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Research Article

A Semianalytical Approach to Large Deflections in Compliant Beams under Point Load

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The deflection of compliant mechanism (CM) which involves geometrical nonlinearity due to large deflection of members continues to be an interesting problem in mechanical systems. This paper deals with an analytical investigation of large deflections in compliant mechanisms. The main objective is to propose a convenient method of solution for the large deflection problem in CMs in order to overcome the difficulty and inaccuracy of conventional methods, as well as for the purpose of mathematical modeling and optimization. For simplicity, an element is considered which is a cantilever beam out of linear elastic material under vertical end point load. This can further be used as a building block in more complex compliant mechanisms. First, the governing equation has been obtained for the cantilever beam; subsequently, the Adomian decomposition method (ADM) has been utilized to obtain a semianalytical solution. The vertical and horizontal displacements of a cantilever beam can conveniently be obtained in an explicit analytical form. In addition, variations of the parameters that affect the characteristics of the deflection have been examined. The results reveal that the proposed procedure is very accurate, efficient, and convenient for cantilever beams, and can probably be applied to a large class of practical problems for the purpose of analysis and optimization.

1. Introduction

In recent years, it has turned out that many phenomena in engineering, physics, chemistry, and other branches of science can be described very successfully by nonlinear models using mathematical tools. Generally, these problems cannot be solved explicitly, and normally fail to yield exact solutions. This is why over time, so many methods have been developed for approximate or numerical solutions. The perturbation method [1] is one of the well-known methods to solve nonlinear equations. However, since using the common perturbation method is based upon the existence of a small parameter, developing methods for the applications which do not contain small parameters is difficult. Very recently, some promising approximate analytical solutions were proposed, such as Exp-function method [2, 3], variational iteration method (VIM) [4–6], homotopy-analysis method (HAM) [6, 7], and homotopy perturbation method (HPM) [6, 8–13]. Other methods are reviewed in [14, 15].

In the beginning of the 1980s, Adomian [16, 17] proposed a new and fruitful method (the so-called Adomian decomposition method, ADM) for solving linear and nonlinear (algebraic, differential, partial differential, integral, etc.) equations. It has been shown [18, 19] that this method yields a rapid convergence of the solutions series to linear and nonlinear deterministic and stochastic equations. In method, the solution of a functional equation is considered as an infinite series which is in practice limited to a finite number of iterations to converge rapidly to accurate solutions. It is well known that this method avoids linearization and unrealistic assumptions, and efficiently provides a numerical solution with high accuracy.

A compliant mechanism (CM) is a mechanism that gains some or all of its motion from the deformation of slender segments rather than from relative motion between rigid-body links connected by joints [20]. CMs have many advantages over their rigid body counterparts such as single-piece production, absence of coulomb friction, no need for lubrication, and compactness. Applications range from surgical instruments to MEMS [20]. However, the design of CMs is complicated by the flexible members which include elastic links and elastic hinges. These usually undergo large deflections which introduce geometric nonlinearities. Therefore, the study of large deflections in elastic beams has long been one of the central themes of interest aiming at accurately describing the deformation in CMs [20, 21]. Since in these applications, curvature is nonlinear due to both material and geometrical nonlinearities, a nonlinear mathematical formulation should be considered. Consequently, deflections are difficult to determine by analytical methods, hence numerical and approximate methods should be employed. Due to the complexity of the nonlinear governing equations, only a few studies have been carried out so far to investigate the nonlinear deformation of beams [22–26].

The principal aim of this work is to investigate the feasibility of the ADM method in analyzing compliant mechanical systems. In particular, this work aims to overcome the difficulty and inaccuracy of conventional methods which depend on elliptic integrals and functions or which are based on linear beam theory. As a secondary aim, the ADM method is applied to a cantilever beam to obtain the approximate analytic expressions for the rotation angle as well as vertical and horizontal end point displacements, in order to use it as a building block for the purpose of mathematical modeling and optimization of more complex CMs such as the work of [27]. Thirdly, we are investigating the accuracy and efficiency of this method for the discussed problem.

The cantilever beam is assumed to be initially straight, inextensible, rigid in shear, of constant cross-section and end point loaded by force. This force is perpendicular to the initial beam axis and keeps this orientation as its magnitude is increased. The material is assumed to follow linear elastic stress-strain behavior and to be isotropic.

The paper is organized as follows: firstly, the moment-curvature relationship and governing equation are presented in Section 2.1. Since often only infinitesimal displacements are considered, the governing equation for small deflection is also presented in this section. In Section 2.2, ADM will be applied to the governing equation in order to obtain a semianalytical solution. The results of ADM are portrayed graphically and presented numerically in Section 3 and discussed in Section 4, and will be been compared with those of the numerical solution using Richardson extrapolation method [28].

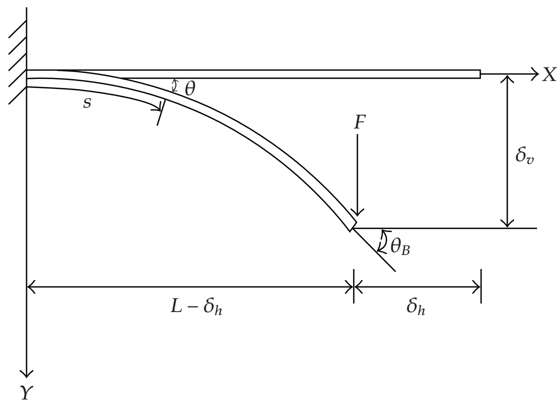

Figure 1: Cantilever beam subjected to a free end point loading.

2. Materials and Methods

2.1. Moment-Curvature Relationship and Governing Equation

Using the Bernoulli-Euler equation [29], the curvature of a prismatic beam, κ, can be written as

$$\kappa = \frac{d\theta}{ds} = \frac{M}{EI}, \tag{2.1}$$

where M is the bending moment, E is Young's modulus, I is the area moment of inertia of the beam, while EI is called the bending stiffness of the beam. Furthermore, $\theta(s)$ is the slope of any point along the arc length with respect to the x-axis, and s is the arc-coordinate on the neutral axis of the beam from the fixed end to the base.

The moment at any point in the beam shown in Figure 1 is given by

$$M = F(L - \delta_h - x), \tag{2.2}$$

where F is the point load at the free end. Thus, the bending equation of a uniform cross-section beam with large deflection is

$$\frac{d\theta}{ds} = \frac{F}{EI}(L - \delta_h - x), \quad \theta(0) = 0, \ \theta'(l) = 0, \tag{2.3}$$

where the prime denotes the differential with respect to s, and where δ_h is the horizontal deflection of beam. The axial elongation of the beam is neglected, because it is much smaller than the lateral deflection at the free end point.

By differentiating (2.1) once with respect to s and rearranging it, we obtain

$$\frac{d^2\theta}{ds^2} = \frac{dM/ds}{EI}, \tag{2.4}$$

By introducing the dimensionless parameter $\zeta = s/L$, the original equation evolves into the governing equation for large deformation of a cantilever beam under free end point load shown in Figure 1 in the following form:

$$\frac{d^2\theta}{d\zeta^2} + \alpha \cos\theta = 0, \quad \theta(0) = 0, \; \theta'(1) = 0 \tag{2.5}$$

where $\alpha = FL^2/EI$ is the dimensionless end point load. The rotation angle of the beam at free end point is denoted by $\theta_B = \theta(1)$. The dimensionless exact vertical and horizontal displacements of the free end point are given by [24]

$$\frac{\delta_v}{L} = 1 - \frac{2}{\sqrt{\alpha}}\left[E(\mu) - E(\varphi, \mu)\right] \tag{2.6}$$

$$\frac{\delta_h}{L} = 1 - \sqrt{\frac{2 \sin\theta_B}{\alpha}}, \tag{2.7}$$

where δ_v is vertical distance of beam, $E(\mu)$ is the complete elliptic integral of the second kind, $E(\varphi, \mu)$ is the elliptic integral of the second kind, θ_B is the rotation angle of the beam free end point, and

$$\mu = \sqrt{\frac{1 + \sin\theta_B}{2}}, \qquad \varphi = \arcsin\left(\frac{1}{\sqrt{2}\mu}\right). \tag{2.8}$$

For infinitesimal deflection, we can assume that the linearized form of (2.5) according to

$$\frac{d^2\theta}{d\zeta^2} + \alpha = 0, \quad \theta(0) = 0, \quad \theta'(1) = 0 \tag{2.9}$$

is sufficiently accurate to model the problem. Solution of (2.9) is

$$\theta(\zeta) = \frac{\alpha}{2}(2 - \zeta)\zeta. \tag{2.10}$$

2.2. Semianalytical Solution

Prior to applying the ADM [16, 17] to (2.5), in order to better demonstrate how ADM works, let us consider general equation $Hu = g$, where H represents a general nonlinear differential operator consisting of both linear and nonlinear terms, g is the source term, and u is the unknown function. The linear terms are further decomposed to $L(u) + R(u)$, where L is easily invertible (usually the highest order derivative) and R comprises the remaining terms of linear operator. Now, the nonlinear term is represented by N. For (2.5), the general equation of $Hu = g$ now can be rewritten as

$$L(\theta) + N(\theta) + R(\theta) = g. \tag{2.11}$$

It should be noted that this work deals with the large deformation of elastic beam, hence the assumption of $\sin(\theta) = \theta$ and $\cos(\theta) = 1$ is ineffectual. So, the Taylor series expansion of $\cos(\theta)$ in (2.5) is considered as follows [15]:

$$\cos(\theta) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!}, \ldots, \tag{2.12}$$

Thus, $N(\theta) = \alpha(1 - \theta^2/2! + \theta^4/4!)$, $R(\theta) = 0$, and $L(\theta) = d^2\theta/d\zeta^2$.

The nonlinear operator $N(\theta)$ can be decomposed into an infinite series of polynomials given by

$$N(\theta) = \sum_{n=0}^{\infty} A_n(\theta_0, \theta_1, \ldots, \theta_n), \tag{2.13}$$

where $A_n(\theta_0, \theta_1, \ldots, \theta_n)$ are the appropriate Adomian's polynomials defined by

$$A_n = \frac{1}{n!}\left\{\frac{d^n}{d\lambda^n} N\left[\sum_{n=0}^{\infty}\left(\lambda^k \theta_k\right)\right]\right\}, \quad n \geq 0, \tag{2.14}$$

where

$$\begin{aligned}
A_0 &= \alpha\left(1 - \frac{\theta_0^2}{2!} - \frac{\theta_0^4}{4!}\right), \\
A_1 &= \frac{\alpha}{2!}(2\theta_0\theta_1) - \frac{\alpha}{4!}\left(4\theta_1\theta_0^3\right), \\
A_2 &= \frac{\alpha}{2!}\left(2\theta_0\theta_2 + \theta_1^2\right) - \frac{\alpha}{4!}\left(4\theta_2\theta_0^3 + 6\theta_1^2\theta_0^2\right), \\
A_3 &= \frac{\alpha}{2!}(2\theta_0\theta_3 + 2\theta_1\theta_2) - \frac{\alpha}{4!}\left(4\theta_3\theta_0^3 + 6\theta_1^3\theta_0 + 12\theta_2\theta_1\theta_0^2\right), \\
A_4 &= \frac{\alpha}{2!}\left(2\theta_0\theta_4 + 2\theta_1\theta_3 + \theta_2^2\right) - \frac{\alpha}{4!}\left[4\theta_4\theta_0^3 + 6\theta_2^2\theta_0^2 + 12\left(\theta_3\theta_1\theta_0^2 + \theta_2\theta_1^2\theta_0\theta_1^4\right)\right], \\
A_5 &= \frac{\alpha}{2!}(\theta_0\theta_5 + 2\theta_1\theta_4 + 2\theta_2\theta_3) - \frac{\alpha}{4!}\left[4\left(\theta_5\theta_0^3 + \theta_2\theta_1^3\right) + 12\left(\theta_4\theta_1\theta_0^2 + \theta_3\theta_2\theta_0^2 + \theta_3\theta_1^2\theta_0 + \theta_2^2\theta_0\theta_1\right)\right], \\
&\vdots
\end{aligned} \tag{2.15}$$

The inverse operator L^{-1} is defined as the integral with respect to s′ from 0 to ζ

$$L^{-1} = \iint_{0}^{\zeta} (\cdot) d\zeta d\zeta, \tag{2.16}$$

If L is a second-order operator, L^{-1} is a twofold indefinite integral:

$$L^{-1}L(\theta) = \theta - \theta(0) - \zeta \frac{\delta\theta(0)}{\delta\zeta}, \tag{2.17}$$

Solving (2.11) for $L(\theta)$ and multiplying by L^{-1} on both sides gives

$$L^{-1}L(\theta) = L^{-1}(g) - L^{-1}[R(\theta)] - L^{-1}[N(\theta)]. \tag{2.18}$$

Comparing (2.17) and (2.18) gives

$$\theta = f(\zeta) = L^{-1}(g) - L^{-1}[R(\theta)] - L^{-1}[N(\theta)], \tag{2.19}$$

where

$$f(\zeta) = f_0(\zeta) + f_1(\zeta) + f_2(\zeta) + \cdots + f_n(\zeta) \tag{2.20}$$

So, (2.20) shows that $f(\zeta)$ is decomposed into $f_n(\zeta)$ components where $n = 0, \ldots, \infty$. Also $f_0(\zeta)$ is 0 for the discussed problem, $f_n(\zeta)$ where $n = 1, \ldots, \infty$ arises from the prescribed initial or boundary conditions.

Using (2.19) and (2.20), we have

$$\begin{aligned}
f_1(\zeta) &= -\frac{1}{2}\alpha\zeta(\zeta - 2) \\
f_2(\zeta) &= 0.4166666667 \times 10^{-2}\zeta\alpha^3\Big(\zeta^5 - 6\zeta^4 + 10\zeta^3 + 0.1295824560 \times 10^{-4}\alpha^{10} \\
&\qquad + 0.1026303239 \times 10^{-2}\alpha^{-14} - 0.5538583175 \times 10^{-3}\alpha^{12} \\
&\qquad -0.2858762790 \times 10^{-3}\alpha^{16} - 16\Big), \\
f_3(\zeta) &= -0.2314814815 \times 10^{-4}\zeta\alpha^5\Big(\zeta^9 - 10\zeta^8 + 35.35714286\zeta^7 - 42.85714286\zeta^7 - 72\zeta^4 + 240\zeta^3
\end{aligned}$$

$$
\begin{aligned}
&- 502.8571429 - 0.2332484208 \times 10^{-2}\alpha^{8} - 0.1847345831\alpha^{12} \\
&+0.9969449715 \times 10^{-1}\alpha^{10} + 0.5145773021 \times 10^{-1}\alpha^{14}\Big), \\
f_4(\zeta) = {}& 0.1112891738 \times 10^{-7} q\alpha^{5}\Big[-10.70320788\alpha^{14} + 38.42479328\alpha^{12} - 20.73645541\alpha^{10} \\
&+ 0.4851567153\alpha^{8} + \alpha^{2}\Big(\zeta^{13} - 14\zeta^{12} + 76.70562771\zeta^{11} \\
&+ 185.7142857\zeta^{9} - 138.6666667\zeta^{8} + 980.5714286\zeta^{7} \\
&- 1782.857143\zeta^{6} - 2614.857143\zeta^{4} + 10380.19048\zeta^{3} \\
&-192.4675325\zeta^{10} - 23055.51515\Big) - 10028.57143\zeta^{7} - 260\zeta^{9} \\
&+2600\zeta^{8} + 9508.571429 - 12480\zeta^{5} + 17828.57143\zeta^{6}\Big] \\
f_5(\zeta) = {}& -0.4970431509 \times 10^{-9}\alpha^{7}\zeta\Big[\alpha^{2}\zeta^{17} - 18\alpha^{2}\zeta^{16}134.0264492\alpha^{2}\zeta^{15} - 512.4231866\alpha^{2}\zeta^{14} \\
&+\Big(1002.160731\,\alpha^{2} - 1119.512195\Big)\zeta^{13} \\
&+ \Big(-1008.627178\alpha^{2} + 15673.17073\Big)\zeta^{12} \\
&+\Big(2289.943617\,\alpha^{2} - 89653.05670\Big)\zeta^{11} \\
&+\Big(-8618.789990\alpha^{2}+260831.8023\Big)\zeta^{10} \\
&+\Big(-385879.8606 + 11088.50174\alpha^{2}\Big)\zeta^{9} \\
&+ \Big(271668.2927 - 5421.045296\alpha^{2}\Big)\zeta^{8} \\
&+ \Big(45652.94574\,\alpha^{2} - 299389.5470\Big)\zeta^{7} \\
&+ \Big(798372.1254 - 96311.55799\alpha^{2}\Big)\zeta^{6} \\
&- 745147.3171\zeta^{5} + \Big(-129054.6519\,\alpha^{2} + 53224.80836\Big)\zeta^{4} \\
&+ \Big(-177416.0279 + 560287.2601\alpha^{2}\Big)\zeta^{3} - 108.6277719\,\alpha^{6} \\
&- 1276774.856\,\alpha^{2} + 4642.942943\,\alpha^{8} - 8603.404934\,\alpha^{10} \\
&+2396.474351\,\alpha^{12} + 1153328.248\Big], \\
&\vdots
\end{aligned}
\tag{2.21}
$$

Table 1: Comparison of large deflection results in cantilever beams under free end point load using a linear model and the presented method, showing the absolute error and accuracy as compared to the well-established Richardson numerical solution (Richardson extrapolation) [24], as well as the number of polynomials, n, required for an accuracy of at least 99.00%.

	Linear solution		Presented method		
α	Absolute error	Accuracy (%)	Absolute error	Accuracy (%)	Number of polynomials, n, required for accuracy of 99%
0.1	5E-5	99.90	4E-10	99.99	1
0.2	4E-4	99.63	5E-9	99.99	1
0.3	11E-3	99.18	2E-8	99.99	1
0.4	34E-3	98.55	3E-8	99.99	2
0.5	56E-3	97.76	3E-8	99.99	3
0.6	96E-3	96.81	2E-8	99.99	3
0.7	1E-2	95.71	2E-7	99.99	4
0.8	3E-2	94.47	6E-7	99.99	4
0.9	3E-2	93.10	2E-6	99.99	4
1.0	4E-2	91.62	1E-5	99.99	4
1.1	5E-2	90.02	5E-5	99.99	5
1.2	6E-2	88.33	2E-4	99.96	5
1.3	8E-2	86.54	7E-4	99.86	6
1.4	9E-2	84.67	2E-3	99.63	7
1.5	1E-1	82.40	5E-3	99.21	8

In the same manner, the rest of the components until $f_{10}(\zeta)$ are obtained. Lastly, in accordance with the standard Adomian method, the solution of $\theta(\zeta)$ is obtained by the series

$$\theta(\zeta) = \sum_{n=1}^{\infty} f_n(\zeta). \tag{2.22}$$

As the number of polynomials of above equation increases from 1 to ∞, the solution changes from the linear solution to the original nonlinear one, while the accuracy increases accordingly (see right column in Table 1). With this result, the vertical and horizontal displacements of the beam can be determined by using (2.6) and (2.7), respectively.

3. Results

To test the validity and accuracy of the method used in this study, the residual rotation angle for the beam shown in Figure 1 was determined as a function of ζ for the linear solution and the presented method. The results are displayed in Figure 2.

For further illustration, the results obtained by ADM are compared with the numerical result obtained by a finite difference technique using Richardson extrapolation [26] as stated in Table 1.

For the purpose of comparison, both the maximum absolute error and accuracy at the free end are included in this table. In addition, the number of polynomials required for an accuracy of at least 99.00% is indicated in this table. The contour of rotation angle of the beam at any point along the beam versus α is portrayed graphically in Figure 3. The trends of rotation angle, vertical and horizontal displacements of the beam at the free end point versus α are illustrated in Figure 4. Table 1 as well as Figures 2–4 are obtained for α from 0 to 1.5.

4. Discussion

This study demonstrates that the ADM method can conveniently be applied to the analysis of compliant mechanical systems. This method not only overcame the inaccuracy problem of the linear solution but also it has eliminated some shortcoming in conventional numerical methods like elliptic integrals, such as implementation difficulties and accuracy problems, due to table look-up. It provides a semiexact closed form solution for both the rotation angle of the beam at any point along the beam length as well as the vertical and horizontal position of the end tip and therefore this can provide a simple way to foretell the physical characteristic of the deflection without using numerical results. On the other hand, it can further be used for mathematical modeling and optimization of more complex CMs such as those in the work of [27].

Figure 2 shows that residuals from the presented method are significantly less than from the linear solution. Practically, they can be considered negligible. This gives us a reason to claim that the presented method is remarkably accurate. Also this figure shows that the method remains accurate if α is varied between 0.5 and 1.5 although the accuracy decreases with increasing α.

As expected, Table 1 shows that higher values of α (larger deflection) result in less accuracy for the linear model while just a slight reduction in accuracy can be seen in the presented method (no more than 0.08%). It also shows that only few polynomials are required to reach the accuracy of 99.00%. Accordingly, the presented method not only is valid for small deflection of the beam but also is highly accurate for large deflection. As shown in this table, the expression for the rotation angle obtained by ADM is very accurate and convenient because the maximum error and the minimum accuracy for presented method is 5.37E-3 and 99.21%, respectively, and the computations are very straightforward using mathematical packages. If more accuracy were needed, the number of orthogonal polynomials used in (2.22) should be increased.

We report that considering computational time required for comparable accuracy, the presented method needs only 2.8 seconds to reach accurate results after 10 iterations using a CPU of 2.0 GHz.

As can be seen clearly in Figure 3, the value of the rotation angle of the beam is increased as the dimensionless displacement ζ and dimensionless applied force α increase. Also, both increasing ζ and α give rise to a more significant increase of rotating angle. This approach presents a simple way to predict the results shown in these figures directly from (2.21): both above results can be predicted due to positive powers of ζ and α.

Now, as a closed-form expression for the rotation angle of any cross-section of the beam is obtained, the vertical and horizontal displacements can readily be calculated from (2.6) and (2.7). As with the previous case, the effect of variations α is significant on all the above cases as illustrated in Figure 4. An increase in α gives rise to an increase in rotation angle, and in vertical and horizontal displacements of the beam at free end point. However, as can be seen in Figure 4, this effect is stronger for the rotational and vertical displacement and less strong for the horizontal displacement. As mentioned previously, these results can be straightforwardly predicted by (2.6), (2.7), and (2.21).

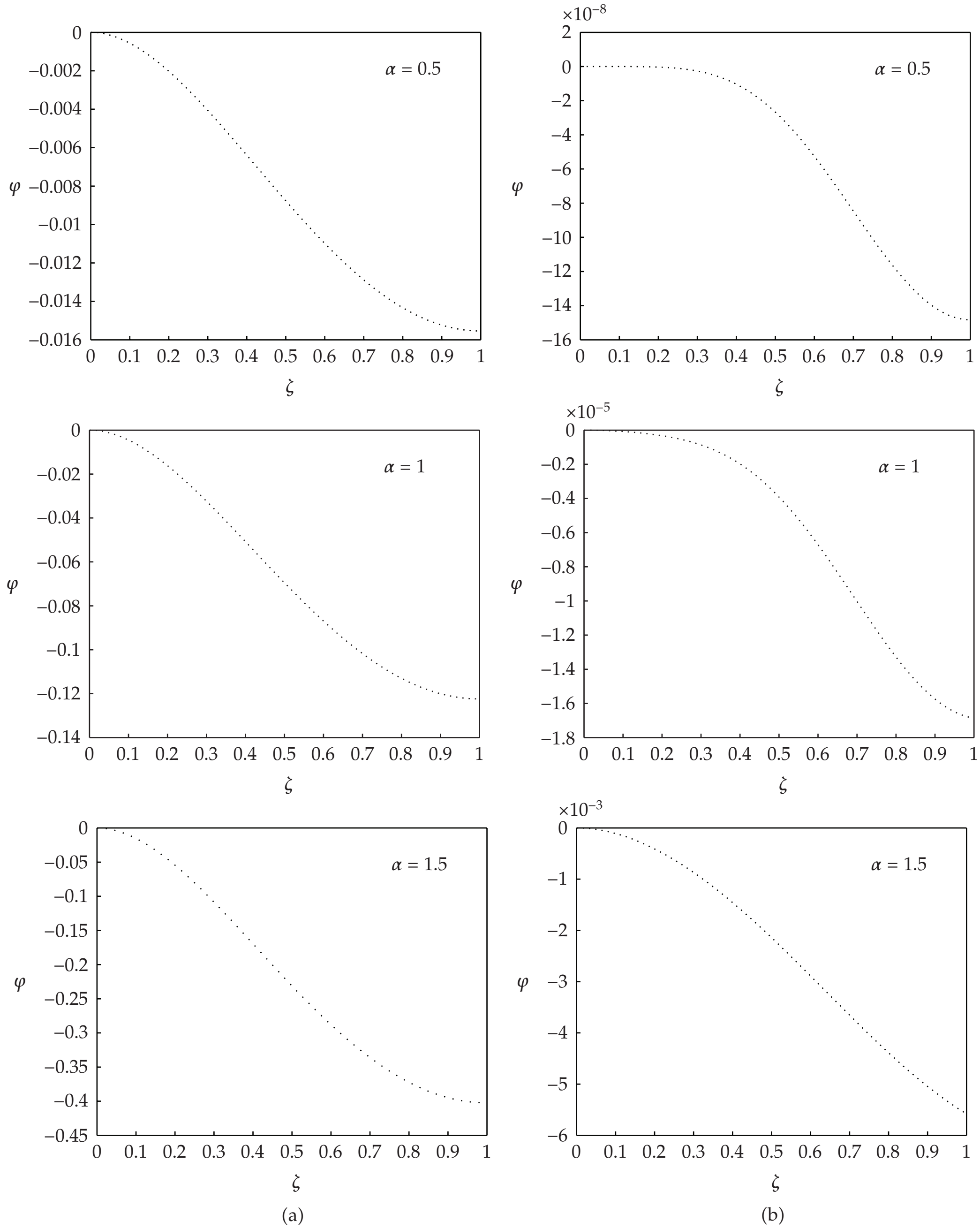

Figure 2: The residual of rotation angle versus ζ from (a) linear solution and (b) presented method.

5. Conclusion

In this paper, we have successfully utilized ADM to handle the geometric nonlinearity caused by large deflection of a cantilever beam under a point load at the free end point. For small deformations, the linear solution is valid, but for larger deformation, we encounter

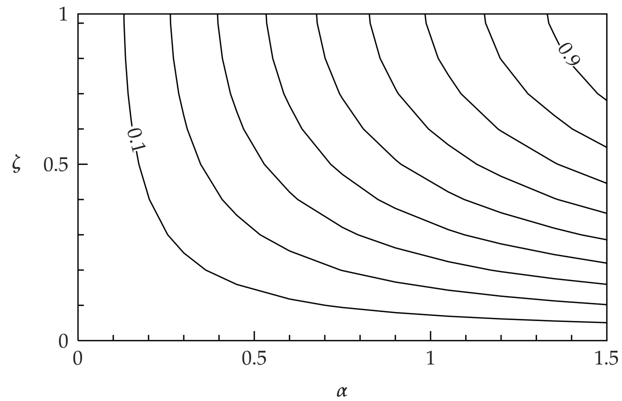

Figure 3: The dimensionless normalized rotation angle of the beam versus α and ζ with respect to a reference value of 0.644930, for α from 0 to 1.5, and ζ from 0 to1.

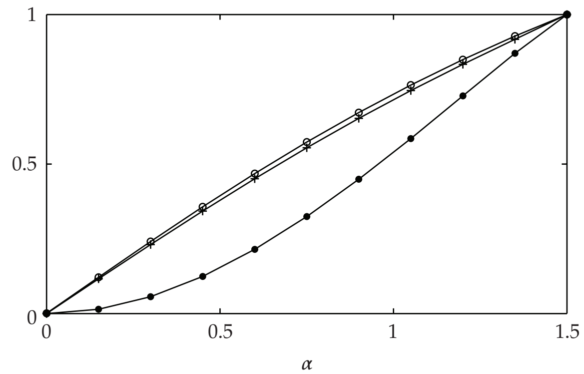

Figure 4: The dimensionless normalized (+) rotation angle (∘) vertical and (•) horizontal displacements of the beam at free end point versus α from 0 to 1.5; the references values are 0.644930, 0.410876 and 0.104729, respectively.

a nonlinear problem. Using ADM, we obtained an explicit expression for the rotation angle along the beam, which lead to a semianalytical solution for vertical and horizontal displacements. It is evident that the method is very powerful and efficient for solving this kind of problems arising in compliant mechanisms and other mechanical systems, and presents a rapid convergence for the solutions. It also overcame the difficulty and inaccuracy in using conventional numerical and linear methods. Besides, as the semiexact solution of the governing equation is obtained, the optimal design of practical problems, prediction of physical characteristic, as well as mathematical modeling of more complex compliant mechanisms can be achieved conveniently.

It was found that the maximum inaccuracy for proposed solutions occurs at the free end point and amounts to less than 0.08% as compared to regular numerical solution methods. The method remains accurate if the dimensionless stiffness is varied although the accuracy decreases with increasing dimensionless stiffness. On the other hand, the dimensionless stiffness has less influence on the horizontal displacement of free end point in comparison to vertical displacement and end point rotation.

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