Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2007, Article ID 78702, 11 pages
doi:10.1155/2007/78702

Research Article

Least Squares Fitting of Piecewise Algebraic Curves

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Received 25 March 2007; Revised 4 June 2007; Accepted 18 October 2007

Recommended by T. Zolezzi

A piecewise algebraic curve is defined as the zero contour of a bivariate spline. In this paper, we present a new method for fitting C^1 piecewise algebraic curves of degree 2 over type-2 triangulation to the given scattered data. By simultaneously approximating points, associated normals and tangents, and points constraints, the energy term is also considered in the method. Moreover, some examples are presented.

1. Introduction

The curve fitting problem is very important in CAD (computer-aided design) and CAGD (computer-aided geometric design). The reconstruction of planar shape from data (possible unorganized or noisy) is a very interesting subject with various applications, for example, in medical imaging.

When compared to these representations, the use of implicitly defined curves offers a number of advantages (see [1]). The main advantages of the implicitly defined algebraic curves over the functional and parametric curves are as follows. (1) The class of algebraic curves is closed under several geometric operations (intersections, union, offset, etc.), often required in a solid modeling system. For example, the offset of a parametric curve may not be parametric but is always algebraic and has an implicit representation. (2) Implicit algebraic curve segments have more degrees of freedom compared with rational function and rational parametric curves of the same degree. Then implicit algebraic curve segments appear to be more flexible to approximate a complicated curve with fewer number of pieces or to achieve higher order of smoothness.

Various methods for the implicit curves and surfaces approximation and fitting have been described in the vast literatures (see [1–10]). Carr et al. [3] used the polyharmonic radial basis functions to reconstruct smooth, manifold surfaces from point-cloud data and to repair incomplete meshes. Pratt [7] and Taubin [8] introduced methods for curve and surface fitting. The methods are based on algebraic distance, combined with suitable normalization of unknown coefficients. Taubin [8] and Tarel [9] method constrained the sum of the squared gradients at the data sites. This leads to a geometrically invariant quadratic normalization. Bajaj et al. [1] and Bajaj and Xu [2] have developed nonproduct algebraic spline curves and surfaces over triangulations, which are called A-splines, into a powerful tool for reconstructing curves and surfaces from measurement data. Their approach focuses on the use of low-degree patches whose coefficients satisfy certain sign conditions in order to guarantee the desired topology of results.

Jüttler [4, 5] described an algorithm for fitting implicit defined tensor-product spline curves and surfaces to scattered data. Wang et al. [10] and Yang et al. [11] used the implicit defined tensor-product spline curves and surfaces for fitting and blending . The main advantage of their methods is being computationally simple, and the main disadvantage is that the degrees of curves and surfaces are high. For example, the degree is 6 totally, in fact, of a tensor-product spline of degree 3 .

In this paper, we present a new technique for curve fitting with piecewise algebraic curves of a lower degree. We consider a set of points

$$\mathbf{p}_i = (p_{i1}, p_{i2}) \in R^2, \quad i = 1,\dots,N \tag{1.1}$$

in the plane. The approximation curve is to be described as the zero contour of a nontensor product C^1 bivariate spline of degree 2 over type-2 triangulation. This method is computationally simple.

This paper is organized as follows. In Section 2, the bivariate spline space $S_2^1(\triangle_{mn}^{(1)})$ and piecewise algebraic curves are introduced. Furthermore, we present the method for fitting piecewise algebraic curves to scattered data in Section 3. At last, some examples are given.

2. Bivariate spline space $S_2^1(\triangle_{mn}^{(1)})$

In this section, we will introduce the bivariate spline space $S_2^1(\triangle_{mn}^{(2)})$ firstly, which is presented in [12, 13].

The type-2 triangulation $\triangle_{mn}^{(2)}$ is yielded by connecting two diagonals at each small rectangular cell, which are based on rectangular regions. Clearly, if the original rectangular partition is uniform, then the induced type-2 triangulation is a cross-cut partition. We only discuss uniform type-2 triangulations in this section.

Without loss of generality, let D be a unit square region as follows:

$$D = [0,1] \otimes [0,1]. \tag{2.1}$$

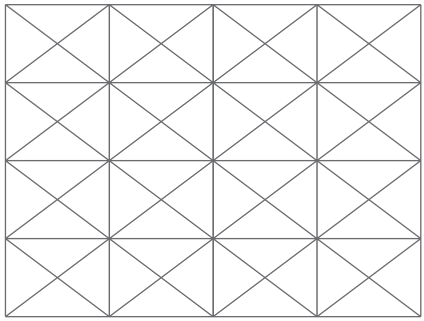
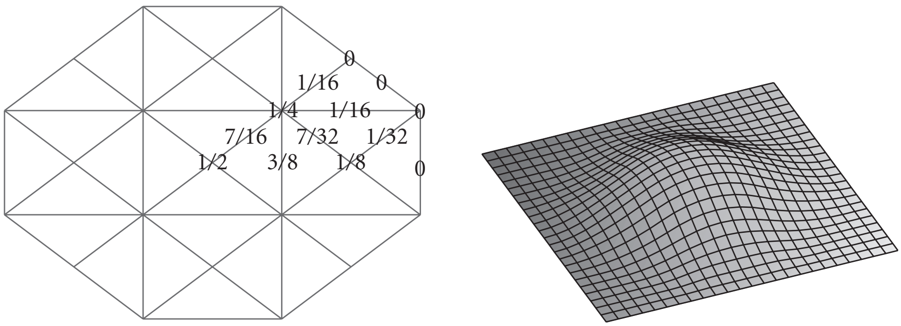


FIGURE 2.1. Type-2 triangulation with $m = 4$, $n = 4$.



(a) Support of $B(x, y)$

(b) B-spline base

FIGURE 2.2. The support and B-spline base of $B(x, y)$ over $\triangle_{mn}^{(2)}$.

The type-2 triangulation $\triangle_{mn}^{(2)}$ is yielded by the following partition lines (see Figure 2.1):

$$mx - i = 0, \qquad ny - i = 0, \qquad ny - mx - i = 0, \qquad ny + mx - i = 0, \tag{2.2}$$

where $i = \dots, -1, 0, 1, \dots$.

Using the dimension formulae on the cross-cut partition in [12, 13], we have

$$\dim S_2^1\left(\triangle_{mn}^{(2)}\right) = (m+2)(n+2) - 1. \tag{2.3}$$

We first introduce a locally supported spline in $S_2^1(\triangle_{mn}^{(2)})$ with its support octagon Q centered at $(0,0)$ as shown in Figure 2.2(a). It is known that a bivariate polynomial of degree 2 on a triangle can be uniquely determined by the values of three vertices and three midpoints of the edges. In Figure 2.2(a), the values are given on some triangles, and other values are obtained by the symmetry of lines $x = 0$, $y = 0$, $x + y = 0$, $x - y = 0$. Let $B(x, y)$ be a piecewise polynomial defined in R^2, that is, zero outside of Q, and let its representation in the every triangle of D be determined by the values.

Clearly, $B(x,y) \in C^1(R^2)$, and $B(x,y) > 0$ inside of Q. Hence, $B(x,y)$ is a bivariate B-spline over the partition as shown in Figure 2.2(b). Making use of conformality conditions at mesh points, we can get that $B(x,y)$ is uniquely determined by the symmetry of lines $x = 0$, $y = 0$, $x + y = 0$, $x - y = 0$, and normalized condition $B(0,0) = 1/2$. By the smoothing cofactor-conformality method, we can point out that the support of $B(x,y)$ is the smallest one [12, 13].

Let

$$B_{ij}(x,y) = B\left(mx - i + \frac{1}{2}, ny - j + \frac{1}{2}\right), \tag{2.4}$$

then collection

$$A = \{B_{ij}(x,y) : i = 0,\dots,m+1,\ j = 0,\dots,n+1\} \tag{2.5}$$

is a subspace of $S_2^1(\triangle_{mn}^{(2)})$. Note that each element of A is a nontrivial element of $S_2^1(\triangle_{mn}^{(2)})$, and the number of elements in A is $(m+2)(n+2)$. By using formulae (2.3), A must be a linearly dependent set. Wang gave the following results in [13].

Theorem 2.1. *The bivariate B-spline functions of A defined by (2.4) satisfy*

$$\sum_{i=0}^{m+1}\sum_{j=0}^{n+1}(-1)^{i+j}B_{ij} = 0. \tag{2.6}$$

For any i_0, j_0, $0 \le i_0 \le m+1$, $0 \le j_0 \le n+1$, the collection

$$A_{i_0 j_0} = \{B_{ij}(x,y) \in A : (i,j) \neq (i_0, j_0)\} \tag{2.7}$$

is a basis of $S_2^1(\triangle_{mn}^{(2)})$.

Theorem 2.2. *Bivariate B-spline functions in A have the properties*

$$\sum_{i=0}^{m+1}\sum_{j=0}^{n+1}(-1)^{i+j}B_{ij}(x,y) \equiv 0, \qquad \sum_{ij}B_{ij}(x,y) \equiv 1. \tag{2.8}$$

By Theorem 2.1, for each bivariate spline $s(x,y) \in S_2^1(\triangle_{mn}^{(2)})$, there must exist $c_{ij} \in R$, $i = 0,\dots,m+1$, $j = 0,\dots,n+1$ such that

$$s(x,y) = \sum_{i=0}^{m+1}\sum_{j=0}^{n+1}c_{ij}B_{ij}(x,y). \tag{2.9}$$

The curve

$$Z(s) := \left\{(x,y) \in D \mid s(x,y) = 0,\ s(x,y) \in S_2^1\left(\triangle_{mn}^{(2)}\right)\right\} \tag{2.10}$$

is called a C^1 *piecewise algebraic curve* of degree 2. For convenience, it is also called a piecewise algebraic curve. It is obvious that the piecewise algebraic curve is a generalization of the classical algebraic curve [13]. The recent researches on the piecewise algebraic curves can be referred to [13–16]. In this paper, we use the piecewise algebraic curves for fitting the given points.

3. Fitting piecewise algebraic curves to data

Let $\Pi = \{\mathbf{p}_i\}_{i=1}^{N} \in \Omega$ be the point set for fitting, where Ω is a square region. First, we partition the region Ω to the uniform type-2 triangulation $\triangle_{mn}^{(2)}$ as described in Section 2. Then, we get the B-spline basis of $S_2^1(\triangle_{mn}^{(2)})$ by (2.4), and a spline $s(x,y) \in S_2^1(\triangle_{mn}^{(2)})$ can be expressed as

$$s(x,y) = \sum_{i=0}^{m+1}\sum_{j=0}^{n+1} c_{ij}B_{ij}(x,y) \tag{3.1}$$

with real coefficients c_{ij}.

Let $\mathbf{c} = (c_{ij})_{(i,j)\in I}$ be the vector obtained by gathering all coefficients of the fitting piecewise algebraic curve, in a suitable order, where $I = \{(i,j) \mid i = 0,\dots,m+1,\ j = 0,\dots,n+1\}$. Its components will be computed by minimizing a quadratic objective function, which is formed as a certain linear combination of 2 terms, and the points constraints.

3.1. Approximation of data. The first term deals with the point set Π. The given data are approximated by minimizing the sum of squared "algebraic distances" (see [7, 8]),

$$L(\mathbf{c}) = \sum_{i=1}^{N} s^2(p_{i1}, p_{i2}). \tag{3.2}$$

The sum L is a homogeneous quadratic form of the unknown coefficients vector $\mathbf{c}$. Hence it is minimized by the null vector $\mathbf{c} = \mathbf{0}$, leading to the spline $s(x,y) = 0$. In order to avoid this result, one has to add other terms or normalizations. Various normalizations have been presented in the literature [7, 8, 11]. Most of them are based on a suitable norm in the coefficient space. Jüttler [4, 5] used the method based on the simultaneous approximation of the data and the associated normal vectors. For being computationally simple and controlling the shape of the fitting curve, our approach is based on the associated normals, tangents, and the points constraints which will be introduced in the next sections.

3.2. Fitting curves to associated normals and tangents. As the first step of this section, we generate the unit normal vectors $\{\mathbf{n}_i = (n_{i1}, n_{i2}),\ i = 1,\dots,N\}$ and tangent vectors $\{\mathbf{s}_i = (s_{i1}, s_{i2}),\ i = 1,\dots,N\}$, which are associated with the given dataset $\{\mathbf{p}_i = (p_{i1}, p_{i2}),\ i = 1,\dots,N\}$. The estimation of normal and tangent vectors from scattered data is a standard problem in curve and surface approximation. In this paper, we use the method presented in [6] and summarize it as follows. More details can be found in [5, 6].

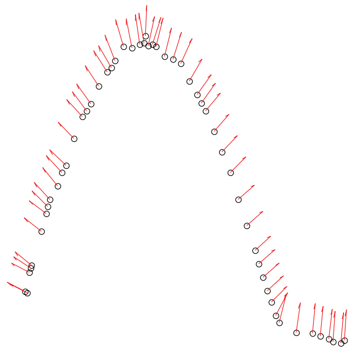


FIGURE 3.1. The estimated normal vectors of the dataset.

Take normal vectors for example. Firstly, for a point $\mathbf{p}_i = (p_{i1}, p_{i2})$, a local regression line $L_i : y = ax + b$ can be computed by minimizing a quadratic function

$$D_l = \sum_{i=1}^{N} (ap_{i1} + b - p_{i2})^2 w_i, \tag{3.3}$$

where w_i is a nonnegative weight for each point $\mathbf{p}_i$ computed by an appropriate weighting function. We can choose any weighting function which generates larger penalties for the points far from $\mathbf{p}_i$. One of our choices is $w_i = e^{-r^2/H^2}$, where $r = \|\mathbf{p}_i - \mathbf{p}_j\|$, and H is a suitable real constant. From this weighted regression, we can compute the local best regression line L_i for $\mathbf{p}_i$ by minimizing the objective function subject to the quadratic equality constraint $\|\mathbf{n}^*\| = 1$, where $\mathbf{n}^*$ is the unit normal vector of L_i.

Secondly, choose a new Cartesian coordinate system whose x axis is parallel to the line L_i, and $\mathbf{p}_i$ is a new origin. Then we fit a local quadratic regression curve $Q_i : \hat{y} = a\hat{x}^2 + b\hat{x} + c$ to the dataset $\{\hat{\mathbf{p}}_i, i = 1, \ldots, N\}$. Its coefficients are computed by minimizing the weighted sum

$$D_q = \sum_{i=1}^{N} \left(a\widehat{p_{i1}}^2 + b\widehat{p_{i1}} + c - \widehat{p_{i2}}\right)^2 w_i. \tag{3.4}$$

The direction $\pm\mathbf{n}_i$ of the unit normal vector which is associated with $\mathbf{p}_i$ is chosen such that it is parallel to the normal of the quadratic polynomial Q_i. In order to get the useful results, we have to guarantee that the neighboring normal vectors $\mathbf{n}_i, \mathbf{n}_j$ have same orientations, that is, $\mathbf{n}_i \cdot \mathbf{n}_j \geq 0$. The unit tangent vector $\mathbf{s}_i$ associated with $\mathbf{p}_i$ can be computed by $\mathbf{n}_i \cdot \mathbf{s}_i = 0$, and the neighboring tangent vectors also have the same orientations. Figure 3.1 shows the estimated normal vectors of the dataset for example.

In addition to the algebraic distance $L(\mathbf{c})$, the following term:

$$M(\mathbf{c}) = \sum_{i=1}^{N} \left(\nabla s(p_{i1}, p_{i2}) \cdot \mathbf{n}_i\right)^2 + \sum_{i=1}^{N} \left(\nabla s(p_{i1}, p_{i2}) \cdot \mathbf{s}_i - 1\right)^2 \tag{3.5}$$

can lead to the nonzero result **c** by minimizing the sum, where $\nabla s(p_{i1}, p_{i2}) = (s_x, s_y)|_{(p_{i1}, p_{i2})}$ is the gradient of the spline $s(x, y)$ at the point $\mathbf{p}_i = (p_{i1}, p_{i2})$, and $\mathbf{n}_i$, $\mathbf{s}_i$ are the unit estimated normal vector and tangent vector at $\mathbf{p}_i$ by above method.

3.3. Points constraints. In order to get results more useful and pleasant, we need to add some constraints to the object function. Our approach adds some point restrictions as an auxiliary means.

A piecewise algebraic curve $Z(s)$ divides Ω into three parts: the curve itself $s(x, y) = 0$, the interior of the surface $s(x, y) < 0$, and the exterior of the surface $s(x, y) > 0$.

For some points in Π are accurate by the measurements, the fitting curve is required to pass through them. On the other hand, for obtaining a desirable shape of the fitting curve, we impose the constraint that a point set is inside (or outside) the piecewise algebraic curve. Thus we need to add the following constraints to the objective function:

$$s(\mathbf{p}_i) = 0, \quad i = i_1, \ldots, i_l, \qquad s(\mathbf{d}_i) \le 0, \quad i = 1, \ldots, k, \qquad s(\mathbf{g}_i) \ge 0, \quad i = 1, \ldots, r. \tag{3.6}$$

Wang gave the following result in [13].

Theorem 3.1. *A point set is the interpolation set for $S_2^1(\triangle_{mn}^{(2)})$ if and only if there is no nonzero spline $h \in S_2^1(\triangle_{mn}^{(2)})$ such that the point set lies on the piecewise algebraic curve $Z(h)$.*

By using the above result, the point set $\{\mathbf{p}_i\}_{i=i_1}^{i_l}$ in (3.6) must be chosen properly such that it does not include an interpolation set for $S_2^1(\triangle_{mn}^{(2)})$ or its subspace. The point sets $\mathbf{d}_i$ and $\mathbf{g}_i$ can be chosen interactively.

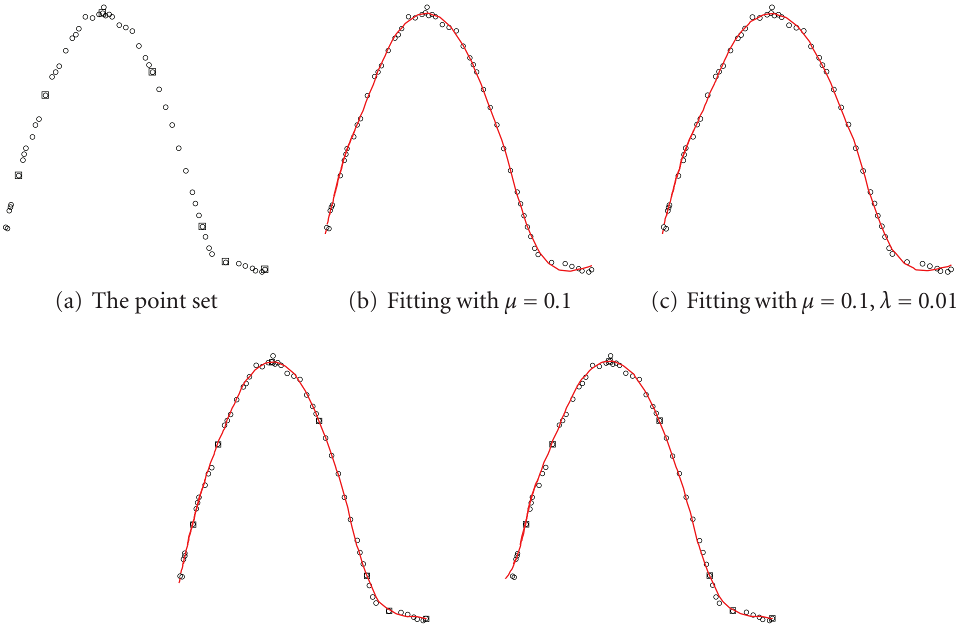
3.4. Energy term. In some cases, the minimizing of the algebraic distances L with the points constraints may produce the fitting piecewise algebraic curve that splits into several disconnected components. There are several possibilities to solve this problem of curve fitting. For example, basing on the signs of the coefficients, one may derive a criteria which can guarantee the desired topology of the result (see [2]).

Since we wish to compute the solution by solving a system of linear equations, we use the simpler approach of adding a suitable energy term that pulls the approximation of the curve towards a simpler shape as described in [5]. If the energy term has a sufficient strong influence, then the fitting curve has the desired topology.

An energy term is given by the quadratic function

$$E(\mathbf{c}) = \int_D \left(s_{xx}^2 + 2s_{xy}^2 + s_{yy}^2\right) dx\, dy, \tag{3.7}$$

which measures the deviation of the spline $s(x, y)$ from a linear function. Hence, by increasing the influence of this term, the resulting curve gets closer to a straight line [5].



(a) The point set (b) Fitting with $\mu = 0.1$ (c) Fitting with $\mu = 0.1, \lambda = 0.01$

(d) Fitting with $\mu = 0.3$ and points interpolation (e) Fitting with $\mu=0.1, \lambda=0.02$ and points interpolation

FIGURE 4.1. Fitting with piecewise algebraic curves.

3.5. Computing the solution. By the above subsections, we compute the fitting piecewise algebraic curve by solving the following optimization problem:

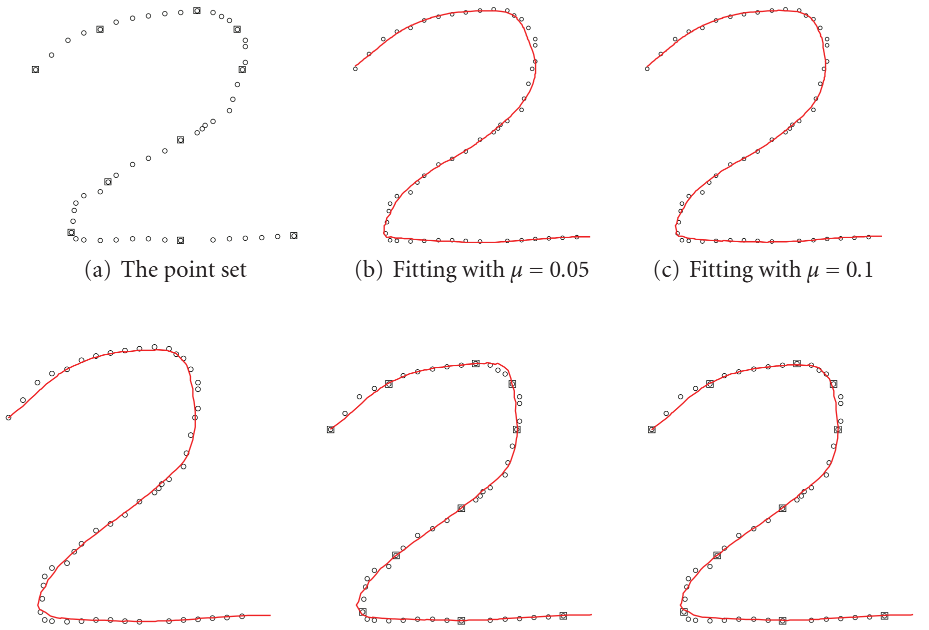
$$\begin{aligned} \min L(\mathbf{c}) + \mu M(\mathbf{c}) + \lambda E(\mathbf{c}) \text{ s.t.} \\ s(\mathbf{p}_i) = 0, \quad i = i_1, \ldots, i_l, \\ s(\mathbf{d}_i) \leq 0, \quad i = 1, \ldots, k, \\ s(\mathbf{g}_i) \geq 0, \quad i = 1, \ldots, r, \end{aligned} \tag{3.8}$$

where μ, λ are nonnegative weights.

As this leads to a quadratic objective function of the unknown coefficients vector $\mathbf{c}$ with the constraints, the solution can be found by solving the quadratic optimization problem. It is easy to prove that the objective function of (3.8) is convex and the $s(\cdot) = 0$ is the linear function of $\mathbf{c}$. Then the optimization problem (3.8) has the nonzero ideal solution [17]. There are a lot of methods [17] and mathematical softwares (e.g., Matlab, Maple) to solve this problem. The weights μ, λ control the influences of the terms M, E.

4. Examples

In this section, we present two examples with different data constraints and the weights μ, λ.



(a) The point set

(b) Fitting with $\mu = 0.05$

(c) Fitting with $\mu = 0.1$

(d) Fitting with $\mu = 0.1, \lambda = 0.01$

(e) Fitting with $\mu = 0.15$ and points interpolation

(f) Fitting with $\mu = 0.1, \lambda = 0.01$ and points interpolation

FIGURE 4.2. Fitting with piecewise algebraic curves.

Example 4.1. As the first example, we approximate the points set (see Figure 4.1(a), boxed points to be interpolated) with the piecewise algebraic curves. We use the type-2 triangulation with $m = 4, n = 4$ in this example. The weights of the objective function are chosen as $\lambda = 0.0, \mu = 0.1, 0.3$ in Figures 4.1(b) and 4.1(d), respectively, whereas the points interpolation are included in Figure 4.1(d). The resulting piecewise algebraic curves are shown in Figures 4.1(c) and 4.1(e) with weights $\mu = 0.1, \lambda = 0.1, 0.2$, respectively, whereas the points interpolation is included in Figure 4.1(e).

Example 4.2. In this example, we approximate the points set (see Figure 4.2(a), boxed points to be interpolated) with the piecewise algebraic curves. We use the type-2 triangulation with $m = 5, n = 5$ in this example. The weights of the objective function are chosen as $\lambda = 0.0, \mu = 0.05, 0.1, 0.15$ in Figures 4.2(b), 4.2(c), and 4.2(e), respectively, whereas the points interpolation are included in Figure 4.2(e). The resulting piecewise algebraic curves are shown in Figures 4.2(d) and 4.2(f) with weights $\mu = 0.1, \lambda = 0.01$, whereas the points interpolation are included in Figure 4.2(f).

5. Summary and conclusions

In this paper, we present a new method for fitting C^1 piecewise algebraic curves of degree 2 over type-2 triangulation to the given scattered data. By simultaneously approximating points, associated normals and tangents, and points constraints, the energy term is also considered in the method. This method is computationally simple. We can extend the method to the spline spaces $S_4^2(\triangle_{mn}^{(2)})$ and $S_3^1(\triangle_{mn}^{(1)})$ directly. For the large number of the scattered data, we can refine the triangulation to approximate the data better.

The objective function can be updated with the weighted least square and adding other terms such as data-dependent term. Future researches will focus on updating the objective function and fitting the piecewise algebraic curves to more complex objects over other partitions.

Acknowledgments

The authors appreciate the comments and suggestions of the anonymous reviewers. The research is supported partially by the National Natural Science Foundation of China (Grants no. 60373093, 60533060, 10726068) and the Research Project of Liaoning Educational Committee of China (Grant no. 2005085).

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