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Research Article

Sliding Mode Control of Uncertain Neutral Stochastic Systems with Multiple Delays

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This paper is concerned with the sliding mode control for uncertain stochastic neutral systems with multiple delays. A switching surface is adopted first. Then, by means of linear matrix inequalities (LMIs), a sufficient condition is derived to ensure the global stochastic stability of the stochastic system in the sliding mode for all admissible uncertainties. The synthesized sliding mode controller guarantees the existence of the sliding mode.

1. Introduction

Time delay occurs due to the finite capabilities of information processing and data transmission among various parts of the system. The phenomena of time delay are often encountered in various relevant systems, such as HIV infection with drug therapy, aircraft stabilization, chemical engineering systems, inferred grinding model, manual control, neural network, nuclear reactor, population dynamic model, rolling mill, ship stabilization, and systems with lossless transmission lines. It is well known that time delay factors always lead to poor performance. Hence, problems of stability analysis and stabilization of dynamical systems with time delays in the state variables and/or control inputs have received considerable interest for more than three decades [1–6].

In practice, systems are almost always innately "noisy". Therefore, in order to model a system realistically, a degree of randomness must be incorporated into the model. Thus, a class of stochastic systems has received great attention in the past decade [7]. On the other hand, it has been shown that a lot of practical systems can be modeled by using functional differential equations of the neutral type [8, 9]. However, the mathematical model always contains some uncertain elements. Therefore, uncertain systems have been extensively studied in the past years [10–12].

To cope with the problem of stability of uncertain stochastic neutral delay systems, most of the research focused on the retarded functional differential equations and it also seems that few results are available on the variable structure control.

Sliding mode control (SMC) is a particular type of variable structure control. It provides an effective alternative to deal with the nonlinear dynamic systems. The main feature of SMC is its easy realization, control of independent motion, insensitivity to variation in plant parameters or external perturbations, and wide variety of operational models [13–15].

The purpose of this paper lies in the design of SMC for a class of uncertain stochastic neutral delay systems. A switching surface, which makes it easy to guarantee the stability of the uncertain stochastic neutral delay systems in the sliding mode, is first proposed. By means of linear matrix inequalities (LMIs), a sufficient condition is given such that the stochastic dynamics in the specified switching surface is globally stochastically stable. And then, based on this switching surface, a synthesized SMC law is derived to guarantee the existence of the composite sliding motion. Finally, a numerical example is illustrated to demonstrate the validity of the proposed SMC.

2. Problem formulation

Consider the following neutral stochastic system with uncertainties and multiple delays:

$$\begin{aligned} d\left[Ex(t)-Cx(t-\tau)\right] &= \left[\left(A+\Delta A(t)\right)x(t)+\left(A_d+\Delta A_d(t)\right)x(t-h(t))+Bu(t)\right]dt \\ &\quad+\left[\Delta E(t)x(t)+\Delta E_d(t)x(t-h(t))\right]dw(t), \\ x(t) &= \phi(t), \quad t\in[-H,0], \end{aligned} \tag{2.1}$$

where $x(t)\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the control input, τ is the constant delay, $h(t)$ is the time-varying differentiable bounded delay satisfying $0\le h(t)\le h_M$, $\overset{\Delta}{h(t)}\le h_D<1$, $w(t)$ is an m-dimensional Brownian motion, $H=\max\{\tau,h_M\}$. It is assumed that $\phi(t)$ is the initial condition which is continuous, $t\in[-H,0]$. In system (2.1), $E\in\mathbb{R}^{n\times n}$, $C\in\mathbb{R}^{n\times n}$, $A\in\mathbb{R}^{n\times n}$, $A_d\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$ are known real constant matrices. $\Delta A(t)$, $\Delta A_d(t)$, $\Delta E(t)$, and $\Delta E_d(t)$ represent the structured uncertainties in (2.1), which are assumed to be of the forms

$$\begin{aligned} \Delta A(t)=MF_1(t)N_A, \qquad \Delta A_d(t)=MF_2(t)N_{A_d}, \\ \left[\Delta E(t),\Delta E_d(t)\right]=MF_3(t)\left[N_E,N_{E_d}\right], \end{aligned} \tag{2.2}$$

M, N_A, N_{A_d}, N_E, and N_{E_d} are some given constant matrices, $F_l(t)$ $(l=1,2,3)$ are unknown real time-varying matrices which have the following structure:

$F_l(t)=\text{blockdiag}\{\delta_{l_1}(t)I_{r_{l_1}},\dots,\delta_{l_k}(t)I_{r_{l_k}},F_{l_1}(t),\dots,F_{l_s}(t)\}$, $\delta_{l_i}\in\mathbb{R}$, $|\delta_{l_i}|\le 1$, $1\le i\le k$, and $F_{l_j}^{\top}F_{l_j}\le I$, $1\le j\le s$.

We define the sets Δ_l as

$$\Delta_l=\left\{F_l^{\top}(t)F_l(t)\le I,\ F_lN_l=N_lF_l,\ \forall N_l\in\Sigma_l\right\}, \tag{2.3}$$

where $\Sigma_l = \{N_l = \text{blockdiag}[N_{l_1}, \ldots, N_{l_k}, n_{l_1} I_{f_{l_1}}, \ldots, n_{l_s} I_{f_{l_s}}]\}$, N_{l_i} are invertible for $1 \leq i \leq k$, and $n_{l_j} \in \mathbb{R}$, $n_{l_j} \neq 0$ for $1 \leq j \leq s$.

The following useful lemmas will be used to derive the desired LMI-based stability criteria.

Lemma 2.1 (see [1]). *The LMI* $\left[\begin{smallmatrix} S_{11} & S_{12} \\ * & S_{22} \end{smallmatrix}\right] < 0$, *with* $S_{11} = S_{11}^\top$, $S_{22} = S_{22}^\top$, *is equivalent to*

$$S_{22} < 0, \qquad S_{11} - S_{12} S_{22}^{-1} S_{12}^\top < 0. \tag{2.4}$$

Lemma 2.2 (see [11]). *Let* D, E, F_l *be real matrices of appropriate dimensions and* $F_l \in \Delta_l$. *Then for any block-structured matrix* $N_l \in \Sigma_l$,

$$DF_lE + (DF_lE)^\top \leq D\left(N_l N_l^\top\right) D^\top + E^\top \left(N_l N_l^\top\right)^{-1} E. \tag{2.5}$$

Lemma 2.3 (see [11]). *Let* A, D, E, F_l, *and* P *be real matrices of appropriate dimensions with* $P = P^\top > 0$ *and* $F_l \in \Delta_l$. *Then for any block-structured matrix* $N_l \in \Sigma_l$ *satisfying* $P^{-1} - D(N_l N_l^\top) D^\top > 0$, *one has*

$$(A + DF_lE)^\top P (A + DF_lE) \leq A^\top \left(P^{-1} - DN_l N_l^\top D^\top\right)^{-1} A + E^\top \left(N_l N_l^\top\right)^{-1} E. \tag{2.6}$$

Lemma 2.4 (see [11]). *For any* $z, y \in \mathbb{R}^n$ *and for any symmetric positive-definite matrix* $X \in \mathbb{R}^{n \times n}$,

$$-2z^\top y \leq z^\top X^{-1} z + y^\top X y. \tag{2.7}$$

Definition 2.5 (see [14]). The nominal stochastic time-delay system of form (2.1) with $u(t) = 0$ is said to be mean-square asymptotically stable if

$$\lim_{t \to \infty} \mathbb{E}|x(t)|^2 = 0. \tag{2.8}$$

Definition 2.6 (see [14]). The uncertain time delay system of the form (2.1) is robustly mean square stabilized if the nominal system is mean-square asymptotically stable for all admissible uncertainties.

In order to simplify the treatment of the problem, the operator $\Im : C([-\tau, 0], \mathbb{R}^n) \to \mathbb{R}^n$ is defined to be

$$\Im(x_t) = Ex(t) - Cx(t - \tau). \tag{2.9}$$

The stability of the operator $\Im$ is defined as follows.

Definition 2.7 (see [9]). The operator $\Im$ is said to be stable if the zero solution of the homogeneous difference equation

$$\begin{gathered} \Im(x_t) = 0, \quad t \geq 0, \\ X_0 = \varphi \in \{\psi \in C[-\tau, 0] : \Im\psi = 0\} \end{gathered} \tag{2.10}$$

is uniformly asymptotically stable.

If $\operatorname{rank}(E) = m < n$, then it is easy to find that there exist nonsingular constant matrices K and S, such that

$$KES^{-1} = \begin{pmatrix} E_1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad KCS^{-1} = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix}, \tag{2.11}$$

where E_1 is an $m \times m$ nonsingular matrix, C_1 and C_2 are $m \times m$ and $(n-m) \times (n-m)$ constant matrices, respectively.

Lemma 2.8 (see [9]). *The operator $\mathfrak{I}$ is stable if $\|E_1^{-1}C_1\| < 1$ and $|C_2| \neq 0$, where E_1, C_1, and C_2 are defined as in* (2.11) *and $\|\cdot\|$ is any matrix norm.*

3. Switching surface and controller design

In this work, we choose the switching function as follows:

$$S(t) = D\big[Ex(t) - Cx(t-\tau)\big] + \sigma(t), \tag{3.1}$$

where the auxiliary variable $\sigma(t)$ satisfies the following:

$$d\sigma = -D\big[(A+BK)x(t) + A_d x(t-h(t))\big]dt - D\big[\Delta E x(t) + \Delta E_d x(t-h(t))\big]dw(t), \tag{3.2}$$

where $D \in \mathbb{R}^{m\times n}$ and $K \in \mathbb{R}^{m\times n}$ are constant matrices. The matrix K is chosen such that the matrix $A + BK$ is Hurwitz, and the matrix D is to be designed later so that DB is nonsingular. As long as the system operates in the sliding mode, it satisfies the equations $S(t) = 0$ and $\overset{\Delta}{S}(t) = 0$ [13].

Therefore, the equivalent control $u_{\mathrm{eq}}(t)$ in the sliding manifold is given by

$$u_{\mathrm{eq}} = -(DB)^{-1}D\big[(\Delta A(t) - BK)x(t) + \Delta A_d(t)x(t-h(t))\big]. \tag{3.3}$$

Substituting (3.3) into system (2.1), the following equivalent sliding mode dynamics can be obtained:

$$\begin{aligned} d\big[Ex(t) - Cx(t-\tau)\big] = \Big[&\big(A + BK + \Delta A(t) - B(DB)^{-1}D\Delta A(t)\big)x(t) \\ &+ \Big(A_d + \Delta A_d(t) - B(DB)^{-1}D\Delta A_d\Big)x(t-h(t))\Big]dt \\ &+ \big[\Delta E x(t) + \Delta E_d x(t-h(t))\big]dw(t). \end{aligned} \tag{3.4}$$

Now, we proceed to the first task which is to analyze the robustly stochastic stability of the sliding motion described by (3.4), and derive a sufficient condition by means of the linear matrix inequality method.

4. Robust stabilization in the mean square sense

Theorem 4.1. *Consider the equivalent sliding mode dynamics* (3.4). *If the operator $\Im$ is stable and there exist symmetric positive-definite matrices X, Q_1, Q_2, T_1, T_2, T_3, T_4, T_5, and T_6 satisfying the following LMIs:*

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & E^\top XB & E^\top XM & E^\top XM & 0 & 0 & 0 \\ * & \Pi_{22} & -C^\top XA_d & 0 & 0 & 0 & C^\top XB & C^\top XM & C^\top XM \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\dfrac{B^\top XB}{2} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -T_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -T_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\dfrac{B^\top XB}{2} & 0 & 0 \\ * & * & * & * & * & * & * & -T_3 & 0 \\ * & * & * & * & * & * & * & 0 & -T_4 \end{bmatrix} < 0, \quad (4.1)$$

where $\Pi_{11} = E^\top X(A+BK) + (A+BK)^\top XE + Q_1 + Q_2 + N_A^\top(T_1+T_3+T_5)N_A + N_E^\top T_6 N_E$, $\Pi_{12} = -(A+BK)^\top XC$, $\Pi_{13} = E^\top XA_d + N_{E_d}^\top T_4 N_{E_d}$, $\Pi_{22} = -Q_1 + N_A^\top T_2 N_A$, $\Pi_{33} = -(1-h_D)Q_1 + N_{A_d}^\top(T_1 + T_3 + T_5)N_{A_d} + N_{E_d}^\top T_4 N_{E_d}$,

$$\begin{bmatrix} -X & XM \\ * & -T_6 \end{bmatrix} < 0, \quad (4.2)$$

$$\begin{bmatrix} -\dfrac{X}{2} & XM \\ * & -T_5 \end{bmatrix} < 0, \quad (4.3)$$

then the uncertain time delay system of the form (2.1) *with the switching surface* (3.1) *is robustly stochastically stable and sliding mode matrix $D = B^\top X$. In the above LMIs, T takes the form of $N_l N_l^T$ for $N_l \in \Sigma_l$.*

Proof. Choose a Lyapunov functional candidate $V(x(t),t)$ as

$$V(x_t) = (\Im x_t)^\top(t)P(\Im x_t) + \int_{t-\tau}^{t} x^\top(s)Q_1 x(s)ds + \int_{t-h(t)}^{t} x^\top(s)Q_2 x(s)ds. \quad (4.4)$$

Then, the averaged derivative is given by the following expression:

$$\begin{aligned} \mathsf{L}V(x_t) = {} & 2[Ex(t) - Cx(t-\tau)]^\top X[(A + BK + \Delta A(t) - B(DB)^{-1}D\Delta A(t))x(t) \\ & \quad + (A_d + \Delta A_d(t) - B(DB)^{-1}D\Delta A_d(t))x(t-h(t))] \\ & + [\Delta E(t)x(t) + \Delta E_d(t)x(t-h(t))]^\top P[\Delta E(t)x(t) + \Delta E_d(t)x(t-h(t))] \\ & + x^\top(t)Q_1x(t) - x^\top(t-\tau)Q_1x(t-\tau) + x^\top(t)Q_2x(t) \\ & - (1-\overset{\Delta}{h}(t))x^\top(t-h(t))Q_2x(t-h(t)). \end{aligned} \quad (4.5)$$

Using Lemma 2.1, inequality (4.2) is equivalent to

$$X^{-1} - MT_6^{-1}M^\top > 0. \tag{4.6}$$

Hence, it follows from Lemma 2.3 that

$$\begin{aligned}
&\left[\Delta E(t)x(t) + \Delta E_d(t)x(t-h(t))\right]^\top X\left[\Delta E(t)x(t) + \Delta E_d(t)x(t-h(t))\right] \\
&\quad = \left\{MF_3(t)\left[N_E x(t) + N_{E_d}x(t-h(t))\right]\right\}^\top X\left\{MF_3(t)\left[N_E x(t) + N_{E_d}x(t-h(t))\right]\right\} \\
&\quad \le \left[N_E x(t) + N_{E_d}x(t-h(t))\right]^\top T_6\left[N_E x(t) + N_{E_d}x(t-h(t))\right] \\
&\quad = x^\top(t)N_E^\top T_6 N_E x(t) + x^\top(t)N_E^\top T_6 N_{E_d}x(t-h(t)) + x^\top(t-h(t))N_{E_d}^\top T_6 N_E x(t) \\
&\qquad + x^\top(t-h(t))N_{E_d}^\top T_6 N_{E_d}x(t-h(t)).
\end{aligned} \tag{4.7}$$

Note that $D = B^\top X$, and it follows form Lemma 2.4 that

$$\begin{aligned}
&-2x^\top(t)E^\top XB(DB)^{-1}D\Delta A(t)x(t) \le x^\top\left[E^\top XB\left(B^\top XB\right)^{-1}B^\top XE + \Delta A^\top(t)X\Delta A(t)\right]x(t), \\
&-2x^\top(t)E^\top XB(DB)^{-1}D\Delta A_d(t)x(t-h(t)) \\
&\qquad \le x^\top E^\top XB\left(B^\top XB\right)^{-1}B^\top XEx(t) + x^\top(t-h(t))\Delta A_d^\top(t)X\Delta A_d(t)x(t-h(t)), \\
&2x^\top(t-\tau)C^\top XB(DB)^{-1}D\Delta A(t)x(t) \\
&\qquad \le x^\top(t-\tau)C^\top XB\left(B^\top XB\right)^{-1}B^\top XCx(t-\tau) + x^\top(t)\Delta A^\top(t)X\Delta A(t)x(t), \\
&2x^\top(t-\tau)C^\top XB(DB)^{-1}D\Delta A_d(t)x(t-h(t)) \\
&\qquad \le x^\top(t-\tau)C^\top XB\left(B^\top XB\right)^{-1}B^\top XCx(t-\tau) + x^\top(t-h(t))\Delta A_d^\top(t)X\Delta A_d(t)x(t-h(t)).
\end{aligned} \tag{4.8}$$

Substituting (4.7) and (4.8) into (4.5), we obtain

$$\mathrm{L}V(x(t)) \le \mathrm{X}^\top(t)\Xi \mathrm{X}(t), \tag{4.9}$$

where $\Xi = \Xi_1 + M_1F_1(t)N_1 + N_1^\top F_1(t)M_1^\top + M_1F_2(t)N_2 + N_2^\top F_2(t)M_1^\top + M_2F_1(t)N_1 + N_1^\top F_1(t)M_2^\top + M_2F_2(t)N_2 + N_2^\top F_2(t)M_2^\top$, $\mathrm{X} = [x^\top(t), x^\top(t-\tau), x^\top(t-h(t))]^\top$,

$$\begin{aligned}
&\Xi_1 = \begin{bmatrix} \Xi_{11} & -(A+BK)^\top XC & E^\top XA_d + N_{E_d}^\top T_6 N_{E_d} \\ * & \Xi_{22} & -C^\top XA_d \\ * & * & \Xi_{33} \end{bmatrix}, \\
&\Xi_{11} = E^\top X(A+BK) + (A+BK)^\top XE + N_E^\top T_6 N_E + 2\Delta A^\top(t)X\Delta A(t) + Q_1 + Q_2 \\
&\qquad + 2E^\top XB\left(B^\top XB\right)^{-1}B^\top XE, \\
&\Xi_{22} = -Q_1 + 2C^\top XB\left(B^\top XB\right)^{-1}B^\top XC, \\
&\Xi_{33} = -(1-h_D)Q_2 + 2\Delta A_d^\top(t)X\Delta A_d(t) + N_{E_d}^\top T_6 N_{E_d}, \\
&M_1 = \begin{bmatrix} E^\top XM \\ 0 \\ 0 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 0 \\ -C^\top XM \\ 0 \end{bmatrix}, \qquad N_1 = \begin{bmatrix} N_A & 0 & 0 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} 0 & 0 & N_{A_d} \end{bmatrix}.
\end{aligned} \tag{4.10}$$

Using Lemmas 2.1 and 2.2, we have

$$
\Xi \le \Xi'
= \begin{bmatrix}
\Xi'_{11} & \Pi_{12} & \Pi_{13} & E^{\top}XB & E^{\top}XM & E^{\top}XM & 0 & 0 & 0 & 0 & 0 \\
* & \Pi_{22} & -C^{\top}XA_d & 0 & 0 & 0 & C^{\top}XB & C^{\top}XM & C^{\top}XM & 0 & 0 \\
* & * & \Xi'_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\dfrac{B^{\top}XB}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -T_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -T_2 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\dfrac{B^{\top}XB}{2} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -T_3 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -T_4 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\dfrac{X}{2} & 0 \\
* & * & * & * & * & * & * & * & * & 0 & -\dfrac{X}{2}
\end{bmatrix}
+ M_3\overline{F}(t)N_3 + N_3^{\top}\overline{F}^{\top}(t)M_3^{\top},
$$

$$
\begin{aligned}
\Xi'_{11} &= E^{\top}X(A+BK) + (A+BK)^{\top}XE + Q_1 + Q_2 + N_A^{\top}(T_1+T_3)N_A + N_E^{\top}T_6N_E, \\
\Xi'_{33} &= -(1-h_D)Q_1 + N_{A_d}(T_2+T_4)N_{A_d} + N_{E_d}^{\top}T_6N_{E_d}, \\
M_3 &= \begin{bmatrix} N_A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & N_{A_d} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{\top}, \qquad \overline{F}(t) = \begin{bmatrix} F_1^{\top}(t) & 0 \\ 0 & F_2^{\top}(t) \end{bmatrix}, \\
N_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M^{\top}X & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M^{\top}X \end{bmatrix}.
\end{aligned}
\tag{4.11}
$$

With Lemma 2.1, we can see that $\Xi' < 0$ is equivalent to LMIs (4.1)–(4.3).

According to Itô's formula, system (2.1) is robustly stochastically stable. This completes the proof. □

5. Sliding mode control

We now design an SMC law such that the reachability of the specified switching surface is ensured.

Theorem 5.1. *Consider the uncertain stochastic time delay system* (2.1). *Suppose that the switching function is given as* (3.1) *with* $D = B^{\top}X$, *where* X *is the solution of LMIs* (4.1)–(4.3). *Then the reachability of the sliding surface* $s(t) = 0$ *can be guaranteed by the following SMC law:*

$$
u(t) = Kx(t) - \rho(t)\mathrm{sgn}(s(t)), \tag{5.1}
$$

where the switching gain $\rho(t)$ *is given as*

$$\rho(t) = \lambda + \|(DB)^{-1}DM\| \times \left(\|N_A x(t)\| + \|N_{A_d} x(t-h(t))\|\right) \tag{5.2}$$

with $\lambda > 0$.

Proof. A Lyapunov functional candidate $V(t)$ is defined as

$$V(t) = \frac{1}{2}S^{\top}(t)(DB)^{-1}S(t). \tag{5.3}$$

Hence we have

$$\begin{aligned} \overset{\Delta}{V}(t) &= S^{\top}(t)(DB)^{-1}\overset{\Delta}{S}(t) \\ &= S^{\top}(t)(DB)^{-1}D\left[\Delta A(t)x(t) + \Delta A_d(t)x(t-h(t)) - B\rho(t)\operatorname{sgn}(s(t))\right] \\ &\le \|S(t)\|\|(DB)^{-1}DM\| \times \left(\|N_A x(t)\| + \|N_{A_d} x(t-h(t))\|\right) - \rho(t)\|S(t)\| \\ &\le -\lambda\|S(t)\| < 0 \quad \text{for } \|S(t)\| \neq 0. \end{aligned} \tag{5.4}$$

This completes the proof. □

6. An illustrative example

Consider neutral stochastic systems (2.1) with

$$\begin{gathered} A = \begin{bmatrix} 2.3 & 1.2 \\ 2 & 3.4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0.3 \\ 0 & 0.1 \end{bmatrix}, \\ M = \begin{bmatrix} 0.2 & 0 \\ 0.3 & -0.01 \end{bmatrix}, \quad E = I, \quad A_d = \begin{bmatrix} 0.5 & 0 \\ 0.2 & 0.3 \end{bmatrix}, \\ N_A = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \quad N_{A_d} = \begin{bmatrix} -0.1 & 0 \\ 0.3 & 0.3 \end{bmatrix}, \quad N_E = \begin{bmatrix} 0.4 & 0 \\ -0.01 & 0.4 \end{bmatrix}, \\ N_{E_d} = \begin{bmatrix} 0.21 & 0 \\ 0.1 & -0.1 \end{bmatrix}, \quad h(t) = 0.1\sin t. \end{gathered} \tag{6.1}$$

We select matrix $K = \begin{bmatrix} -11.3000 & -1.2000 \\ -2.0000 & -12.4000 \end{bmatrix}$. Using Matlab LMI control toolbox to solve the LMIs (4.1)–(4.3), we obtain the following:

$$\begin{gathered} X = \begin{bmatrix} 0.1752 & -0.0038 \\ -0.0038 & 0.1985 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 1.1255 & -0.0186 \\ -0.0186 & 1.2138 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 1.0613 & -0.0605 \\ -0.0605 & 1.2145 \end{bmatrix}, \\ T_1 = \begin{bmatrix} 1.1670 & 0.0001 \\ 0.0001 & 1.1623 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 1.1661 & -0.0007 \\ -0.0007 & 1.1652 \end{bmatrix}, \quad T_3 = \begin{bmatrix} 1.1670 & 0.0001 \\ 0.0001 & 1.1623 \end{bmatrix}, \\ T_4 = \begin{bmatrix} 1.1647 & 0.0025 \\ 0.0025 & 1.1700 \end{bmatrix}, \quad T_5 = \begin{bmatrix} 1.1670 & 0.0001 \\ 0.0001 & 1.1623 \end{bmatrix}, \quad T_6 = \begin{bmatrix} 1.1580 & -0.0046 \\ -0.0046 & 1.1705 \end{bmatrix}. \end{gathered} \tag{6.2}$$

7. Conclusions

In this paper, we have investigated the sliding mode control problem for uncertain stochastic neutral systems with multiple delays. The stability criteria are expressed by means of LMIs, which can be readily tested by some standard numerical packages. Therefore, the developed result is practical.

Acknowledgments

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