying parameter uncertainty of an interval system, and uncertainty of the controller. Also the controller guarantees disturbance attenuation of the closed loop system from w(t) to z(t).

Lemma 2.1 (see [26]). Let Y_1 , M, N, and ψ be matrices of appropriate dimensions and assume ψ is symmetric, satisfying $\psi^T \psi \leq I$, then

$$Y_1 + M\psi N + N^T \psi^T M^T < 0 \tag{2.12}$$

if and only if there exists a scalar $\varepsilon > 0$ *satisfying*

$$Y_1 + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0. \tag{2.13}$$

3. Main Results

Defining $h_a = (h_M + h_m)/2$, $\delta = (h_M - h_m)/2$, where h_a and δ can be taken as the mean value and range of variation of the time-varying delay. First consider a delay-dependent stability for the following nominal system of (2.1) ($w(t) \equiv 0$):

$$\dot{x}(t) = A_0 x(t) + A_{d0} x(t - h(t)) + B_0 u(t).$$
(3.1)

Using the Leibniz-Newton formula, we can write

$$x(t - h_a) - x(t - h(t)) = \int_{t - h(x)}^{t - h_a} \dot{x}(s) ds$$
(3.2)

and system (3.1) can be rewritten as

$$\dot{x}(t) = \left(A_0 + B_0 \hat{K}\right) x(t) + A_{d0} x(t - h_a) - A_{d0} \int_{t - h(x)}^{t - h_a} \dot{x}(s) ds.$$
(3.3)

The following theorem presents a sufficient condition for the existence of the nonfragile guaranteed cost controller.

Theorem 3.1. A control law $u = \hat{K}x$ is said to be a non-fragile guaranteed cost control associated with cost matrix P, Q, R, S > 0 and N_i (i = 1, 2) of appropriate dimensions for the system (3.1) and cost function (2.9) and given scalars h_m and h_M . Suppose that the disturbance input is zero for all times ($w(t) \equiv 0$), if the following matrix inequality

$$\Gamma = \begin{bmatrix}
\Delta_{1} \ PA_{d0} - N_{1}^{T} + N_{2} \ -h_{a}N_{1}^{T} \ \delta PA_{d0} \ h_{a}\left(A_{0} + B_{0}\hat{K}\right)^{T}R \ \delta\left(A_{0} + B_{0}\hat{K}\right)^{T}S \\
* \ -Q - N_{2}^{T} - N_{2} \ -h_{a}N_{2}^{T} \ 0 \ h_{a}A_{d0}^{T}R \ \delta A_{d0}^{T}S \\
* \ * \ -h_{a}R \ 0 \ 0 \ 0 \\
* \ * \ * \ -h_{a}R \ 0 \\
* \ * \ * \ * \ -h_{a}R \ 0 \\
* \ * \ * \ * \ * \ -h_{a}R \ 0 \\
* \ * \ * \ * \ * \ -\delta S
\end{bmatrix} < 0,$$
(3.4)

where

$$\Delta_{1} = Q + P(A_{0} + B_{0}\hat{K}) + (A_{0} + B_{0}\hat{K})^{T}P + (A_{0} + B_{0}\hat{K})^{T}(h_{a}R + \delta S)(A_{0} + B_{0}\hat{K}) + (R_{1} + \hat{K}^{T}R_{2}\hat{K}) + N_{1}^{T} + N_{1}$$
(3.5)

holds for all admissible uncertainty (2.7), (2.8), and any $\tau(t)$ satisfying (2.2). The closed-loop cost function satisfies

$$J \leq J^{*} = x_{0}^{T} P x_{0} + \int_{-h_{a}}^{0} \varphi^{T}(t) Q \varphi(t) dt + \int_{-h_{a}}^{0} d\beta \int_{\beta}^{0} \dot{\varphi}^{T}(t) R \dot{\varphi}(t) dt + \int_{-h_{a}}^{-h_{m}} d\beta \int_{\beta}^{0} \dot{\varphi}^{T}(t) S \dot{\varphi}(t) dt.$$
(3.6)

Proof. Choose a Lyapunov function as

$$V(x(t)) = V_{1}(x(t)) + V_{2}(x(t)),$$

$$V_{1}(x(t)) = x^{\mathrm{T}}(t)Px(t) + \int_{t-h_{a}}^{t} x^{\mathrm{T}}(\alpha)Qx(\alpha)d\alpha + \int_{-h_{a}}^{0} d\beta \int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\theta)R\dot{x}(\theta)d\theta,$$

$$V_{2}(x(t)) = \begin{cases} \int_{-h_{a}}^{-h_{m}} d\beta \int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\theta)S\dot{x}(\theta)d\theta, & h_{m} \le h(t) < h_{a}, \\ 0 & h(t) < h_{a}, \\ \int_{-h_{M}}^{-h_{a}} d\beta \int_{t+\beta}^{t} \dot{x}^{\mathrm{T}}(\theta)S\dot{x}(\theta)d\theta, & h_{m} < h(t) \le h_{M}, \end{cases}$$

$$(3.7)$$

where P > 0, Q > 0, R > 0, and S > 0.

Case 1 ($h_m \le h(t) \le h_a$). Taking the time derivative of $V(\cdot)$ along any trajectory of the closed-loop system (3.3) is given by

$$\dot{V}(x(t)) = x^{T}(t) \left[P\left(A + B\hat{K}\right) + \left(A + B\hat{K}\right)^{T} P \right] x(t) + x^{T}(t)Qx(t) + 2x^{T}(t)PA_{d0}(t)x(t - h_{a}) - 2x^{T}(t)PA_{d0} \int_{t-h(x)}^{t-h_{a}} \dot{x}(s)ds - x^{T}(t - h_{a})Qx(t - h_{a}) + \dot{x}^{T}(t)(h_{a}R + \delta S)\dot{x}(t) - \int_{t-h_{a}}^{t} \dot{x}^{T}(s)R\dot{x}(s)ds - \int_{t-h_{a}}^{t-h_{m}} \dot{x}^{T}(\theta)S\dot{x}(\theta)d\theta.$$
(3.8)

It is easy to see that

$$-\int_{t-h_a}^{t-h_m} \dot{x}^{\mathrm{T}}(\theta) S \dot{x}(\theta) d\theta \leq -\int_{t-h_a}^{t-h(t)} \dot{x}^{\mathrm{T}}(\theta) S \dot{x}(\theta) d\theta.$$
(3.9)

According to (3.2), for any matrices N_i , i = 1, 2, with appropriate dimensions, the following equations hold:

$$2\left[x^{T}(t)N_{1}^{T} + x^{T}(t-h_{a})N_{2}^{T}\right] \times \left[x(t) - x(t-h_{d}) - \int_{t-h_{a}}^{t} \dot{x}(s)ds\right] = 0.$$
(3.10)

Then we have

$$\begin{split} \dot{V}(x(t)) &\leq x^{T}(t) \left[P\left(A + B\hat{K}\right) + \left(A + B\hat{K}\right)^{T} P + Q \right] x(t) + 2x^{T}(t) PA_{d0}(t) x(t - h_{a}) \\ &- 2x^{T}(t) PA_{d0} \int_{t-h(t)}^{t-h_{a}} \dot{x}(s) ds - x^{T}(t - h_{a}) Qx(t - h_{a}) + \dot{x}^{T}(t) (h_{a}R + \delta S) \dot{x}(t) \\ &- \int_{t-h_{a}}^{t} \dot{x}^{T}(s) R \dot{x}(s) ds - \int_{t-h_{a}}^{t-h(t)} \dot{x}^{T}(\theta) S \dot{x}(\theta) d\theta + 2 \left[x^{T}(t) N_{1}^{T} + x^{T}(t - h_{a}) N_{2}^{T} \right] \\ &\times \left[x(t) - x(t - h_{d}) - \int_{t-h_{a}}^{t} \dot{x}(s) ds \right] \leq \frac{1}{h_{a}} \int_{t-h_{a}}^{t} \eta^{T}(t, s) \Phi_{1} \eta(t, s) ds \\ &+ \frac{1}{h_{a} - h(t)} \int_{t-h_{a}}^{t-h(t)} \eta^{T}(t, s) \Phi_{2} \eta(t, s) ds, \end{split}$$
(3.11)

where

$$\eta^{T}(t,s) = \left[x^{T}(t)x^{T}(t-h_{a})\dot{x}^{T}(s)\right],$$

$$\Phi_{1} = \begin{bmatrix}\mathfrak{A} PA_{d0} + \left(A_{0} + B_{0}\hat{K}\right)^{T}(h_{a}R + \delta S)A_{d0} - N_{1}^{T} + N_{2} + Z_{12} - h_{a}N_{1}^{T}\\ * -Q + A_{d0}^{T}(h_{a}R + \delta S)A_{d0} - N_{2}^{T} - N_{2} + Z_{22} - h_{a}N_{2}^{T}\\ * & & -h_{a}R\end{bmatrix}, \quad (3.12)$$

$$\Phi_{2} = \begin{bmatrix} -Z_{11} & -Z_{12} & \delta \left[PA_{d0} + \left(A_{0} + B_{0} \hat{K} \right)^{T} (h_{a}R + \delta S) A_{d0} \right] \\ * & -Z_{22} & \delta A_{d0}^{T} (h_{a}R + \delta S) A_{d0} \\ * & * & \delta^{2} A_{d0}^{T} (h_{a}R + \delta S) A_{d0} - \delta S \end{bmatrix},$$

where \mathfrak{A} denotes $Q + P(A_0 + B_0 \hat{K}) + (A_0 + B_0 \hat{K})^T P + (A_0 + B_0 \hat{K})^T (h_a R + \delta S)(A_0 + B_0 \hat{K}) + N_1^T + N_1 + Z_{11}$. With $Z_{11} > 0$, $Z_{22} > 0$, Z_{12} are some parameter matrices of appropriate dimensions. The asymptotic stability is achieved if $\Phi_1 < 0$ and $\Phi_2 < 0$ which is equivalent. From Schur complement, (3.4) holds if and only if $\Phi_1 < 0$ and $\Phi_2 < 0$ simultaneously.

From (3.11), we have

$$\dot{V}(x(t)) + x^{T}(t) \left(R_{1} + \hat{K}^{T} R_{2} \hat{K} \right) x(t) \leq \eta^{T}(t, s) \Gamma \eta(t, s).$$

$$(3.13)$$

According to inequality (3.4), Γ < 0 implies Φ_1 < 0 and Φ_2 < 0, then

$$\dot{V}(x(t)) < -x^{\mathrm{T}}(t)R_{1} + ((K + \Delta K)^{\mathrm{T}}R_{2}(K + \Delta K))x(t) < 0$$
 (3.14)

can be obtained so that

$$\dot{V}(x(t)) < -\left[x(t)^T R_1 x(t) + u(t)^T R_2 u(t)\right] < 0.$$
 (3.15)

Therefore, the closed-loop system (3.1) is asymptotically stable. Furthermore, by integrating both sides of the above inequality from 0 to *T* and using the initial condition,

$$J = \int_{0}^{\infty} \left[x^{T}(t) R_{1}x(t) + u^{T}(t) R_{2}u(t) \right] dt$$

$$\leq -\int_{0}^{\infty} \dot{V}(x(t)) dt = V(x(0)) \leq x_{0}^{T} P x_{0} + \int_{-h_{a}}^{0} \varphi^{T}(t) Q \varphi(t) dt \qquad (3.16)$$

$$+ \int_{-h_{a}}^{0} d\beta \int_{\beta}^{0} \dot{\varphi}^{T}(t) R \dot{\varphi}(t) dt + \int_{-h_{a}}^{-h_{m}} d\beta \int_{\beta}^{0} \dot{\varphi}^{T}(t) S \dot{\varphi}(t) dt$$

is obtained. Thus, Theorem 3.1 is true.

Case 2 ($h(t) = h_a$). Similar to the above analysis, it is easy to see that Φ_1 , Φ_2 are the leading minor of obtained (3.11) and (3.13), respectively. Therefore, system (3.1) is asymptotically stable if and only if (3.4) holds. The closed-loop cost function satisfies (3.6).

Case 3 ($h_m \le h(t) \le h_M$). Similar to Case 1, one can prove that system (3.1) is asymptotically stable.

The objective of this paper is to develop a procedure for determining a state feedback gain matrix \hat{K} which contains controller gain perturbation such that the control law $u = \hat{K}x$ is a non-fragile guaranteed cost H_{∞} control of the system (2.1), cost function (2.9), and disturbance attenuation γ .

Theorem 3.2. A control law $u = \hat{K}x$ is said to be a non-fragile guaranteed cost H_{∞} control associated with cost matrix P, Q, R, S > 0, and N_i (i = 1, 2) of appropriate dimensions for the system (2.1) and cost function (2.9), disturbance attenuation $\gamma > 0$, and given scalars h_m and h_M , if the following matrix inequality

$$\Sigma = \begin{bmatrix} \Delta_{1} \ PA_{d0} - N_{1}^{T} + N_{2} \ -h_{a}N_{1}^{T} \ \delta PA_{d0} \ h_{a}\mathfrak{B}R \ \delta \mathfrak{B}S \ \left(C + D\hat{K}\right)^{T} \ PB_{1} \\ * \ -Q - N_{2}^{T} - N_{2} \ -h_{a}N_{2}^{T} \ 0 \ h_{a}A_{d0}^{T}R \ \delta A_{d0}^{T}S \ 0 \ 0 \\ * \ * \ -h_{a}R \ 0 \ 0 \ 0 \ 0 \ 0 \\ * \ * \ * \ -h_{a}R \ 0 \ 0 \ 0 \\ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \ 0 \\ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ * \ * \ * \ * \ -h_{a}R \ 0 \ 0 \\ * \ (3.17)$$

where \mathfrak{B} denotes $(A_0 + B_0 \widehat{K})^T$, and

$$\Delta_1 = Q + P\left(A_0 + B_0\hat{K}\right) + \left(A_0 + B_0\hat{K}\right)^T P + \left(R_1 + \hat{K}^T R_2\hat{K}\right) + N_1^T + N_1$$
(3.18)

holds for all admissible uncertainty (2.7) and (2.8). The closed-loop cost function satisfies (3.6).

Proof. It has been noticed that (3.17) implies (3.4). Therefore (3.17) ensures the asymptotic stability of the closed loop system (2.1). Under zero initial condition $x(t) = 0, t \in [-h_M, 0]$,

$$J = \int_0^\infty \left[z(t)^T z(t) - \gamma^2 w(t)^T w(t) \right] dt$$
(3.19)

can be introduced. Then for any nonzero $w(t) \in L_2[0, \infty]$,

$$J \leq \int_0^\infty \left[z(t)^T z(t) - \gamma^2 w(t)^T w(t) + \dot{V} x(t) \right] dt = \int_0^\infty \phi(t)^T \psi \phi(t) dt,$$
(3.20)

where $\phi(t) = \left[\eta(t)^T \ w(t)^T \right]$ and $\psi = \Sigma$. The condition of $\psi < 0$ implies $w(t) \in L_2[0, \infty]$. Therefore when $\psi < 0$, the closed loop system (2.1) is asymptotically stable with disturbance attenuation $\gamma > 0$ and nonfragility $\mu > 0$.

Theorem 3.3. There exist non-fragile guaranteed cost H_{∞} controllers for the system (2.1) and the cost function (2.9), disturbance attenuation $\gamma > 0$, h_m , h_M , and $\mu > 0$, $\alpha > 0$, if there exists scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\rho_1 > 0$, $\rho_2 > 0$, symmetric positive matrices U, \tilde{S}, \tilde{Q} and, \tilde{N}_i (i = 1, 2) of appropriate dimensions and a matrix W such that

Δ_{11}	Δ_{12}	$-h_a \widetilde{N}_1^T$	$\delta A_{d0} \tilde{S}$	$h_a \mathfrak{C}$	бC	W^T	\mathfrak{D}	B_0D_k	D_d	UE_k^T	0	D_1	D_2	UE_1^T	$W^T E_2^T$	и	B_1
*	Δ_{22}	$-h_a \widetilde{N}_2$	0	$h_a U A_{d0}^T$	$\delta U A_{d0}^T$	0	0	0	0	0	UE_d^T	0	0	0	0	0	0
*	*	$-\alpha h_a U$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
*	*	*	$-\delta \widetilde{S}$	$\delta h_a \widetilde{S} A_{d0}^T$	$\delta^2 \tilde{S} A^T_{d0}$	0	0	0	0	0	$\delta^{-1} \widetilde{S} E_d^T$	0	0	0	0	0	0
*	*	*	*	$-\alpha^{-1}h_a U$	0	0	0	$h_a B_0 D_k$	$h_a D_d$	0	0	$h_a D_1$	$h_a D_2$	0	0	0	0
*	*	*	*	*	$-\delta \tilde{S}$	0	0	$\delta B_0 D_k$	δD_d	0	0	δD_1	δD_2	0	0	0	0
*	*	*	*	*	*	$-R_{2}^{-1}$	0	D_k	0	0	0	0	0	0	0	0	0
*	*	*	*	*	*	*	-I	DD_k	0	0	0	0	0	0	0	0	0
*	*	*	*	*	*	*	*	$-\rho_1^{-1}I$	0	0	0	0	0	0	$D_k^T E_2^T$	0	0
*	*	*	*	*	*	*	*	*	$-\rho_2^{-1}I$	0	0	0	0	0	0	0	0
*	*	*	*	*	*	*	*	*	*	$-\mu^{-1}\rho_1 I$	0	0	0	0	0	0	0
*	*	*	*	*	*	*	*	*	*	*	$-\rho_2 I$	0	0	0	0	0	0
*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_1^{-1}I$	0	0	0	0	0
*	*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_2^{-1}I$	0	0	0	0
*	*	*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_1 I$	0	0	0
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_2 I$	0	0
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	$-R_1^{-1}$	0
L *	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	$-\gamma^2 I$

< 0,

where \mathfrak{C} denotes $(A_0 U + B_0 W)^T$, \mathfrak{D} denotes $(CU + DW)^T$, and

$$\Delta_{11} = \widetilde{Q} + (A_0 U + B_0 W) + (A_0 U + B_0 W)^T + \widetilde{N}_1^T + \widetilde{N}_1,$$

$$\Delta_{12} = A_{d0} U - \widetilde{N}_1^T + \widetilde{N}_2,$$

$$\Delta_{22} = -\widetilde{Q} - \widetilde{N}_2^T - \widetilde{N}_2.$$
(3.22)

Furthermore, if $(\varepsilon_i, \rho_i, U, \tilde{S}, \tilde{N}_i, W)$, i = 1, 2 is a feasible solution to the inequality (3.21), then $u = \hat{K}x$ is a non-fragile guaranteed cost H_{∞} controller of the system (2.1), where the feedback gain matrix \hat{K} is given by $\hat{K} = WU^{-1}$ and the corresponding closed-loop cost function satisfies (3.6).

Proof. Let $A_{d0} = A_{d0} + D_d F_d E_d$, $\hat{K} = K + D_k F_k E_k$.

By manipulating the left-hand side in inequality (3.17), it follows that the inequality (3.17) is equivalent to

$$Y_1 + \Sigma_1 + \Sigma_1^{\rm T} + \Sigma_2 + \Sigma_2^{\rm T} < 0, \tag{3.23}$$

where

 $\widetilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0 K) + \mathfrak{E}P + N_1^T + N_1,$

$$\Sigma_{1} = \begin{bmatrix} PB_{0}D_{k} \\ 0 \\ 0 \\ 0 \\ 0 \\ h_{a}R^{T}B_{0}D_{k} \\ \delta S^{T}B_{0}D_{k} \\ D_{k} \\ DD_{k} \\ 0 \end{bmatrix} F_{k}[E_{k}\ 0\ 0\ 0\ 0\ 0\ 0\ 0], \qquad \Sigma_{2} = \begin{bmatrix} PD_{d} \\ 0 \\ 0 \\ 0 \\ h_{a}R^{T}D_{d} \\ \delta S^{T}D_{d} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_{d}[0\ E_{d}\ 0\ \delta E_{d}\ 0\ 0\ 0\ 0\ 0], \qquad (3.24)$$

where \mathfrak{E} denotes $(A_0 + B_0 K)^T$.

By applying Lemma 2.1, the above inequality (3.23) holds for all F_k , F_d , satisfying $F_k(t)F_k^{T}(t) < \mu I$, $F_dF_d^{T} \le I_{n^2}$ if and only if there exists a constant $\rho_1 > 0$, $\rho_2 > 0$ such that

It follows from the Schur complement that the above inequality is further equivalent to the following inequality:

٢Â	$\tilde{\Delta}_1 PA_{d0} - N_1^T + N_2$	$-h_a N_1^T$	δPA_{d0}	$h_a \mathfrak{E} R$	$\delta \mathfrak{E}S$	K^T	Ŧ	PB_0D_k	PD_d	E_k^T	0	PB_1
,	$* -Q - N_2^T - N_2$	$-h_a N_2^T$	0	$h_a A_{d0}^T R$	$\delta A_{d0}^T S$	0	0	0	0	0	E_d^T	0
,	* *	$-h_a R$	0	0	0	0	0	0	0	0	0	0
,	* *	*	$-\delta S$	$\delta h_a A_{d0}^T R$	$\delta^2 A_{d0}^T S$	0	0	0	0	0	$\delta^{-1}E_d^T$	0
, I	k *	*	*	$-h_a R$	0	0	0	$h_a R^T B_0 D_k$	$h_a R^T D_d$	0	0	0
,	k *	*	*	*	$-\delta S$	0	0	$\delta S^T B_0 D_k$	$\delta S^T D_d$	0	0	0
,	* *	*	*	*	*	$-R_2^{-1}$	0	D_k	0	0	0	0
,	k *	*	*	*	*	*	-I	DD_k	0	0	0	0
,	* *	*	*	*	*	*	*	$- ho_1^{-1}I$	0	0	0	0
,	* *	*	*	*	*	*	*	*	$- ho_2^{-1}I$	0	0	0
,	* *	*	*	*	*	*	*	*	*	$-\mu^{-1} ho_1 I$	0	0
,	* *	*	*	*	*	*	*	*	*	*	$-\rho_2 I$	0
Ľ,	k *	*	*	*	*	*	*	*	*	*	*	$-\gamma^2 I$

< 0,

where \mathfrak{F} denotes $(C + DK)^T$, and

$$\widetilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0 K) + (A_0 + B_0 K)^T P + N_1^T + N_1.$$
(3.27)

Let $A_0 = A_0 + D_1F_1E_1$, $B_0 = B_0 + D_2F_2E_2$. By manipulating the left-hand side in inequality (3.17) again, it follows that the above inequality is equivalent to

$$Y_2 + \Xi_1 + \Xi_1^T + \Xi_2 + \Xi_2^T < 0, (3.28)$$

where

	$\widetilde{\Delta}_1$	$PA_{d0} - N_1^T + N_2$	$-h_a N_1^T$	δPA_{d0}	$h_a \mathfrak{E} R$	$\delta \mathfrak{E} S$	K^T	\mathfrak{F}	PB_0D_k	PD_d	E_k^T	0	PB_1	
	*	$-Q - N_2^T - N_2$	$-h_a N_2^T$	0	$h_a A_{d0}^T R$	$\delta A_{d0}^T S$	0	0	0	0	0	E_d^T	0	
	*	*	$-h_a R$	0	0	0	0	0	0	0	0	0	0	
	*	*	*	$-\delta S$	$\delta h_a A_{d0}^T R$	$\delta^2 A_{d0}^T S$	0	0	0	0	0	$\delta^{-1} E_d^T$	0	
	*	*	*	*	$-h_a R$	0	0	0	$h_a R^T B_0 D_k$	$h_a R^T D_d$	0	0	0	
	*	*	*	*	*	$-\delta S$	0	0	$\delta S^T B_0 D_k$	$\delta S^T D_d$	0	0	0	
$Y_2 =$	*	*	*	*	*	*	$-R_2^{-1}$	0	D_k	0	0	0	0	,
	*	*	*	*	*	*	*	-I	DD_k	0	0	0	0	
	*	*	*	*	*	*	*	*	$-\rho_1^{-1}I$	0	0	0	0	
	*	*	*	*	*	*	*	*	*	$- ho_2^{-1}I$	0	0	0	
	*	*	*	*	*	*	*	*	*	*	$-\mu^{-1} ho_1 I$	0	0	
	*	*	*	*	*	*	*	*	*	*	*	$-\rho_2 I$	0	
	*	*	*	*	*	*	*	*	*	*	*	*	$-\gamma^2 I$	
													(3.29	1)

and

$\widetilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0 K) + (A_0 + B_0 K)^T P + N_1^T + N_1,$

1	PD_1	l	ΓPD_2 -	1
	0		0	
	0		0	
	0		0	
	$h_a R^T D_1$		$h_a R^T D_2$	
	$\delta S^T D_1$		$\delta S^T D_2$	
$\Xi_1 =$	0	$F_1[E_1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ $	0	$F_2[E_2K \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ E_2D_K \ 0 \ 0].$
	0		0	
	0		0	
	0		0	
	0		0	
	0		0	
	0			1
				(3.30)

Applying Lemma 2.1 again, the inequality (3.28) holds for all F_1, F_2 satisfying $F_1^T(t)F_1(t) \le I_{n^2}, F_2^T(t)F_2(t) \le I_{n \times p}$, if and only if there exists a constant $\varepsilon_1 > 0, \varepsilon_2 > 0$, such that

	PD_2	PD_2	
	0	0	
	0	0	
	0	0	
	$h_a R^T D_2$	$h_a R^T D_2$	
	$\delta S^T D_2$	$\delta S^T D_2$	
$+ \varepsilon_2$	0	0	
	0	0	
	0	0	
	0	0	
	0	0	
	0	0	
	0	0	

 $+ \varepsilon_2^{-1} \begin{bmatrix} E_2 K & 0 & 0 & 0 & 0 & 0 & 0 & E_2 D_K & 0 & 0 \end{bmatrix}^T \begin{bmatrix} E_2 K & 0 & 0 & 0 & 0 & 0 & E_2 D_K & 0 & 0 \end{bmatrix}$ < 0.

(3.31)

It follows from the Schur complement again that the above inequality is further equivalent to the following inequality:

$\Gamma^{\tilde{\Delta}_1}$	$PA_{d0} - N_1^T + N_2$	$-h_a N_1^T$	δPA_{d0}	$h_a \mathfrak{E} R$	$\delta \mathfrak{E}S$	K^T	\mathfrak{F}	PB_0D_k	PD_d	E_k^T	0	PD_1	PD_2	E_1^T	$K^T E_2^T$	PB_1	
*	$-Q - N_2^T - N_2$	$-h_a N_2^T$	0	$h_a A_{d0}^T R$	$\delta A_{d0}^T S$	0	0	0	0	0	E_d^T	0	0	0	0	0	
*	*	$-h_a R$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
*	*	*	$-\delta S$	$\delta h_a A_{d0}^T R$	$\delta^2 A_{d0}^T S$	0	0	0	0	0	$\delta^{-1} E_d^T$	0	0	0	0	0	
*	*	*	*	$-h_a R$	0	0	0	$h_a RB_0 D_k$	$h_a RD_d$	0	0	$h_a RD_1$	$h_a RD_2$	0	0	0	
*	*	*	*	*	$-\delta S$	0	0	δSB_0D_k	δSD_d	0	0	δSD_1	δSD_2	0	0	0	
*	*	*	*	*	*	$-R_2^{-1}$	0	D_k	0	0	0	0	0	0	0	0	
*	*	*	*	*	*	*	-I	DD_k	0	0	0	0	0	0	0	0	
*	*	*	*	*	*	*	*	$-\rho_1^{-1}I$	0	0	0	0	0	0	$D_k^T E_2^T$	0	
*	*	*	*	*	*	*	*	*	$-\rho_2^{-1}I$	0	0	0	0	0	0	0	
*	*	*	*	*	*	*	*	*	*	$-\mu^{-1}\rho_1 I$	0	0	0	0	0	0	
*	*	*	*	*	*	*	*	*	*	*	$-\rho_2 I$	0	0	0	0	0	
*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_1^{-1}I$	0	0	0	0	
*	*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_2^{-1}I$	0	0	0	
*	*	*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_1 I$	0	0	
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	$-\varepsilon_2 I$	0	
L *	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	$-\gamma^2 I$	I
<	0,																
																(3.32)	2)

and

$$\widetilde{\Delta}_1 = Q + R_1 + P(A_0 + B_0 K) + (A_0 + B_0 K)^T P + N_1^T + N_1.$$
(3.33)

By pre- and postmultiplying the left-hand side matrix in the above inequality by the matrix diag{ p^{-1} p^{-1} p^{-1} s^{-1} R^{-1} I \cdots p^{-1} I}, respectively, and defining the matrix $U = P^{-1}$, W = KU, $\widetilde{N}_1 = UN_1U$, $\widetilde{N}_2 = UN_2U$, $\widetilde{Q} = UQU$ and, $\widetilde{S} = S^{-1}$. If we set $R = \alpha U^{-1}$, then $URU = \alpha U$, and $R^{-1} = \alpha^{-1}U$, and it can be concluded that the above matrix inequality is equivalent to (3.21). The proof is complete.

4. A Numerical Example

Consider system (2.1) with [17]

$$A = \begin{bmatrix} 4 & 0.1 & -0.3 \\ -0.2 & 3 & -0.2 \\ 0.2 & -0.3 & 2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.4 & 0.1 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & 0.2 & 0 \\ 0 & 3 & 1 \\ 0.1 & 0 & 3 \end{bmatrix},$$
$$B_w = \begin{bmatrix} 0.2 & 0 \\ 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad (4.1)$$
$$E = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0.1 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad E_b = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix},$$

and $1.2 \le h(t) \le 1.8$. By applying Theorem 3.3 with $\alpha = 0.16$ and $\gamma = 1$, the controller gain is obtained:

$$K = \begin{bmatrix} 2.5536 & -0.1039 & -0.2143 \\ 0.1895 & -2.5017 & 1.1060 \\ -0.1513 & 0.4258 & -2.4936 \end{bmatrix},$$
(4.2)

where it is also a result $\varepsilon_1 = 3$, $\varepsilon_2 = 4$, $\rho_1 = 5$, $\rho_2 = 6$:

$$U = \begin{bmatrix} 0.2911 & 0.0213 & 0.0045 \\ 0.0213 & 0.3233 & 0.0392 \\ 0.0045 & 0.0392 & 0.351 \end{bmatrix}, \qquad W = \begin{bmatrix} 0.6318 & 0.0036 & -0.0402 \\ 0.0036 & -0.7133 & -0.0251 \\ -0.0402 & -0.0251 & -0.5452 \end{bmatrix}, \qquad \tilde{S} = 8.$$
(4.3)

For $h_m = 1.2$, the maximum allowable upper bound of the delay is $h_M = 3.0679$ which is larger than $h_M = 1.8467$ derived in Jiang and Han in [17]. This means that any h(t) satisfyies $1.2 \le h(t) \le 3.0679$.

The associated upper bound over the closed-loop cost function is $J^* = 24.5368$.

The obtained robust and non-fragile optimal guaranteed cost H_{∞} controller guarantees the asymptotic stability, disturbance attenuation γ , $||z(t)||_2 < 0.385 ||w(t)||_2$, and the upper bound of cost function J^* .

5. Conclusion

This paper has considered the problem of delay-dependent stability and guaranteed cost H_{∞} control with interval time-varying delay for an interval system based on Lyapunov–Krasovskii functional approach. The delay-dependent stabilization criterion for guaranteed cost H_{∞} control has been formulated in terms of LMIs. The derivative of the interval time-varying delay is not a restriction, which allows a fast time-varying delay and also is more close to practices control object. A numerical example has shown the effectiveness of the method.

Acknowledgment

This work is supported by the National Science Foundation of China (no. 60134010).

References

- S. Białas, "A necessary and sufficient condition for the stability of interval matrices," International Journal of Control, vol. 37, no. 4, pp. 717–722, 1983.
- [2] M. E. Sezer and D. D. Šiljak, "On stability of interval matrices," IEEE Transactions on Automatic Control, vol. 39, no. 2, pp. 368–371, 1994.
- [3] K. Wang, A. N. Michel, and D. R. Liu, "Necessary and sufficient conditions for the Hurwitz and Schur stability of interval matrices," *IEEE Transactions on Automatic Control*, vol. 39, no. 6, pp. 1251–1255, 1994.

- [4] S. Hu and J. Wang, "On stabilization of a new class of linear time-invariant interval systems via constant state feedback control," *IEEE Transactions on Automatic Control*, vol. 45, no. 11, pp. 2106–2111, 2000.
- [5] K. H. Wei, "Stabilization of linear time-invariant interval systems via constant state feedback control," IEEE Transactions on Automatic Control, vol. 39, no. 1, pp. 22–32, 1994.
- [6] F. X. Wu, Z. K. Shi, and G. Z. Dai, "H_∞ robust control for interval systems," Acta Automatica Sinica, vol. 25, no. 5, pp. 705–708, 1999.
- [7] W.-J. Mao, "Necessary and sufficient condition for robust H_∞ control of dynamic interval systems," in *Proceedings of the 5th World Congress on Intelligent Control and Automation (WCICA '04)*, vol. 1, pp. 733–737, Hangzhou, China, June 2004.
- [8] L. Yu and J. Chu, "An LMI approach to guaranteed cost control of linear uncertain time-delay systems," Automatica, vol. 35, no. 6, pp. 1155–1159, 1999.
- [9] A. Fischman, J. M. Dion, L. Dugard, et al., "A linear matrix inequality approach for guaranteed cost control," in *Proceedings of the 13th IFAC World Congress*, pp. 197–202, International Federation of Automatic Control, San Francisco, Calif, USA, 1996.
- [10] C.-T. Qin and G.-R. Duan, "Optimal robust guaranteed cost control of uncertain linear continuous time systems via dynamical output feedback," in *Proceedings of the 6th World Congress on Intelligent Control and Automation (WCICA '06)*, vol. 1, pp. 2441–2445, Dalian, China, June 2006.
- [11] L. H. Keel and S. P. Bhattacharyya, "Robust, fragile, or optimal?" IEEE Transactions on Automatic Control, vol. 42, no. 8, pp. 1098–1105, 1997.
- [12] W. M. Haddad and J. R. Gorrado, "Robust resilient dynamic controllers for systems with parametric uncertainty and controller gain variations," in *Proceedings of the American Control Conference*, vol. 5, pp. 2837–2841, Philadephia, Pa, USA, 1998.
- [13] A. Jadbabaie, C. T. abdallah, D. Famularo, and P. Dorato, "Robust, non-fragile and optimal controller design via linear matrix inequalities," in *Proceedings of the American Control Conference*, vol. 5, pp. 2842–2846, Philadelphia, Pa, USA, 1998.
- [14] G.-H. Yang, J. L. Wang, and C. Lin, " H_{∞} control for linear systems with additive controller gain variations," *International Journal of Control*, vol. 73, no. 16, pp. 1500–1506, 2000.
- [15] G.-H. Yang and J. L. Wang, "Non-fragile H_∞ control for linear systems with multiplicative controller gain variations," *Automatica*, vol. 37, no. 5, pp. 727–737, 2001.
- [16] J. H. Lee, S. W. Kim, and W. H. Kwon, "Memoryless H_∞ controllers for state delayed systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 1, pp. 159–162, 1994.
- [17] X. Jiang and Q.-L. Han, "On H_{∞} control for linear systems with interval time-varying delay," *Automatica*, vol. 41, no. 12, pp. 2099–2106, 2005.
- [18] X. Jiang and Q.-L. Han, "Delay-dependent robust stability for uncertain linear systems with interval time-varying delay," *Automatica*, vol. 42, no. 6, pp. 1059–1065, 2006.
- [19] Y. He, Q.-G. Wang, and C. Lin, "An Improved H_∞ filter design for systems with time-varying interval delay," *IEEE Transactions on Circuits and Systems II*, vol. 53, no. 11, pp. 1235–1239, 2006.
- [20] Y. He, Q.-G. Wang, C. Lin, and M. Wu, "Delay-range-dependent stability for systems with timevarying delay," Automatica, vol. 43, no. 2, pp. 371–376, 2007.
- [21] X.-G. Liu, M. Wu, R. Martin, and M.-L. Tang, "Delay-dependent stability analysis for uncertain neutral systems with time-varying delays," *Mathematics and Computers in Simulation*, vol. 75, no. 1-2, pp. 15– 27, 2007.
- [22] H. Gao and C. Wang, "Comments and further results on: "a descriptor system approach to H_" control of linear time-delay systems"," *IEEE Transactions on Automatic Control*, vol. 48, no. 3, pp. 520–525, 2003.
- [23] S. Xu and J. Lam, "Improved delay-dependent stability criteria for time-delay systems," IEEE Transactions on Automatic Control, vol. 50, no. 3, pp. 384–387, 2005.
- [24] Y. He, M. Wu, J.-H. She, and G.-P. Liu, "Delay-dependent robust stability criteria for uncertain neutral systems with mixed delays," Systems & Control Letters, vol. 51, no. 1, pp. 57–65, 2004.
- [25] W. Xie, "New LMI-based conditions for quadratic stabilization of LPV systems," Journal of Inequalities and Applications, vol. 2008, Article ID 563845, 12 pages, 2008.
- [26] P. P. Khargonekar, I. R. Petersen, and K. Zhou, "Robust stabilization of uncertain linear systems: quadratic stabilizability and H_{∞} control theory," *IEEE Transactions on Automatic Control*, vol. 35, no. 3, pp. 356–361, 1990.