Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2008, Article ID 867906, 11 pages
doi:10.1155/2008/867906

Research Article

Steady Flow of a Second-Grade Fluid in an Annulus with Porous Walls

M. Emin Erdoğan and C. Erdem İmrak

Mechanical Engineering Department, Faculty of Mechanical Engineering, Istanbul Technical University, Gümüşsuyu 34437, Istanbul, Turkey

Correspondence should be addressed to C. Erdem İmrak, imrak@itu.edu.tr

Received 22 June 2007; Accepted 9 February 2008

Recommended by Kumbakonam Rajagopal

An exact solution of an incompressible second-grade fluid for flow between two coaxial cylinders with porous walls is given. It is assumed that the inner cylinder is rotating with a constant angular velocity and the outer one is at rest. The solution is expressed in terms of the confluent hypergeometric functions and it is valid for all values of the cross-Reynolds number and the elastic number. The solutions for -2, $+\infty$, and $-\infty$ values of the cross-Reynolds number are obtained and a comparison with those of the Newtonian fluid is given. Furthermore, the torque exerted by the fluid on the inner cylinder is calculated. It is shown that the moment coefficient depends on the cross-Reynolds number, the elastic number, and the ratio of the radii of the cylinders. The variation of the moment coefficient with these numbers is discussed.

1. Introduction

The flow considered in this paper is for flow of a second-grade fluid in an annulus with porous walls. For a Newtonian fluid, the flow in an annulus has been investigated by Berman [1, 2] and Yuan [3]. The extensions of this flow to non-Newtonian fluids have been studied by many authors [4–6]. However, the results for non-Newtonian fluids have been given by using a perturbation method. Therefore, it is not possible to compare with the exact solution of a Newtonian fluid for all values of the cross-Reynolds number which is defined by the suction velocity at the surface of the cylinder, the radius of the cylinder, and the kinematic viscosity of the fluid. The exact solution given in this paper provides comparison with the exact solution of a Newtonian fluid for all values of the cross-Reynolds number. The non-Newtonian fluid model in this paper is that of the second-grade fluid [7]. Although there are some criticisms on the applications of this model, many papers have been published and a listing of some of them may be found in the literature. Furthermore, it has been shown by Walters [8] that

for many types of problems in which the flow is slow enough in the viscoelastic sense, the results given by Oldroyd's constitutive equation will be substantially similar to those of fluids of the second or third grade. Therefore, it would seem reasonable to use the second- or third-grade models in carrying out the calculations. This is particularly so in view of the fact that the calculation is generally simpler. The constitutive equation for an incompressible second-grade fluid has three coefficients. There are some restrictions on these coefficients due to the Clausius-Duhem inequality and due to the assumption that the Helmhotlz free energy is minimum in equilibrium. A comprehensive discussion on the restrictions for these coefficients has been given by Dunn and Fosdick [9] and Dunn and Rajagopal [10]. The sign of the material moduli is the subject of much controversy [11, 12].

The no-slip boundary condition is sufficient for a Newtonian fluid, but may not be sufficient for a fluid of second grade. Therefore, one needs an additional condition at boundary. However, if one uses a perturbation expansion in terms of the coefficient appearing in the higher-order derivative of the governing equation, the no-slip boundary condition is sufficient. A critical review on the boundary conditions, the existence, and uniqueness of the solution has been given by Rajagopal [13]. In order to overcome the difficulty on the boundary condition, several workers have studied to give an additional condition. Frater [14] has studied the problem without using a perturbation expansion in terms of the coefficient of the higher-order term of the governing equation. He imposed an additional condition so that the solution tends to the Newtonian value as the coefficient of the higher derivative in the equation approaches zero. This can be used provided the explicit solution or the numerical solution should exist. In this paper, the solution is obtained in terms of the confluent hypergeometric functions and has three coefficients: two of them can be determined by the no-slip boundary condition and the other can be determined by using the properties of the confluent hypergeometric functions.

The problem considered in this paper is an extension of a viscous fluid in an annulus with uniform porous walls to the problem for the flow of a second-grade fluid. The fluid injection rate at one wall is taken equal to the withdrawal rate at the outer wall. Further, because of this restriction on the cross wall, the velocity components do not vary along the axis of the cylinder. The flow in an annulus with porous walls has been investigated by many authors [4–6]. In order to obtain the solution in [4, 6], a perturbation method has been used, therefore, the results obtained can be applicable for small values of the cross-Reynolds number. The axial flow in an annulus with porous walls of a second-grade fluid has been investigated in [15]. It can be shown that the axial and the azimuthal velocities are superposable. The solution for the flow of a second-grade fluid between two parallel porous plates has been given in [16]. The flow of a second-grade fluid generated by a rotating porous cylinder has been investigated in [17]. The flow of a second-grade fluid between eccentric rotating cylinders was examined in [18]. Assuming that the eccentricity is small, the secondary flow for a third-grade fluid between two eccentrically placed cylinders has been investigated in [19].The flow of a second-grade fluid in a porous cylindrical tube was examined for suction or injection through the surface of the tube, under the assumption that the second-order parameter is sufficiently small [20]. In this paper, the solution is obtained in terms of the confluent hypergeometric functions and it is valid for all values of the cross-Reynolds number and the elastic number. The solutions for $+\infty$, $-\infty$, and -2 values of the cross-Reynolds number are discussed. The solution for $+\infty$ value of the cross-Reynolds number is given in terms of the modified Bessel functions of the first kind of order one. The solution for $-\infty$ value of the cross-Reynolds number is given in terms of the modified Bessel function of the second kind of order one. The

solution for −2 value of the cross-Reynolds number is given in terms of the exponential integral.

The torque exerted by the fluid on the inner cylinder is calculated. This can be used to obtain the values of the material coefficients. It is shown that the moment coefficient depends on the cross-Reynolds number, the elastic number, and the ratio of the radii of the cylinders. The variation of the moment coefficient with these numbers is discussed. There are appreciable differences between that of the Newtonian fluid and that of the second-grade fluid.

2. Basic equations

The equation of motion for a fluid in the absence of body forces is

$$\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot \boldsymbol{\sigma}, \tag{2.1}$$

where ρ is the density of the fluid, $\mathbf{u}$ is the velocity, $\boldsymbol{\sigma}$ is the stress tensor, and D/Dt represents the material derivative. The continuity equation for the velocity is

$$\nabla \cdot \mathbf{u} = 0. \tag{2.2}$$

Equations (2.1) and (2.2) can be applied to all types of incompressible Newtonian and non-Newtonian fluids. The stress depends on the local properties of the fluid. The relation between the stress and the local properties of the fluid is called constitutive equation. The constitutive equation for an incompressible second-grade fluid is in the following form [7]:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{2.3}$$

where μ, α_1, and α_2 are material constants, and $\mathbf{A}_n$ represents the Rivlin-Eriksen tensor defined as [21]

$$\begin{aligned} &\mathbf{A}_0 = \mathbf{I}, \qquad \mathbf{A}_1 = \nabla\mathbf{u} + (\nabla\mathbf{u})^T, \\ &\mathbf{A}_2 = \left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{A}_1 + \mathbf{A}_1\cdot(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T\cdot\mathbf{A}_1, \end{aligned} \tag{2.4}$$

where t is time, p is pressure, and $\mathbf{I}$ is the identity tensor. The Clausius-Duhem inequality and the condition that Helmholtz free energy is minimum in equilibrium provide the following restrictions [9, 10]:

$$\mu \geq 0, \quad \alpha_1 + \alpha_2 = 0, \quad \alpha_1 \geq 0. \tag{2.5}$$

A comprehensive discussion on the restriction for μ, α_1, and α_2 can be found in the work by Dunn and Rajagopal [10]. The sign of the material moduli α_1 and α_2 is the subject of much controversy [11, 12]. The experiments have not confirmed these restriction on α_1 and α_2. Thus, the conclusion is that the fluids which have been tested are not fluids of second grade and characterized by a different constitutive structure.

Fully developed laminar flow of an incompressible fluid of second grade in an annulus with porous walls is considered. The cylindrical polar coordinates are used. The radii of the

porous cylinders are a_1 and $b_1(a_1 < b_1)$. The rate of fluid withdrawal at one wall of the annulus is assumed to be equal to the rate of injection of fluid at the other wall, and that these rates are independent of axial position in annulus. The condition that the suction rate at wall is equal to the injection rate at the outer is satisfied if

$$b_1 u_b = a_1 u_a, \tag{2.6}$$

where u_a and u_b are the radial velocities at walls of smaller and larger pipes, respectively. Equation (2.6) provides that the velocity is independent of the axial position in annulus. The velocity field is assumed to be in the following form:

$$u_r = \frac{\alpha}{r}, \qquad u_\theta = u(r), \qquad u_z = 0 \tag{2.7}$$

where u_r, u_θ, u_z are components of the velocity in cylindrical polar coordinates, α is positive for injection at the inner cylinder and negative for suction at the inner one. The velocity field given by (2.7) is not a special case of the flow that is given in [22]. The velocity field in [22] is a superposition of the stagnation point flow and the rotating flow. Equation (2.2) is satisfied identically by the velocity given by (2.7). Inserting the velocity given by (2.7) into the expression of the stress, the components of the stress tensor, in cylindrical polar coordinates, can be written in the following forms:

$$\begin{aligned}
\sigma_{rr} &= -p - \frac{2\alpha\mu}{r^2} + \alpha_1\left[\frac{8\alpha^2}{r^4} + 2u'^2 - 2\left(\frac{u^2}{r}\right)'\right] + \alpha_2\left[\frac{4\alpha^2}{r^4} + \left(u' - \frac{u}{r}\right)^2\right],\\
\sigma_{r\theta} &= \mu\left(u' - \frac{u}{r}\right) + \frac{\alpha_1\alpha}{r}\left(u'' + \frac{u'}{r} - \frac{u}{r^2}\right),\\
\sigma_{rz} &= 0,\\
\sigma_{\theta\theta} &= -p + 2\mu\frac{\alpha}{r^2} + \alpha_2\left[4\frac{\alpha^2}{r^4} + \left(u' - \frac{u}{r}\right)^2\right],\\
\sigma_{\theta z} &= 0,\\
\sigma_{zz} &= -p,
\end{aligned} \tag{2.8}$$

where $\sigma_{r\theta} = \sigma_{\theta r}$, $\sigma_{rz} = \sigma_{zr}$, $\sigma_{\theta z} = \sigma_{z\theta}$; the primes denote differentiation with respect to r. Inserting the stress components and the velocity given by (2.7) into (2.1), one obtains

$$\alpha_1\alpha\left(\frac{u'''}{r} + 2\frac{u''}{r^2} - \frac{u'}{r^3} + \frac{u}{r^4}\right) + \mu\left(u'' + \frac{u'}{r} - \frac{u}{r^2}\right) - \rho\frac{\alpha}{r}\left(u' + \frac{u}{r}\right) = 0 \tag{2.9}$$

and the boundary conditions are

$$u(a_1) = \Omega a_1, \qquad u(b_1) = 0, \tag{2.10}$$

where a_1 is the radius of the inner cylinder and b_1 is the radius of the outer cylinder. The solution to (2.9) has three coefficients: two of them can be determined by the no-slip condition (2.10) and the other can be determined by the confluent hypergeometric functions. It is

expected that when the viscoelastic parameter goes to zero, the solution should give the Newtonian case [23], leading to entirely two different kinds of flow behavior. Similar discussions can be found in [24–26].

3. Solution of the problem

After some manipulations, (2.9) takes the form

$$xy'' + (1-x)y' + \frac{R}{2}y = 0, \tag{3.1}$$

where

$$u = \frac{g(r)}{r}, \qquad g' = \frac{r}{b_1^2}y(x), \qquad x = -\frac{\xi^2}{2\varepsilon R}, \qquad \xi = \frac{r}{b_1}, \qquad R = \frac{\alpha}{v}, \qquad \varepsilon = \frac{\alpha_1/\rho}{b_1^2}, \tag{3.2}$$

where prime denotes the differentiation with respect to r, $y(x)$ is the new function, x is the new variable, R is the cross-Reynolds number, and ε is the elastic number and the boundary conditions are

$$g(a_1) = \Omega a_1^2, \qquad g(b_1) = 0. \tag{3.3}$$

Equation (3.3) denotes that the inner cylinder is rotating with angular velocity Ω and the outer one is at rest. The solution of (3.1) can be written in the following form:

$$y = C_1 M\left(-\frac{R}{2}, 1, x\right) + C_2 U\left(-\frac{R}{2}, 1, x\right), \tag{3.4}$$

where $M(a,b,x)$ and $U(a,b,x)$ are the confluent hypergeometric functions [27, 28]. The solution has three coefficients; therefore, one needs three boundary conditions. Thus, unless an additional condition is prescribed over the conditions (2.10), one has a parametric solution. However, the third coefficient can be determined by the properties of the confluent hypergeometric functions. For $R > 0$, x becomes negative; then for $x < 0$, $U(a,b,x)$ is not acceptable and therefore C_2 must be zero and (3.4) takes the form

$$\frac{dg}{dr} = C_1 \frac{r}{b_1^2} M\left(-\frac{R}{2}, 1, -\frac{r^2/b_1^2}{2\varepsilon R}\right). \tag{3.5}$$

Using the identity in [28],

$$M\left(-\frac{R}{2}, 1, -\frac{r^2/b_1^2}{2\varepsilon R}\right) = e^{-r^2/b_1^2/2\varepsilon R} M\left(1+\frac{R}{2}, 1, \frac{r^2/b_1^2}{2\varepsilon R}\right). \tag{3.6}$$

The integration gives

$$g = C\int e^{-z} M\left(1+\frac{R}{2}, 1, z\right) dz + D, \tag{3.7}$$

where $z = (r^2/b_1^2)/2\varepsilon R$. Using the identity in [28] which is

$$\int e^{-z} M\left(1 + \frac{R}{2}, 1, z\right) dz = z e^{-z} M\left(2 + \frac{R}{2}, 2, z\right) + \text{constant}, \tag{3.8}$$

and by the boundary condition (3.3), one obtains

$$\frac{g}{\Omega a_1^2} = \frac{\xi^2 e^{-\xi^2/2\varepsilon R} M(2 + R/2, 2, \xi^2/2\varepsilon R) - e^{-1/2\varepsilon R} M(2 + R/2, 2, 1/2\varepsilon R)}{\sigma^2 e^{-\sigma^2/2\varepsilon R} M(2 + R/2, 2, \sigma^2/2\varepsilon R) - e^{-1/2\varepsilon R} M(2 + R/2, 2, 1/2\varepsilon R)}, \tag{3.9}$$

where $u/\Omega a_1 = (\sigma/\xi)(g/\Omega a_1^2)$ and $\sigma = a_1/b_1$. When ε goes to zero, using the asymptotic expression of $M(a, b, x)$ in [28], $g/\Omega a_1^2$ becomes

$$\lim_{\varepsilon \to 0} \frac{g}{\Omega a_1^2} = \lim_{\varepsilon \to 0} \frac{(r^2/b_1^2)(\sigma^2/b_1^2/2\varepsilon R)^{R/2} - (1/2\varepsilon R)^{R/2}}{\sigma^2(\sigma^2/2\varepsilon R)^{R/2} - (1/2\varepsilon R)^{R/2}} = \frac{1 - \xi^{R+2}}{1 - \sigma^{R+2}}, \tag{3.10}$$

or

$$\frac{u}{\Omega a_1} = \frac{\sigma}{1 - \sigma^{2+R}} \left(\frac{1}{\xi} - \xi^{1+R}\right), \tag{3.11}$$

which is the expression of the velocity of a Newtonian fluid. The variation of $u/\Omega a_1$ with respect to $\zeta = (\xi - \sigma)/(1 - \sigma)$ for various values of R and ε is illustrated in Figure 1. The value of σ is taken as 0.2 and the values of ε are taken as 0 and 1. Equation (3.9) is valid for all values of R and ε. When R goes to infinity, using the expression given in [28] which is

$$\lim_{a \to \infty} \frac{1}{\Gamma(b)} M\left(a, b, \frac{x}{a}\right) = x^{1/2 - (1/2)b} I_{b-1}\left(2\sqrt{x}\right), \tag{3.12}$$

equation (3.9) takes the following form:

$$\frac{u}{\Omega a_1} = \frac{\sigma}{\xi} \frac{\xi I_1(\varepsilon^{-1/2}\xi) - I_1(\varepsilon^{-1/2})}{\sigma I_1(\varepsilon^{-1/2}\sigma) - I_1(\varepsilon^{-1/2})}, \tag{3.13}$$

where $\Gamma(x)$ is the Gamma function and $I_1(x)$ is the modified Bessel function of the first kind of order one. The variation of $u/\Omega a_1$ with respect to ζ for various values of ε is illustrated in Figure 1. Since the asymptotic form of $I_1(x)$ is $e^x/\sqrt{2\pi x}$ when ε goes to zero, (3.13) becomes

$$\frac{u}{\Omega a_1} = \frac{\sigma}{\xi}. \tag{3.14}$$

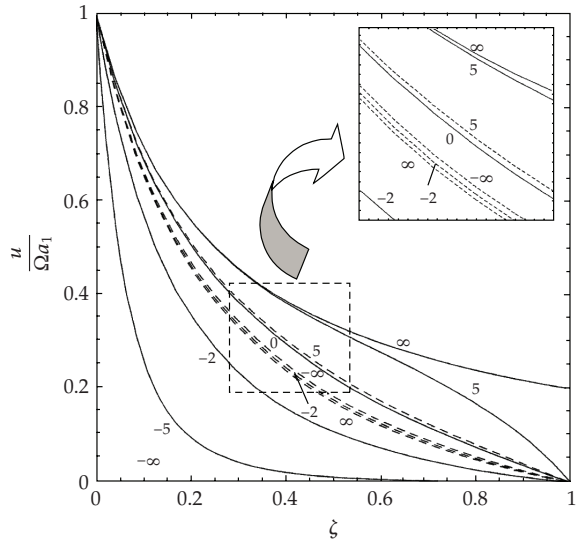

Figure 1: The variation of the azimuthal velocity for various values of the cross-Reynolds number and the elastic number. ($\varepsilon = 0$); ($\varepsilon = 1$), $\zeta = (\xi - \sigma)/(1 - \sigma)$; $\sigma \le \xi \le r/b_1$, $\sigma = 0.2$.

Equation (3.14) satisfies the boundary condition at $\xi = \sigma$, but it does not satisfy the boundary condition at $\xi = 1$. It is clearly seen from Figure 1 that when R goes to $+\infty$, the flow shows a boundary layer character near the outer cylinder for a Newtonian fluid, but it does not behave the same way for a fluid of second grade. A similar situation occurs for the axial flow between two circular cylinders with porous walls of a second-grade fluid [15].

For $R < 0$, x is positive; then for $x > 0$, $M(a, b, x)$ becomes an increasing function of x; therefore, C_1 must be zero, then (3.4) takes the form

$$\frac{dg}{dr} = C_2 \frac{r}{b_1^2} \left(\frac{N}{2}, 1, \frac{r^2/b_1^2}{2\varepsilon N} \right), \tag{3.15}$$

where $N = -R(> 0)$. Using the identity in [28] which is

$$\int U(a, 1, z)dz = \frac{1}{1-a} U(a - 1, 0, z) + \text{constant}, \tag{3.16}$$

and by the boundary condition, the integration gives

$$\frac{g}{\Omega a_1^2} = \frac{U\left(N/2 - 1, 0, r^2/b_1^2/2\varepsilon R\right) - U\left(N/2 - 1, 0, 1/2\varepsilon N\right)}{U\left(N/2 - 1, 0, \sigma^2/2\varepsilon N\right) - U\left(N/2 - 1, 0, 1/2\varepsilon N\right)}, \tag{3.17}$$

where $u/\Omega a_1 = (\sigma/\xi)(g/\Omega a_1^2)$. When ε goes to zero, using the asymptotic expression of $U(a, b, x)$ in [28], one obtains

$$\frac{u}{\Omega a_1} = \frac{\sigma}{1 - \sigma^{2-N}} \left(\frac{1}{\xi} - \xi^{1-N} \right) \tag{3.18}$$

which is the expression of the velocity for a Newtonian fluid. The variation of $u/\Omega a_1$, with respect to $\zeta = (\xi - \sigma)/(1-\sigma)$ for various values of $-R$ and ε, is illustrated in Figure 1. The value of σ is taken as 0.2 and the values of ε are taken as 0 and 1. Equation (3.17) is valid for all values of $-R$ and ε. When N goes to infinity, using the asymptotic expression given in [28] which is

$$\lim_{a\to\infty} \Gamma(1+a-b)\mathrm{U}\left(a, b, \frac{x}{a}\right) = 2x^{1/2-(1/2)b} K_{b-1}\left(2\sqrt{x}\right). \tag{3.19}$$

Equation (3.17) takes the following form:

$$\frac{u}{\Omega a_1} = \frac{\sigma}{\xi} \frac{\xi K_1\left(\varepsilon^{-1/2}\xi\right) - K_1\left(\varepsilon^{-1/2}\right)}{\sigma K_1\left(\varepsilon^{-1/2}\sigma\right) - K_1\left(\varepsilon^{-1/2}\right)}, \tag{3.20}$$

where $K_1(x)$ is the modified Bessel function of the second kind of order one. The variation of $u/\Omega a_1$ with respect to ζ for various values of ε is illustrated in Figure 1. Since the asymptotic form of $K_1(x)$ is $\sqrt{\pi/2x}e^{-x}$ when ε goes to zero, (3.20) becomes

$$\frac{u}{\Omega a_1} = 0. \tag{3.21}$$

Equation (3.21) satisfies the boundary condition at $\xi = 1$, but it does not satisfy the boundary condition at $\xi = \sigma$. The reason is that a boundary layer occurs at $\xi = \sigma$ for a Newtonian fluid, but it does not behave the same way for a fluid of second grade.

The case of $R = -2$, namely $N = 2$, cannot be obtained by (3.17). For $N = 2$, by the definition of $U(a, b, x)$, $U(1, 1, x)$ can be found as

$$U(1, 1, x) = e^x Ei(x), \tag{3.22}$$

where $Ei(x)$ is the exponential integral defined in the following form

$$Ei(x) = \int_x^\infty \frac{e^{-t}}{t} dt. \tag{3.23}$$

Inserting $U(1, 1, x)$ into the expression of dg/dr and by integration and by using the boundary conditions, one finds

$$\frac{u}{\Omega a_1} = \frac{\sigma}{\xi} \frac{e^{\xi^2/4\varepsilon} Ei\left(\xi^2/4\varepsilon\right) - e^{1/4\varepsilon} Ei(1/4\varepsilon) + \ln\left(\xi^2\right)}{e^{\sigma^2/4\varepsilon} Ei\left(\sigma^2/4\varepsilon\right) - e^{1/4\varepsilon} Ei(1/4\varepsilon) + \ln\left(\sigma^2\right)}. \tag{3.24}$$

The variation of Ωa_1 with respect to ζ for various values of ε is illustrated in Figure 1. The value of σ is taken as 0.2 and the values of ε are taken as 0 and 1. When ε goes to zero, the argument of the exponential integral becomes large, then $Ei(x)$ can be written as e^{-x}/x and (3.24) becomes

$$\frac{u}{\Omega a_1} = \frac{\sigma}{\xi} \frac{\ln \xi}{\ln \sigma}. \tag{3.25}$$

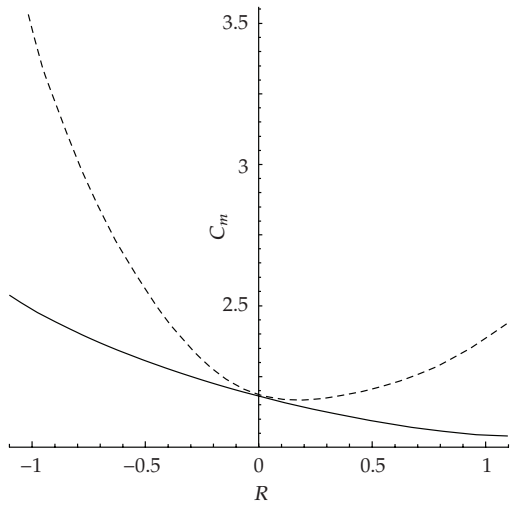

Figure 2: The variation of the moment coefficient with the cross-Reynolds number. The ratio of the radii of the cylinders is taken as 0.2. ($\varepsilon = 0$); ($\varepsilon = 1$),

4. The torque exerted by the fluid on the inner cylinder

The torque exerted by the flow on the inner cylinder is given in the following form:

$$M = 2\pi h a_1^2 \left[\mu \left(\frac{du}{dr} - \frac{u}{r} \right) + \frac{\alpha \alpha_1}{r} \frac{d}{dr} \left(\frac{du}{dr} + \frac{u}{r} \right) \right]_{r=a_1}, \tag{4.1}$$

where h is the length of the inner cylinder. The moment coefficient is

$$(C_m)_{r=a_1} = \sigma \left[\left(\frac{du}{d\xi} - \frac{u}{\xi} \right) + \frac{\varepsilon R}{\xi} \frac{d}{d\xi} \left(\frac{du}{d\xi} + \frac{u}{\xi} \right) \right]_{\xi=\sigma}. \tag{4.2}$$

Equation (4.2) shows that the moment coefficient depends on the cross-Reynolds number, the elastic number, and the ratio of the radii of the cylinders. The variation of the moment coefficient with these numbers is illustrated in Figure 2. There are appreciable differences between that of the Newtonian fluid and that of the second-grade fluid. The curves in Figure 2 show the effect of the second-grade fluid. The moment coefficient decreases with the cross-Reynolds number for a Newtonian fluid and increases with the cross-Reynolds number for a second-grade fluid, for all values of the ratio of the radii of the cylinders.

5. Conclusions

An exact solution of the governing equation for the flow of a fluid of second grade between two coaxial cylinders with porous walls is given. The flow of a second-grade fluid for which the inner cylinder is rotating and the outer one is at rest is considered. The solution is given in terms of the confluent hypergeometric functions and is valid for all values of the cross-Reynolds number and the elastic parameter. The boundary conditions are the no-slip boundary conditions and an additional boundary condition. The additional boundary condition is that the

solution tends to the Newtonian value as the elastic number in the solution approaches zero. The solution is discussed for positive, negative, infinite, and −2 values of the cross-Reynolds number. The torque exerted by the fluid on the inner cylinder is calculated. It is shown that the moment coefficient depends on the cross-Reynolds number, the elastic number, and the ratio of the radii of the cylinders. For a given value of this ratio, it decreases with the cross-Reynolds number for a Newtonian fluid and increases with the cross-Reynolds number for a second-grade fluid.

Acknowledgments

It is pleasure to acknowledge many stimulating correspondences with Professor K. R. Rajagopal and the authors would like to express their thanks to the referees for the valuable suggestions.

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