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Research Article

Stability Analysis and Intermittent Control Synthesis of a Class of Uncertain Nonlinear Systems

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This paper investigates the problem of exponential stabilization for a class of uncertain nonlinear systems by means of periodically intermittent control. Several sufficient conditions of exponential stabilization for this class of uncertain nonlinear systems are formulated in terms of a set of linear matrix inequalities by using quadratic Lyapunov function and inequality analysis technique. Also, the synthesis of stabilization periodically intermittent state feedback controllers is present such that the close-loop system is exponentially stable. A simulation example is given to illustrate the effectiveness of the proposed approach.

1. Introduction

In recent years, significant interest in the study of stability analysis and control design of nonlinear systems has aroused [1–5]. In [4], the problem of the stabilization of affine nonlinear control systems via the center manifold approach was considered. In [5], a stabilizing output feedback model with a predictive control algorithm was proposed for linear systems with input constraints. Recently, incontinuous control techniques such as impulsive control [6] and piecewise feedback control [7] have attracted much attention. In [6], the impulsive control, which makes use of linear static measurement feedback instead of full state feedback for master-slave synchronization schemes that consist of identical chaotic Lur'e systems, was considered. Especially, the recent paper [7] has studied the output regulation problem for a class of discrete-time nonlinear systems under periodic disturbances generated from the so-called exosystems. Furthermore, by exploiting the structural information encoded in the fuzzy rules, a piecewise state feedback and a piecewise error-feedback control laws were constructed to achieve asymptotic rejecting of the unwanted disturbances and/or tracking of the desired trajectories.

Besides these control methods for nonlinear systems mentioned above, intermittent control is a special form of switching control [8]. It has been used for a variety of purposes in engineering fields such as manufacturing, transportation, air-quality control, and communication. Recently, intermittent control has been introduced to chaotic dynamical systems [9–11], in which the method of synchronizing slave-to-master trajectory using intermittent coupling was proposed. However, [9] gave little theoretical analysis of intermittent control systems but only many numerical simulations. In [10], the authors investigated the exponential stabilization problem for a class of chaotic systems with delay by means of periodically intermittent control. In [11], the quasi-synchronization problem for chaotic neural networks with parameter mismatch was formulated via periodically intermittent control. In [12], the problem of the robust stabilization for a class of uncertain linear systems with multiple time-varying delays was investigated. A memoryless state-feedback controller for the robust stabilization of the system was proposed. Based on the Lyapunov method and the linear matrix inequality (LMI) approach, two sufficient conditions for the stability were derived. In [13], a new delay-dependent stability criterion for dynamic systems with time-varying delays and nonlinear perturbations was proposed.

Motivated by the aforementioned discussion, in this paper, we investigate the problem of exponential stabilization of a class of uncertain nonlinear systems by using periodically intermittent control, which is activated in certain nonzero time intervals, and off in other time intervals. Based on Lyapunov stability theory, some exponential stability criteria for this class of uncertain nonlinear systems are given, which have been expressed in terms of linear matrix inequalities (LMIs). A numerical example is given to demonstrate the validity of the result.

The rest of this paper is organized as follows. In Section 2, the intermittent control problem is formulated and some notations and lemmas are introduced. In Section 3, the exponential stabilization problem for a class of uncertain nonlinear systems is investigated by means of periodically intermittent control, and some exponential stability criteria are established. Finally, some conclusions and remarks are drawn in Section 4.

2. Problem Formulation and Preliminaries

Consider a class of nonlinear uncertain systems described as

$$\begin{gathered}\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + f(x(t)),\\ x(t_0) = x_0,\end{gathered} \tag{2.1}$$

where $x \in R^n$ is state vector, and $u \in R^m$ is the external input of system (2.1). $f : R^n \to R^n$ is a continuous nonlinear function with $f(0) = 0$, and there exists a positive definite matrix Q such that $\|f(x)\|^2 \le x^T Q x$ for $x \in R^n$. $\Delta A(t)$ and $\Delta B(t)$ are time-varying uncertainties, which satisfy the following conditions:

$$\Delta A(t) = D_1 F(t) E_1, \qquad \Delta B(t) = D_2 F(t) E_2, \tag{2.2}$$

where D_i, E_i, $i = 1, 2$ are real constant matrices of appropriate dimensions and $F(t)$ is an unknown time-varying matrix with $F^T(t)F(t) \leq I$.

The following lemmas are useful in the proof of our main results.

Lemma 2.1 (see [14]). *Let D, E, and F be real matrices of appropriate dimensions with $F^T F \leq I$, then for any scalar $\varepsilon > 0$, one has the following inequality:*

$$DFE + E^T F^T D^T \leq \varepsilon^{-1} DD^T + \varepsilon E^T E. \tag{2.3}$$

Lemma 2.2 (see [15]). *Let M, N be real matrices of appropriate dimensions. Then, for any matrix $Q > 0$ of appropriate dimension and any scalar $\beta > 0$, the following inequality holds:*

$$MN + N^T M^T \leq \beta^{-1} M Q^{-1} M^T + \beta N^T Q N. \tag{2.4}$$

Lemma 2.3 (see [16]). *Given constant symmetric matrices S_1, S_2, S_3, and $S_1 = S_1^T < 0$, $S_3 = S_3^T > 0$, then $S_1 + S_2 S_3^{-1} S_2^T < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & -S_3 \end{bmatrix} < 0. \tag{2.5}$$

In order to stabilize the system (2.1) by means of periodically intermittent feedback control, we assume that the control imposed on the system is of the following form:

$$u(t) = \begin{cases} Kx(t), & nT \leq t < nT + \tau, \\ 0 & nT + \tau \leq t < (n+1)T, \end{cases} \tag{2.6}$$

where $K \in R^{m \times n}$ is the control gain matrix, $T > 0$ denotes the control period, and $\tau > 0$ is called the control width. Our objective is to design suitable T, τ, and K such that the system (2.1) can be stabilized.

With control law (2.6), system (2.1) can be rewritten as

$$\begin{gathered} \dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))Kx(t) + f(x(t)), \quad nT \leq t < nT + \tau, \\ \dot{x}(t) = (A + \Delta A(t))x(t) + f(x(t)), \quad nT + \tau \leq t < (n+1)T. \end{gathered} \tag{2.7}$$

The above system is classical uncertain switched one where the switching rule only depends on time. Although there are many successful applications of intermittent control, the theoretical analysis on intermittent control system has received little attention. In this paper, we will make a contribution to this issue.

Throughout this paper, we use P^T, $\lambda_{\min}(P)$ $(\lambda_{\max}(P))$ to denote the transpose and the minimum (maximum) eigenvalue of a square matrix P, respectively. The vector (or matrix) norm is taken to be Euclidian, denoted by $\|\cdot\|$. We use $P > 0$ $(< 0, \ \leq 0, \ \geq 0)$ to denote a positive (negative, seminegative, and semipositive) definite matrix P.

3. Exponential Stabilization of a Class of Uncertain Nonlinear System

This section addresses the exponential stability problem of the switched system (2.7). The main result is stated as follows.

Theorem 3.1. *The system* (2.7) *is exponentially stable, if there exists a positive definite matrix* $P > 0$*, scalar constants* $\eta > 0$*,* $\delta > 0$*,* $\varepsilon_{ij} > 0$ $(i = 1, 2,\ j = 1, 2)$*,* $\varepsilon_{13} > 0$*, such that the following LMIs hold:*

$$\begin{bmatrix} \Xi_1 & P & PD_1 & PD_2 \\ P & -\varepsilon_{11}^{-1} I & 0 & 0 \\ D_1^T P & 0 & -\varepsilon_{12}^{-1} I & 0 \\ D_2^T P & 0 & 0 & -\varepsilon_{13}^{-1} \end{bmatrix} < 0, \tag{3.1}$$

$$\begin{bmatrix} A^T P + PA + \varepsilon_{21}^{-1} Q + \varepsilon_{22}^{-1} E_1^T E_1 + \delta I & P & PD_1 \\ P & -\varepsilon_{21}^{-1} I & 0 \\ D_1^T P & 0 & -\varepsilon_{22}^{-1} I \end{bmatrix} < 0, \tag{3.2}$$

where

$$\Xi_1 = A^T P + PA + PBK + K^T B^T P^T + \varepsilon_{11}^{-1} Q + \varepsilon_{12}^{-1} E_1^T E_1 + \varepsilon_{13}^{-1} K^T E_2^T E_2 K + \eta I. \tag{3.3}$$

Moreover, the solution $x(t)$ *satisfies the condition*

$$\|x(t)\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \|x_0\| e^{-((\eta\tau + \delta(T-\tau))/2T\lambda_{\max}(P))(t-\tau)}, \quad \forall t > 0. \tag{3.4}$$

Proof. Consider the following candidate Lyupunov function

$$V(x(t)) = x^T(t) P x(t), \tag{3.5}$$

which implies that

$$\lambda_{\min}(P) \|x(t)\|^2 \le V(x(t)) \le \lambda_{\max}(P) \|x(t)\|^2. \tag{3.6}$$

When $nT \le t < nT + \tau$, the derivative of formula (3.5) with respect to time t along the trajectories of the first subsystem of system (2.7) is calculated and estimated as follows:

$$\begin{aligned} \dot{V}(x(t)) &= x^T(t) \left[(A + \Delta A(t))^T P + P(A + \Delta A(t)) \right] x(t) + x^T(t) P (B + \Delta B(t)) u(t) \\ &\quad + u^T(t) (B + \Delta B(t))^T P x(t) + 2x^T(t) P f(x(t)) \\ &= x^T(t) \left[A^T P + PA + PBK + K^T B^T P \right] x(t) + 2x^T(t) P f(x(t)) \\ &\quad + x^T(t) \left[E_1^T F^T(t) D_1^T P + PD_1 F(t) E_1 + PD_2 F(t) E_2 K + K^T E_2^T F^T(t) D_2^T P \right] x(t). \end{aligned} \tag{3.7}$$

Using Lemmas 2.1 and 2.2, we get

$$
\begin{aligned}
\dot{V}(x(t)) \le{}& x^T(t)\left[A^T P + PA + PBK + K^T B^T P\right]x(t) + \varepsilon_{11}x^T(t)PPx(t) + \varepsilon_{11}^{-1}\left\|f(x(t))\right\|^2 \\
&+ x^T(t)\left[\varepsilon_{12}^{-1}E_1^T E_1 + \varepsilon_{12}PD_1D_1^T P + \varepsilon_{13}^{-1}K^T E_2^T E_2 K + \varepsilon_{13}PD_2D_2^T P\right]x(t) \\
\le{}& x^T(t)\Big[A^T P + PA + PBK + K^T B^T P + \varepsilon_{12}^{-1}E_1^T E_1 + \varepsilon_{12}PD_1D_1^T P + \varepsilon_{13}^{-1}K^T E_2^T E_2 K \\
&+\varepsilon_{13}PD_2D_2^T P + \varepsilon_{11}PP + \varepsilon_{11}^{-1}Q\Big]x(t).
\end{aligned}
\tag{3.8}
$$

From formula (3.1) and Lemma 2.3, we have

$$
\Xi_1 + \varepsilon_{11}PP + \varepsilon_{12}PD_1D_1^T P + \varepsilon_{13}PD_2D_2^T P < 0. \tag{3.9}
$$

Hence, we get

$$
\begin{aligned}
\dot{V}(x(t)) &\le -\eta x^T(t)x(t) \\
&\le -c_1 V(x(t)),
\end{aligned}
\tag{3.10}
$$

where $c_1 = \eta/\lambda_{\max}(P)$.

Thus, we have

$$
\dot{V}(x(t)) \le -c_1 V(x(t)), \quad nT \le t < nT + \tau, \tag{3.11}
$$

which implies that when $nT \le t < nT + \tau$

$$
V(x(t)) \le V(x(nT))e^{-c_1(t-nT)}. \tag{3.12}
$$

Similarly, when $nT + \tau \le t < (n+1)T$, we have

$$
\begin{aligned}
\dot{V}(x(t)) &= x^T(t)\left[(A + \Delta A(t))^T P + P(A + \Delta A(t))\right]x(t) + 2x^T(t)Pf(x(t)) \\
&= x^T(t)\left[A^T P + PA\right]x(t) + 2x^T(t)Pf(x(t)) + x^T(t)\left[E_1^T F^T(t)D_1^T P + PD_1F(t)E_1\right]x(t) \\
&\le x^T(t)\left[A^T P + PA + \varepsilon_{21}PP + \varepsilon_{21}^{-1}Q + \varepsilon_{22}^{-1}E_1^T E_1 + \varepsilon_{22}PD_1D_1^T P\right]x(t).
\end{aligned}
\tag{3.13}
$$

From formula (3.2) and Lemma 2.3, we have

$$
A^T P + PA + \varepsilon_{21}PP + \varepsilon_{21}^{-1}Q + \varepsilon_{22}^{-1}E_1^T E_1 + \varepsilon_{22}PD_1D_1^T P + \delta I < 0, \tag{3.14}
$$

Hence, it is obtained that

$$\begin{aligned}\dot{V}(x(t)) &\leq -\delta x^T(t)x(t)\\ &\leq -c_2 V(x(t)),\end{aligned} \tag{3.15}$$

where $c_2 = \delta/\lambda_{\max}(P)$.

So, we derive that when $nT + \tau \leq t < (n+1)T$,

$$\dot{V}(x(t)) \leq -c_2 V(x(t)), \tag{3.16}$$

$$V(x(t)) \leq V(x(nT+\tau))e^{-c_2(t-nT-\tau)}. \tag{3.17}$$

From inequalities (3.12) and (3.17), we have the following.

When $0 \leq t < \tau$, $V(x(t)) \leq V(x_0)e^{-c_1 t}$ and $V(x(\tau)) \leq V(x_0)e^{-c_1\tau}$.

When $\tau \leq t < T$,

$$\begin{aligned}V(x(t)) &\leq V(x(\tau))e^{-c_2(t-\tau)}\\ &\leq V(x_0)e^{-(c_1\tau + c_2(t-\tau))},\\ V(x(T)) &\leq V(x_0)e^{-(c_1\tau + c_2(T-\tau))}.\end{aligned} \tag{3.18}$$

When $T \leq t < T + \tau$,

$$\begin{aligned}V(x(t)) &\leq V(x(T))e^{-c_1(t-T)}\\ &\leq V(x_0)e^{-(c_1\tau + c_2(T-\tau) + c_1(t-T))},\\ V(x(T+\tau)) &\leq V(x_0)e^{-(2c_1\tau + c_2(T-\tau))}.\end{aligned} \tag{3.19}$$

When $T + \tau \leq t < 2T$,

$$\begin{aligned}V(x(t)) &\leq V(x(T+\tau))e^{-c_2(t-T-\tau)}\\ &\leq V(x_0)e^{-(2c_1\tau + c_2(T-\tau) + c_2(t-T-\tau))},\\ V(x(2T)) &\leq V(x_0)e^{-(2c_1\tau + 2c_2(T-\tau))}.\end{aligned} \tag{3.20}$$

When $2T \leq t < 2T + \tau$,

$$\begin{aligned}V(x(t)) &\leq V(x(2T))e^{-c_1(t-2T)}\\ &\leq V(x_0)e^{-(2c_1\tau + 2c_2(T-\tau) + c_1(t-2T))},\\ V(x(2T+\tau)) &\leq V(x_0)e^{-(3c_1\tau + 2c_2(T-\tau))}.\end{aligned} \tag{3.21}$$

When $2T + \tau \leq t < 3T$,

$$\begin{aligned} V(x(t)) &\leq V(x(2T+\tau))e^{-c_2(t-2T-\tau)} \\ &\leq V(x_0)e^{-(3c_1\tau+2c_2(T-\tau)+c_2(t-2T-\tau))}, \\ V(x(3T)) &\leq V(x_0)e^{-(3c_1\tau+2c_2(T-\tau)+c_2(T-\tau))} \\ &= V(x_0)e^{-(3c_1\tau+3c_2(T-\tau))}. \end{aligned} \tag{3.22}$$

When $nT \leq t < nT + \tau$, that is, $(t-\tau)/T < n \leq t/T$,

$$\begin{aligned} V(x(t)) &\leq V(x(nT))e^{-c_1(t-nT)} \\ &\leq V(x_0)e^{-(nc_1\tau+nc_2(T-\tau))}e^{-c_1(t-nT)} \\ &\leq V(x_0)e^{-(nc_1\tau+nc_2(T-\tau))} \\ &\leq V(x_0)e^{-((c_1\tau+c_2(T-\tau))/T)(t-\tau)}. \end{aligned} \tag{3.23}$$

When $nT + \tau \leq t < (n+1)T$, that is, $t/T < n+1 < (t-\tau+T)/T$,

$$\begin{aligned} V(x(t)) &\leq V(x(nT+\tau))e^{-c_2(t-nT-\tau)} \\ &\leq V(x_0)e^{-((c_1\tau+c_2(T-\tau))/T)(t-\tau)}e^{-c_2(t-nT-\tau)} \\ &\leq V(x_0)e^{-((c_1\tau+c_2(T-\tau))/T)(t-\tau)}. \end{aligned} \tag{3.24}$$

From inequalities (3.23) and (3.24), it follows that for any $t > 0$,

$$\begin{aligned} x^T(t)x(t) &\leq \frac{1}{\lambda_{\min}(P)}V(x_0)e^{-((c_1\tau+c_2(T-\tau))/T)(t-\tau)} \\ &\leq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\|x_0\|^2e^{-((c_1\tau+c_2(T-\tau))/T)(t-\tau)}. \end{aligned} \tag{3.25}$$

Hence, we get

$$\|x(t)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|x_0\|e^{-((c_1\tau+c_2(T-\tau))/2T)(t-\tau)}, \quad \forall t > 0, \tag{3.26}$$

that is,

$$\|x(t)\| \leq \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|x_0\|e^{-(\eta\tau+\delta(T-\tau)/2T\lambda_{\max}(P))(t-\tau)}, \quad \forall t > 0, \tag{3.27}$$

which concludes the proof. □

Remark 3.2. In [17], the problem of an exponential stability for time-delay systems with interval time-varying delays and nonlinear perturbations was investigated. Based on the Lyapunov method, a new delay-dependent criterion for exponential stability is established in terms of LMI. However, in [17], the control is not concerned in the systems. In our paper, as $\tau \to T$, the periodic feedback will be reduced to the general continuous feedback. In this case, formula (3.1) gives an exponential stability criterion for the system (2.1) with continuous feedback control $u(t) = Kx(t)$. Hence, our result have a wider area of applications.

Corollary 3.3. *If there exist a symmetric and positive definite matrix $P > 0$, scalar constants $\eta > 0$, $\delta > 0$, $\varepsilon_j > 0$ $(j = 1, 2, 3)$, such that the following LMIs hold:*

$$\begin{bmatrix} PBK + K^T B^T P + \varepsilon_3^{-1} K^T E_2^T E_2 K + \eta I - \delta I & PD_2 \\ D_2^T P & -\varepsilon_3^{-1} I \end{bmatrix} < 0, \tag{3.28}$$

$$\begin{bmatrix} A^T P + PA + \varepsilon_1^{-1} Q + \varepsilon_2^{-1} E_1^T E_1 + \delta I & P & PD_1 \\ P & -\varepsilon_1^{-1} I & 0 \\ D_1^T P & 0 & -\varepsilon_2^{-1} I \end{bmatrix} < 0, \tag{3.29}$$

then the system (2.7) *is exponentially stable, and moreover,*

$$\|x(t)\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \|x_0\| e^{-(\eta\tau + \delta(T-\tau)/2T\lambda_{\max}(P))(t-\tau)}, \quad \forall t > 0. \tag{3.30}$$

Proof. Set $\varepsilon_{11} = \varepsilon_{21} = \varepsilon_1$, $\varepsilon_{12} = \varepsilon_{22} = \varepsilon_2$, and $\varepsilon_{13} = \varepsilon_3$. From (3.29) and Lemma 2.3, we get

$$\begin{aligned} &A^T P + PA + \varepsilon_{21} PP + \varepsilon_{21}^{-1} Q + \varepsilon_{22}^{-1} E_1^T E_1 + \varepsilon_{22} PD_1 D_1^T P \\ &\quad = A^T P + PA + \varepsilon_1 PP + \varepsilon_1^{-1} Q + \varepsilon_2^{-1} E_1^T E_1 + \varepsilon_2 PD_1 D_1^T P \\ &\quad < -\delta I. \end{aligned} \tag{3.31}$$

So, formula (3.2) holds. From formulae (3.31), (3.28), and Lemma 2.3, we obtain

$$\begin{aligned} &A^T P + PA + PBK + K^T B^T P + \varepsilon_{11} PP + \varepsilon_{11}^{-1} Q + \varepsilon_{12}^{-1} E_1^T E_1 + \varepsilon_{12} PD_1 D_1^T P \\ &\quad + \varepsilon_{13}^{-1} K^T E_2^T E_2 K + \varepsilon_{13} PD_2 D_2^T P + \eta I \\ &\quad = A^T P + PA + PBK + K^T B^T P + \varepsilon_1 PP + \varepsilon_1^{-1} Q + \varepsilon_2^{-1} E_1^T E_1 + \varepsilon_2 PD_1 D_1^T P \\ &\quad + \varepsilon_3^{-1} K^T E_2^T E_2 K + \varepsilon_3 PD_2 D_2^T P + \eta I \\ &\quad < PBK + K^T B^T P + \varepsilon_3^{-1} K^T E_2^T E_2 K + \varepsilon_3 PD_2 D_2^T P + \eta I - \delta I < 0. \end{aligned} \tag{3.32}$$

So, formula (3.1) holds. According to Theorem 3.1, the conclusion is obtained. □

Now, we consider the following uncertain nonlinear system

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (I + \Delta F(t))Bu(t) + f(x(t)),$$
$$x(t_0) = x_0, \tag{3.33}$$

where $x \in R^n$, $u \in R^n$, B is inverse. $\Delta A(t)$ and $\Delta F(t)$ are time-varying uncertainties with $\Delta F^T(t)\Delta F(t) \le I$ and satisfy $\Delta A(t) = D\Delta FE$, in which D and E are real constant matrices of appropriate dimensions. $f : R^n \to R^n$ is a continuous nonlinear function satisfying $f(0) = 0$, and there exists a positive definite matrix Q such that $\|f(x)\|^2 \le x^T Qx$ for $x \in R^n$.

Consider the following control law:

$$u(t) = \begin{cases} kB^{-1}x(t), & nT \le t < nT + \tau, \\ 0, & nT + \tau \le t < (n+1)T, \end{cases} \tag{3.34}$$

where $k \in R$. Then, the system (3.33) with formula (3.34) can be rewritten as

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (I + \Delta F(t))kx(t) + f(x(t)), \quad nT \le t < nT + \tau,$$
$$\dot{x}(t) = (A + \Delta A(t))x(t) + f(x(t)), \quad nT + \tau \le t < (n+1)T. \tag{3.35}$$

Theorem 3.4. *If there exist a symmetric and positive definite matrix $P > 0$, scalar constants $\eta > 0$, $\delta > 0$, $\varepsilon_j > 0$ $(i, j = 1, 2)$, $\varepsilon_{13} > 0$, k, such that the following LMIs hold:*

$$\begin{bmatrix} A^T P + PA + 2kP + \varepsilon_{11} Q + \varepsilon_{12}^{-1} E_1^T E_1 + \varepsilon_{13}^{-1} k^2 I + \eta I & P & PD \\ P & -\left(\varepsilon_{13} + \varepsilon_{11}^{-1}\right)^{-1} I & 0 \\ D^T P & 0 & -\varepsilon_{12}^{-1} I \end{bmatrix} < 0, \tag{3.36}$$

$$\begin{bmatrix} A^T P + PA + \varepsilon_{21} Q + \varepsilon_{22}^{-1} E^T E + \delta I & P & PD \\ P & \varepsilon_{21} I & 0 \\ D^T P & 0 & \varepsilon_{22}^{-1} \end{bmatrix} < 0, \tag{3.37}$$

then the system (3.35) *is exponentially stable, and moreover,*

$$\|x(t)\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \|x_0\| e^{-((\eta\tau + \delta(T-\tau))/2T\lambda_{\max}(P))(t-\tau)}, \quad \forall t > 0. \tag{3.38}$$

Proof. Consider the candidate Lyupunov function (3.5).

When $nT \le t < nT + \tau$, the derivative of Lyupunov function (3.5) with respect to time t along the trajectories of the first subsystem of system (3.35) is calculated and estimated as follows:

$$\begin{aligned}\dot{V}(x(t)) &= \left[(A + \Delta A(t))x(t) + (I + \Delta F(t))kx(t) + f(x(t))\right]^T Px(t) \\ &\quad + x^T(t)P\left[(A + \Delta A(t))x(t) + (I + \Delta F(t))kx(t) + f(x(t))\right] \\ &= x^T(t)\left[A^T P + PA + 2kP\right]x(t) + 2x^T(t)Pf(x(t)) \\ &\quad + x^T(t)\left[E^T \Delta F^T(t)D^T P + PD\Delta F(t)E\right]x(t) + 2kx^T(t)P\Delta F(t)x(t) \\ &\le x^T(t)\left[A^T P+PA+2kP+\varepsilon_{11}^{-1}PP+\varepsilon_{11}Q+\varepsilon_{12}^{-1}E^T E+\varepsilon_{12}PDD^T P+\varepsilon_{13}^{-1}k^2 I+\varepsilon_{13}PP\right]x(t).\end{aligned} \tag{3.39}$$

From formula (3.36) and Lemma 2.3, we have

$$\begin{aligned}\dot{V}(x(t)) &\le -\eta x^T(t)x(t), \\ &\le -c_1 V(x(t)),\end{aligned} \tag{3.40}$$

where $c_1 = \eta/\lambda_{\max}(P)$.

Thus, we have

$$\dot{V}(x(t)) \le -c_1 V(x(t)), \quad nT \le t < nT + \tau, \tag{3.41}$$

which implies that when $nT \le t < nT + \tau$,

$$V(x(t)) \le V(x(nT))e^{-c_1(t-nT)}. \tag{3.42}$$

Similarly, when $nT + \tau \le t < (n + 1)T$, we have

$$\begin{aligned}\dot{V}(x(t)) &= x^T(t)\left[A^T P + PA\right]x(t) + 2x^T(t)Pf(x(t)) + x^T(t)\left[(\Delta A(t))^T P + P\Delta A(t)\right]x(t) \\ &\le x^T(t)\left[A^T P + PA + \varepsilon_{22}^{-1}E^T E + \varepsilon_{22}PDD^T P\right]x(t) + \varepsilon_{21}^{-1}x^T(t)PPx(t) + \varepsilon_{21}\|f(x(t))\|^2 \\ &\le x^T(t)\left[A^T P + PA + \varepsilon_{21}^{-1}PP + \varepsilon_{21}Q + \varepsilon_{22}^{-1}E^T E + \varepsilon_{22}PDD^T P\right]x(t) \\ &\le -c_2 V(x(t)),\end{aligned} \tag{3.43}$$

where $c_2 = \delta/\lambda_{\max}(P)$.

So, we derive that when $nT + \tau \le t < (n+1)T$,

$$\begin{aligned} V(x(t)) &\le -c_2 V(x(t)), \\ V(x(t)) &\le V(x(nT+\tau))e^{-c_2(t-nT-\tau)}. \end{aligned} \tag{3.44}$$

Similar to the proof in Theorem 3.1, we can get

$$\|x(t)\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|x_0\|e^{-((c_1\tau+c_2(T-\tau))/2T)(t-\tau)}, \quad \forall t > 0, \tag{3.45}$$

that is,

$$\|x(t)\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\|x_0\|e^{-((\eta\tau+\delta(T-\tau))/2T\lambda_{\max}(P))(t-\tau)}, \quad \forall t > 0, \tag{3.46}$$

which completes the proof. □

Example 3.5. Consider the system (2.1) with

$$\begin{aligned} &A = \begin{pmatrix} -10 & 2 \\ 2 & -10 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad f(x) = \begin{pmatrix} x_2(t)\sin x_1(t) \\ x_1(t)\cos x_2(t) \end{pmatrix}, \quad K = \begin{pmatrix} 0.01 & 0.2 \end{pmatrix}, \\ &E_1 = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}, \quad D_1 = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}, \quad D_2 = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}. \end{aligned} \tag{3.47}$$

It is obvious that $Q = I$.

For the positive numbers $\eta = 0.5$, $\delta = 2$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, by solving LMIs of Corollary 3.3, we obtain

$$P = \begin{pmatrix} 0.6474 & 0.0745 \\ 0.0745 & 0.5723 \end{pmatrix}. \tag{3.48}$$

Therefore, the system is robustly exponentially stabilizable with feedback control

$$u(t) = \begin{cases} 0.01x_1(t) + 0.2x_2(t), & nT \le t < nT+\tau, \\ 0, & nT+\tau \le t < (n+1)T, \end{cases} \tag{3.49}$$

and the solution of the system satisfies

$$\|x(t)\| \le 1.1476\|x_0\|e^{-((2T-1.5\tau)/1.3866T)(t-\tau)}, \quad \forall t > 0. \tag{3.50}$$

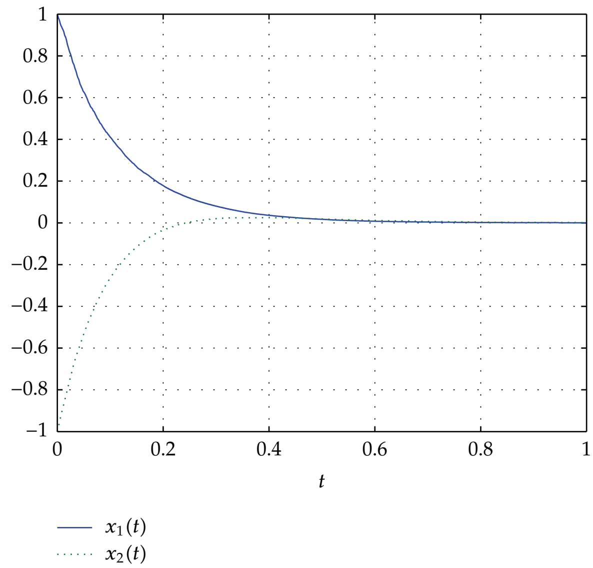

Figure 1: The state x_1 and x_2 of the closed-loop system in Example 3.5.

Simulation result is shown in Figure 1 for the initial condition $x_0 = (1 \quad -1)^T$, $T = 0.2$, $\tau = 0.1$, and $F(t) = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$, where α and β are random constants between 0 and 1. It is seen from Figure 1 that the closed-loop system is exponentially stable.

4. Conclusions

In this paper, we deal with the exponential stabilization problem of a class of uncertain nonlinear systems by means of periodically intermittent control. Based on Lyapunov function approach, several stability criteria have been given in terms of a set of linear matrix inequalities, and stabilization periodically intermittent state feedback controllers are proposed. Finally, a numerical example is provided to show the high performance of the proposed approach.

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