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Research Article

Chaos Control and Hybrid Projective Synchronization of a Novel Chaotic System

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Adaptive feedback controllers based on Lyapunov's direct method for chaos control and hybrid projective synchronization (HPS) of a novel 3D chaotic system are proposed. Especially, the controller can be simplified ulteriorly into a single scalar one to achieve complete synchronization. The HPS between two nearly identical chaotic systems with unknown parameters is also studied, and adaptive parameter update laws are developed. Numerical simulations are demonstrated to verify the effectiveness of the control strategies.

1. Introduction

Since chaotic attractors were found by Lorentz in 1963, many chaotic systems have been constructed, such as the Lorentz system, Chen system, and Lü system [1–8]. Because of the potential applications in engineering, the study of chaotic systems has attracted more and more researchers' attention. Chaos control and synchronization of chaotic systems have been interesting research fields since the pioneering work of Ott, Grebogi, and Yorke and the seminal work of Pecora and Carroll, which are simultaneously reported in 1990. Ever since, various types of synchronization phenomena have been found such as complete synchronization [9, 10], phase synchronization [11, 12], partial synchronization [13], generalized synchronization [14], projective synchronization [15–17], and so forth. They are applied in many fields, such as secure communication, neural networks, optimization of nonlinear system performance, ecological systems, modeling brain activity, system identification and pattern recognition, and so on.

Recently, hybrid projective synchronization (HPS) was proposed. It can be considered as an extension of projective synchronization because complete synchronization and anti-synchronization are both its special cases. It is worthy of study because the response signals can be any proportional to the drive signals by adjusting the factors and it can be used to extend binary digital to variety M-nary digital communications for achieving fast communication. However, the controllers based on different control methods in the existing literatures, such as nonlinear feedback control [18–20], active control [21–23], adaptive control [24–26], and so forth, are mostly vectorial and they are difficult to be put into practice. So, the controllers which are simple, efficient, and easy to implement are required to be designed for both chaos control and HPS between two chaotic systems.

Chen-Lee system [27] is a new 3D chaotic system which was proposed by Chen and Lee. It takes the following form:

$$\begin{aligned}\dot{x}_1 &= ax_1 - x_2x_3,\\ \dot{x}_2 &= -bx_2 + x_1x_3,\\ \dot{x}_3 &= -cx_3 + \frac{x_1x_2}{3},\end{aligned} \tag{1.1}$$

where $x_1,\ x_2,\ x_3$ are state variables, a, b, c are positive constant parameters, and $0 < a < b + c$ to guarantee the system to generate chaos. System (1.1) is symmetrical about three coordinate axes, $x_1,\ x_2,\ x_3$, respectively, and these symmetries persist for all values of the system parameters. This chaotic system is robust to various small perturbations due to its highly symmetric structure, and it is dissipative. Its chaotic attractor is shown in Figure 1 for $a = 5,\ b = 10$, and $c = 3.8$. Due to the fact that the new system has five equilibrium points, some larger chaotic regions, and more complex bifurcation behaviors compared with Lorenz system, Chen system [1], and Lü system [2], it may have good application prospects. So we study chaos control and HPS of the new chaotic system motivated by the idea of designing simple and efficient controller for application. Firstly, the system converges to its unstable equilibrium point by designing an adaptive linear feedback controller which only includes single-state variable. Moreover, the control method proposed is generalized to a class of chaotic systems. Secondly, HPS between two identical systems is studied based on Lyapunov's direct method. An adaptive nonlinear feedback vectorial controller is derived to guarantee HPS, which can degenerate into a single scalar one in the case of complete synchronization. Furthermore, the problem of adaptive hybrid projective synchronization between two nearly identical novel chaotic systems with unknown parameters is also studied, and adaptive parameter update laws are developed. Finally, numerical simulation results illustrate the effectiveness of the proposed control strategies. The approaches in our paper have certain significance for reducing the cost and complexity for controller implementation.

2. Chaos Control of the Novel Chaotic System

In this section, chaotic system (1.1) will be controlled to its unstable equilibrium point $O(0,0,0)$ via an adaptive linear feedback controller which only includes one-state variable. The controller can be designed as

$$u_i = -kx_i, \qquad u_j = 0, \quad j \neq i,\ i, j = 1, 2, 3, \tag{2.1}$$

where feedback gain k is adapted according to the following update law:

$$\dot{k} = k_1 x_i^2, \quad i = 1, 2, 3,\ k(0) = 0,\ k_1 > 0. \tag{2.2}$$

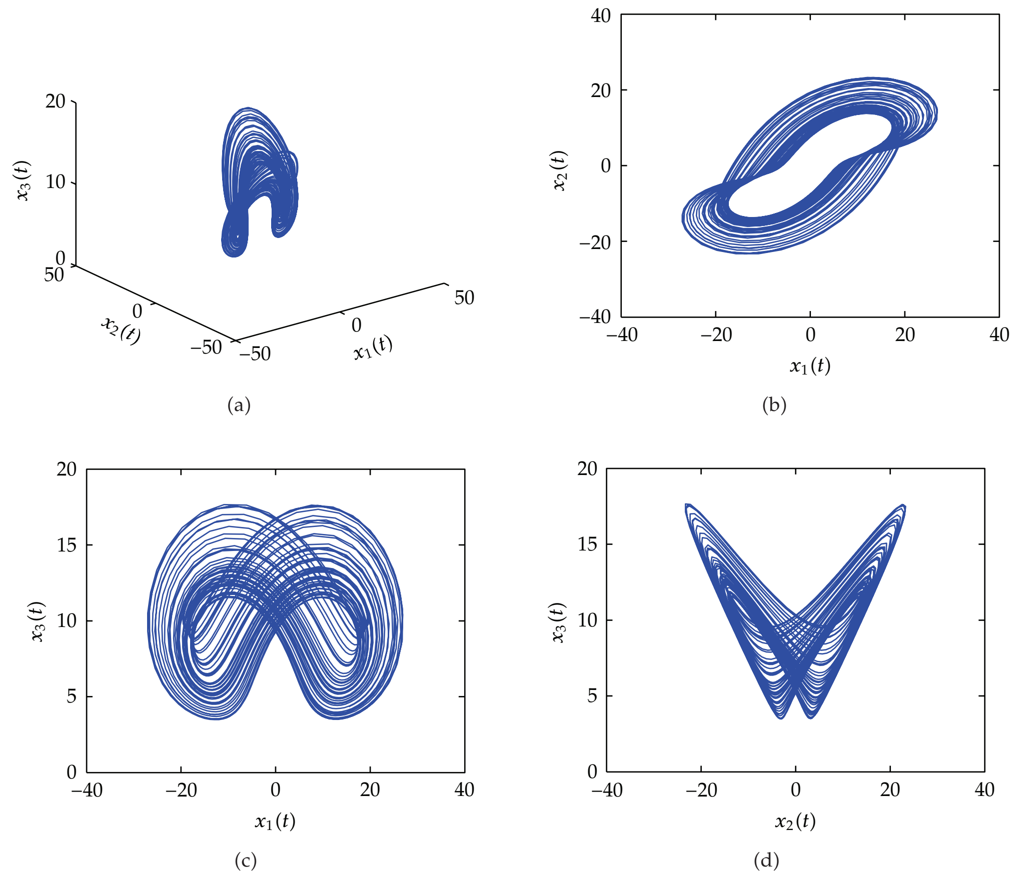

Figure 1: The chaotic attractor of system (1.1) with $a = 5,\ b = 10,\ c = 3.8$.

According to (2.1) and (2.2), the controller associated with adaptive update law can be chosen as

$$u_1 = -kx_1, \qquad u_2 = u_3 = 0, \qquad \dot{k} = k_1 x_1^2, \quad k(0) = 0,\ k_1 > 0, \tag{2.3}$$

and the controlled system is considered as

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 + u_1, \\ \dot{x}_2 &= -bx_2 + x_1x_3, \\ \dot{x}_3 &= -cx_3 + \frac{x_1x_2}{3}. \end{aligned} \tag{2.4}$$

Then, we have the following theorem on stabilizing the origin of system.

Theorem 2.1. *The controlled chaotic system* (2.4) *will globally and asymptotically converge to the unstable equilibrium point* $O(0, 0, 0)$ *under the controller with the update law* (2.3).

Proof. Introducing a candidate Lyapunov function as

$$V(t) = \frac{1}{2}\left(\frac{4}{3}x_1^2 + x_2^2 + x_3^2\right) + \frac{1}{2k_1}(k - k^*)^2, \tag{2.5}$$

where k^* is a sufficiently large constant to be determined. Then, it is positive definite and

$$\begin{aligned} \dot{V}(t) &= \frac{4}{3}x_1\dot{x}_1 + x_2\dot{x}_2 + x_3\dot{x}_3 + \frac{1}{k_1}(k - k^*)\dot{k} \\ &= -\left(k^* - \frac{4}{3}a + \frac{1}{3}k\right)x_1^2 - bx_2^2 - cx_3^2 \\ &= -x^T P x, \end{aligned} \tag{2.6}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \qquad P = \begin{bmatrix} k^* - \dfrac{4a}{3} + \dfrac{k}{3} & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}. \tag{2.7}$$

We can choose the undetermined $k^* > 4a/3 - k/3$ so that the symmetric matrix P is positive definite. Then, $\dot{V}(t)$ is negative semidefinite since $a > 0,\ b > 0,\ c > 0$. According to

$$\int_0^t \lambda_{\min}(P)\|x\|^2 dt \le \int_0^t x^T P x dt = -\int_0^t \dot{V}(t)dt \le V(0), \tag{2.8}$$

where $\lambda_{\min}(P)$ is the smallest eigenvalue of P, x and dx/dt are both bounded. It follows that $\dot{V}(t)$ is uniformly continuous. Based on Barbalat's Lemma, $\dot{V}(t) \to 0$ as $t \to \infty$. So, the system (1.1) converges to $O(0,0,0)$ as t tends to infinity under the controller with the update law (2.3). This completes the proof. □

Numerical simulation demonstrates the performance of the system controlled by the proposed method. The system parameters are chosen to be $a = 5,\ b = 10$, and $c = 3.8$ so that the system (1.1) has a chaotic attractor. The initial conditions are set to be $x_1(0) = 1, x_2(0) = 2, x_3(0) = 3$. The initial condition of the adaptive feedback gain is set to be $k(0) = 0$, and the constant coefficient k_1 is set to be 20. Figure 2 shows the time responses of states $x_1,\ x_2,\ x_3$ for the controlled system (2.4). It is observed that chaotic system is suppressed to its unstable equilibrium point $O(0,0,0)$ under the single scalar controller $u_1 = -kx_1$ with the feedback gain adaptive update law $\dot{k} = k_1 x_1^2$.

The single scalar adaptive control strategy (2.1) with adaptive update law (2.2) proposed above can be applied to a class of general 3D chaotic system

$$\dot{x} = Ax + G(x), \tag{2.9}$$

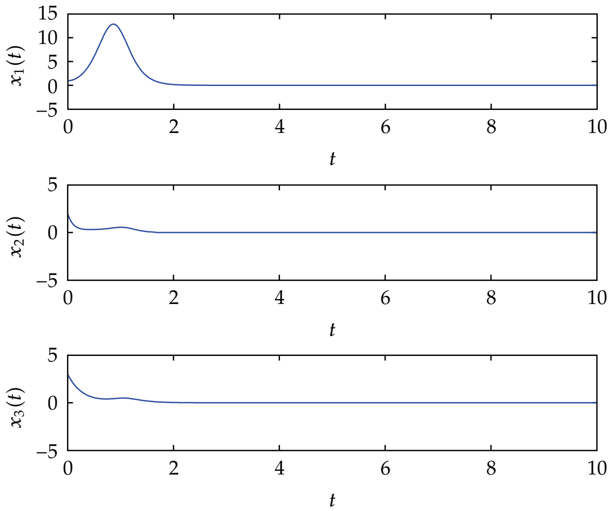

Figure 2: State trajectories of the controlled chaotic system (2.4) when $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 1$, $k(0) = 0$, and $k_1 = 20$.

where $x = (x_1, x_2, x_3)^T \in R^3$ is the state vector, $A \in R^{3\times 3}$ is a constant matrix, and $G(x) = (g_1(x), g_2(x), g_3(x))^T$ is a vector nonlinear term which satisfies

$$a_1 x_1 g_1(x) + a_2 x_2 g_2(x) + a_3 x_3 g_3(x) = 0, \tag{2.10}$$

where a_1, a_2, a_3 are constants. For example, the well-known Lorentz system, Chen system, Lü system, and so on [1–8] all have the forms and they all can be controlled by single scalar adaptive control method.

3. Hybrid Projective Synchronization

3.1. Hybrid Projective Synchronization by Adaptive Feedback Control Law

For two dynamical systems

$$\dot{x} = f(x), \tag{3.1}$$

$$\dot{y} = g(y) + u(x, y), \tag{3.2}$$

where $x = (x_1, x_2, \ldots, x_n)^T, y = (y_1, y_2, \ldots, y_n)^T \in R^n$ are state variables of the drive system (3.1) and the response system (3.2), respectively, $f : R^n \to R^n$ and $g : R^n \to R^n$ are nonlinear vectorial functions, $u(x, y) = (u_1(x, y), u_2(x, y), \ldots, u_n(x, y))^T$ is the nonlinear control vector. If there exists a nonzero constant matrix $\alpha = \mathrm{diag}(\alpha_1, \alpha_2, \ldots, \alpha_n)$ such that $\lim_{t\to\infty}|y - \alpha x| = 0$, namely, $\lim_{t\to\infty}|y_i - \alpha_i x_i| = 0, (i = 1, 2, \ldots, n)$, then the response system and the drive system are said to be in HPS. In particular, the drive-response systems achieve complete synchronization when all values of α_i are equal to 1 and the two chaotic systems are said to be in antisynchronization when all values of α_i are equal to −1.

In this section, we study the hybrid projection synchronization of two identical chaotic systems. The response system corresponding to the drive system (1.1) is defined as follows:

$$\begin{aligned}\dot{y}_1 &= ay_1 - y_2y_3 + u_1,\\ \dot{y}_2 &= -by_2 + y_1y_3 + u_2,\\ \dot{y}_3 &= -cy_3 + \frac{y_1y_2}{3} + u_3,\end{aligned} \tag{3.3}$$

where $(u_1, u_2, u_3)^T$ is the nonlinear control vector. System (1.1) and system (3.3) are in HPS as long as

$$\lim_{t\to\infty} \left| y_i - \alpha_i x_i \right| = 0, \qquad i = 1, 2, 3. \tag{3.4}$$

Define the state error vector as $e = y - \alpha x$, namely,

$$\begin{aligned}e_1 &= y_1 - \alpha_1 x_1,\\ e_2 &= y_2 - \alpha_2 x_2,\\ e_3 &= y_3 - \alpha_3 x_3,\end{aligned} \tag{3.5}$$

where $\alpha = \operatorname{diag}(\alpha_1, \alpha_2, \alpha_3)$ and $\alpha_1, \alpha_2, \alpha_3$ are different, desired scaling factors for HPS. The error dynamical system between system (1.1) and system (3.3) can be written as

$$\begin{aligned}\dot{e}_1 &= ae_1 - e_2e_3 - \alpha_2x_2e_3 - \alpha_3x_3e_2 - (\alpha_2\alpha_3 - \alpha_1)x_2x_3 + u_1,\\ \dot{e}_2 &= -be_2 + e_1e_3 + \alpha_1x_1e_3 + \alpha_3x_3e_1 + (\alpha_1\alpha_3 - \alpha_2)x_1x_3 + u_2,\\ \dot{e}_3 &= -ce_3 + \frac{e_1e_2}{3} + \frac{\alpha_1x_1e_2}{3} + \frac{\alpha_2x_2e_1}{3} + \frac{(\alpha_1\alpha_2 - \alpha_3)x_1x_2}{3} + u_3.\end{aligned} \tag{3.6}$$

The target is to find a controller such that the state errors satisfy

$$\lim_{t\to\infty} e_1(t) = 0, \qquad \lim_{t\to\infty} e_2(t) = 0, \qquad \lim_{t\to\infty} e_3(t) = 0, \tag{3.7}$$

then the global and asymptotical stability of system (3.6) means system (1.1) and (3.3) are in HPS. Choose the control functions u_1, u_2, and u_3 as follows:

$$\begin{aligned}u_1 &= (\alpha_2\alpha_3 - \alpha_1)x_2x_3,\\ u_2 &= -(\alpha_1\alpha_3 - \alpha_2)x_1x_3,\\ u_3 &= -\frac{(\alpha_1\alpha_2 - \alpha_3)x_1x_2}{3} - \frac{2\alpha_1x_1e_2}{3} - \frac{e_1e_3}{3} - ke_3,\end{aligned} \tag{3.8}$$

where k is the feedback gain and is adapted according to the following update law:

$$\dot{k} = k_2 e_1^2, \qquad k(0) = 0,\ k_2 > 0. \tag{3.9}$$

We have the following result.

Theorem 3.1. *For any initial conditions, the drive system* (1.1) *and the response system* (3.3) *are globally and asymptotically hybrid projective synchronized by nonlinear feedback controller* (3.8) *with the update law* (3.9).

Proof. Construct a candidate Lyapunov function

$$V_1(t) = \frac{1}{2}\left(\frac{1}{3}e_1^2 + \frac{1}{3}e_2^2 + e_3^2\right) + \frac{1}{2k_2}(k - k^*)^2. \tag{3.10}$$

It is clear that $V_1(t)$ is a positive definite function. By applying the controller (3.8) to (3.6), the error dynamics can be written as

$$\begin{aligned}
\dot{e}_1 &= ae_1 - e_2e_3 - \alpha_2 x_2 e_3 - \alpha_3 x_3 e_2,\\
\dot{e}_2 &= -be_2 + e_1e_3 + \alpha_1 x_1 e_3 + \alpha_3 x_3 e_1,\\
\dot{e}_3 &= -ce_3 - \frac{\alpha_1 x_1 e_2}{3} + \frac{\alpha_2 x_2 e_1}{3} - ke_3.
\end{aligned} \tag{3.11}$$

The time derivative of $V_1(t)$ along the solution of error dynamical system (3.6) is as follows:

$$\begin{aligned}
\dot{V}_1(t) &= \frac{1}{3}e_1\dot{e}_1 + \frac{1}{3}e_2\dot{e}_2 + e_3\dot{e}_3 + \frac{1}{k_2}(k - k^*)\dot{k}\\
&= -\left(k^* - k - \frac{1}{3}a\right)e_1^2 - \frac{1}{3}be_2^2 - (c + k)e_3^2\\
&= -e^T Pe,
\end{aligned} \tag{3.12}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}, \qquad P = \begin{bmatrix} k^* - k - \dfrac{a}{3} & 0 & 0 \\ 0 & \dfrac{b}{3} & 0 \\ 0 & 0 & c + k \end{bmatrix}. \tag{3.13}$$

The symmetric matrix P should be positive definite when P satisfies the following conditions

$$\begin{aligned}
&k^* - k - \frac{a}{3} > 0,\\
&\frac{b(k^* - k - a/3)}{3} > 0,\\
&\frac{b(c + k)(k^* - k - a/3)}{3} > 0.
\end{aligned} \tag{3.14}$$

Since $a > 0,\ b > 0,\ c > 0,\ k \geq 0$, the symmetric matrix P is positive when $k^* > k + a/3$, then $\dot{V}_1(t)$ is negative semidefinite. Based on Barbalat's Lemma, $\dot{V}_1(t) \to 0$ as $t \to \infty$. It follows that the error variables become zero as t tends to infinity, namely, $\lim_{t\to\infty}|y_i - \alpha_i x_i| = 0, i = 1,2,3$. This means that the two chaotic systems (1.1) and (3.3) are in HPS under the controller (3.8). □

Remark 3.2. In case of $\alpha_i = 1,\ i = 1,2,3$, two chaotic systems are in complete synchronization, the controller can be simplified ulteriorly into $u_1 = 0,\ u_2 = 0,\ u_3 = -2\alpha_1 x_1 e_2/3 - e_1 e_3/3 - k e_3$, namely, it can be simplified to a single scalar form.

3.2. *Adaptive Hybrid Projective Synchronization with Unknown Parameters of the Response System*

In practical applications, the response system parameters are partially or entirely unknown in advance. Therefore, it is necessary to investigate the synchronization problem of chaotic systems with unknown system parameters. In the following, we adopt the adaptive control laws (3.8) to drive two nearly identical chaotic systems with the unknown response system parameters and different initial conditions in HPS. The drive system is designed as (1.1) and the response system is modeled as follows:

$$\begin{aligned} \dot{y}_1 &= \hat{a} y_1 - y_2 y_3 + u_1, \\ \dot{y}_2 &= -\hat{b} y_2 + y_1 y_3 + u_2, \\ \dot{y}_3 &= -\hat{c} y_3 + \frac{y_1 y_2}{3} + u_3, \end{aligned} \tag{3.15}$$

where $u_i(t)$ $(i = 1,2,3)$ are the controllers to be designed. The drive system parameters $a,\ b,\ c$ are known, but the response system parameters $\hat{a},\ \hat{b},\ \hat{c}$ which need to be identified are unknown. Define the error vector as (3.5), then the error dynamical system with unknown parameters is as follows:

$$\begin{aligned} \dot{e}_1 &= \overline{a} e_1 + \overline{a}\alpha_1 x_1 + a e_1 - e_2 e_3 - \alpha_2 x_2 e_3 - \alpha_3 x_3 e_2 - (\alpha_2\alpha_3 - \alpha_1) x_2 x_3 + u_1, \\ \dot{e}_2 &= -\overline{b} e_2 - \overline{b}\alpha_2 x_2 - b e_2 + e_1 e_3 + \alpha_1 x_1 e_3 + \alpha_3 x_3 e_1 + (\alpha_1\alpha_3 - \alpha_2) x_1 x_3 + u_2, \\ \dot{e}_3 &= -\overline{c} e_3 - \overline{c}\alpha_3 x_3 - c e_3 + \frac{e_1 e_2}{3} + \frac{\alpha_1 x_1 e_2}{3} + \frac{\alpha_2 x_2 e_1}{3} + \frac{(\alpha_1\alpha_2 - \alpha_3) x_1 x_2}{3} + u_3, \end{aligned} \tag{3.16}$$

where $\overline{a} = \hat{a} - a,\ \overline{b} = \hat{b} - b,\ \overline{c} = \hat{c} - c$ are the unknown error system parameters.

Theorem 3.3. *For the given drive system* (1.1)*, the response system* (3.15)*, and the corresponding error dynamical system* (3.16)*, if the adaptive control law* (3.8) *associated with* (3.9) *is applied to system* (3.15) *and estimated update laws of the response system parameters satisfy*

$$\begin{aligned} \dot{\overline{a}} &= \frac{-e_1^2 - \alpha_1 x_1 e_1}{3}, \\ \dot{\overline{b}} &= \frac{e_2^2 + \alpha_2 x_2 e_2}{3}, \\ \dot{\overline{c}} &= e_3^2 + \alpha_3 x_3 e_3, \end{aligned} \tag{3.17}$$

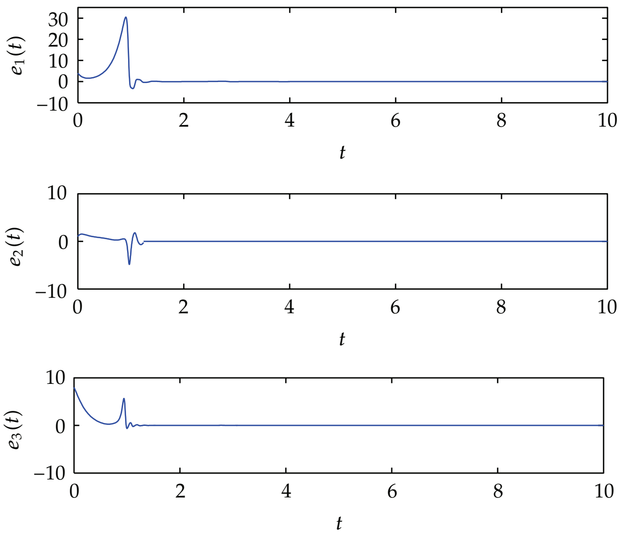

Figure 3: The HPS errors $e_1,\ e_2,\ e_3$ between the two identical chaotic systems (1.1) and (3.3) with $\alpha_1 = 1,\ \alpha_2 = 3$, and $\alpha_3 = -1$.

then the trivial solution of the error dynamical system (3.16) *is globally and asymptotically stable so that systems* (1.1) *and* (3.15) *achieve HPS.*

Proof. Construct a candidate Lyapunov function

$$V_2(t) = V_1(t) + \frac{1}{2}\left(\overline{a}^2 + \overline{b}^2 + \overline{c}^2\right) = \frac{1}{2}\left(\frac{1}{3}e_1^2 + \frac{1}{3}e_2^2 + e_3^2\right) + \frac{1}{2}\left(\overline{a}^2 + \overline{b}^2 + \overline{c}^2\right) + \frac{1}{2k_2}(k - k^*)^2, \tag{3.18}$$

where $V_1(t)$ has been defined in (3.10). Taking the time derivative of $V_2(t)$ along the solution of error system (3.16) and applying the control law (3.8) associated with (3.9) and parameter estimated update laws (3.17), we have

$$\begin{aligned}\dot{V}_2(t) &= \frac{1}{3}e_1\dot{e}_1 + \frac{1}{3}e_2\dot{e}_2 + e_3\dot{e}_3 + \overline{a}\,\dot{\overline{a}} + \overline{b}\,\dot{\overline{b}} + \overline{c}\,\dot{\overline{c}} + \frac{1}{k_2}(k - k^*)\dot{k} \\ &= -\left(k^* - k - \frac{1}{3}a\right)e_1^2 - \frac{1}{3}be_2^2 - (c + k)e_3^2 \\ &= -e^T Pe,\end{aligned} \tag{3.19}$$

where e, P are defined in (3.13). If the conditions in (3.14) hold, $\dot{V}_2(t)$ is negative semidefinite, it follows that $\dot{V}_2(t) \to 0$ as $t \to \infty$ according to Barbalat's Lemma. So, the equilibrium point $e_1 = 0,\ e_2 = 0,\ e_3 = 0,\ k = k^*,\ \overline{a} = 0,\ \overline{b} = 0,\ \overline{c} = 0$ is globally and asymptotically stable. Thus, the two chaotic systems with unknown parameters are in HPS. This completes the proof. □

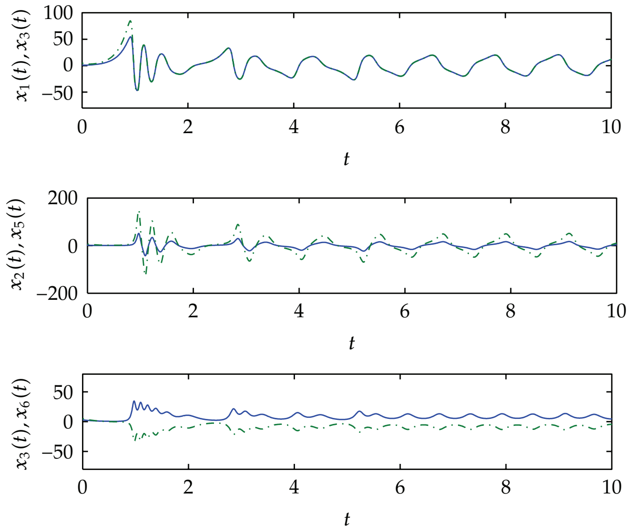

Figure 4: State trajectories of the drive system and the response system, x_1, and y_1 with $\alpha_1 = 1$, x_2, and y_2 with $\alpha_2 = 3$, x_3, and y_3 with $\alpha_3 = -1$.

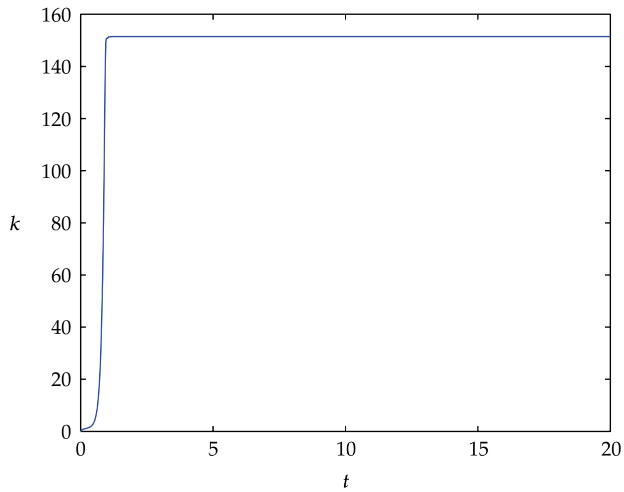

Figure 5: Adaptive feedback control gain k with time history.

3.3. *Numerical Simulations*

To verify and demonstrate the effectiveness of the proposed adaptive control laws for HPS, we will display the numerical simulation results. Let the system parameters be $a = 5$, $b = 10$, and $c = 3.8$, the initial states of the drive system and the response system $x_1(0) = 1$, $x_2(0) = 2$, and $x_3(0) = 3$ and $y_1(0) = 5$, $y_2(0) = 7$, and $y_3(0) = 5$, respectively, the initial condition of the adaptive feedback gain $k(0) = 0$, the constant coefficient $k_2 = 1$, and the synchronization factors be $\alpha_1 = 1$, $\alpha_2 = 3$, and $\alpha_3 = -1$. Fourth-order Runge-Kutta method is used to solve the system with time step size 0.001.

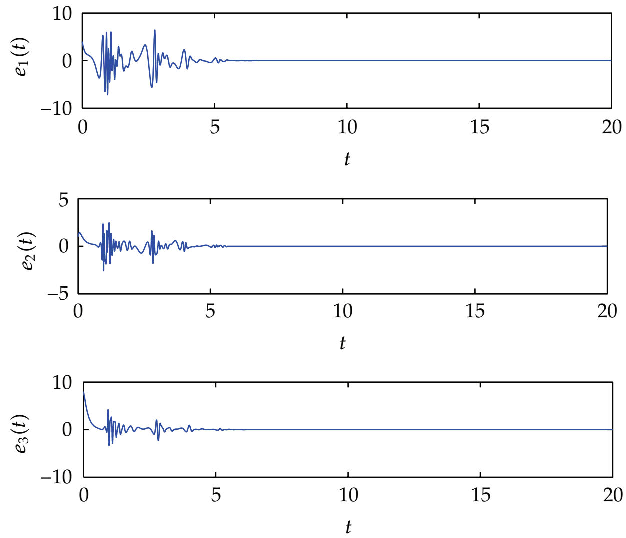

Figure 6: The HPS errors $e_1, \; e_2, \; e_3$ of system (3.16) with unknown parameters of the response system.

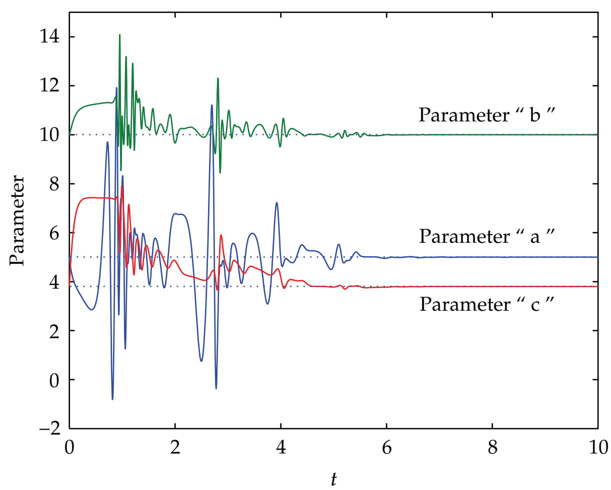

Figure 7: Estimated curves of parameters of the response system (3.15) under the update law (3.17) with $\overline{a}(0) = 0, \; \overline{b}(0) = 0$, and $\overline{c}(0) = 0$.

(1) The HPS errors between the two identical chaotic systems (1.1) and (3.3) are displayed in Figure 3. From these curves, we can see that each error converges to 0. For further observations, the state trajectories of the two systems are depicted in Figure 4. It is shown that variables x_1 and y_1 display a synchronization phenomenon, y_2 finally converges to three times the value of x_2, x_3 and y_3 show antisynchronization behavior. It is clear that the two chaotic systems achieve HPS. Figure 5 shows the curve of adaptive feedback control gain k with time history.

(2) Let the initial parameter errors of system (3.16) be $\overline{a}(0) = 0, \; \overline{b}(0) = 0, \; \overline{c}(0) = 0$. The evolution of the synchronized errors with unknown parameters of the response system is

illustrated in Figure 6. It is shown that synchronized errors between the two chaotic systems converge to zero asymptotically. It means the two nearly identical chaotic systems are in HPS. Figure 7 illustrates the estimated curves of parameters of the response system (3.15) under the update law (3.17).

4. Conclusion

Chaos control and hybrid projective synchronization between two chaotic systems are addressed. The single scalar adaptive feedback control method is proposed and numerical simulation results are demonstrated to verify the effectiveness and efficiency of the control strategies. Based on the control method, single scalar adaptive feedback controllers overcome the shortcomings of the controllers in the existing literature. They have less parameters, simpler form, and are easier to implement compared with other controllers. They are valuable to be applied to the realization in engineering.

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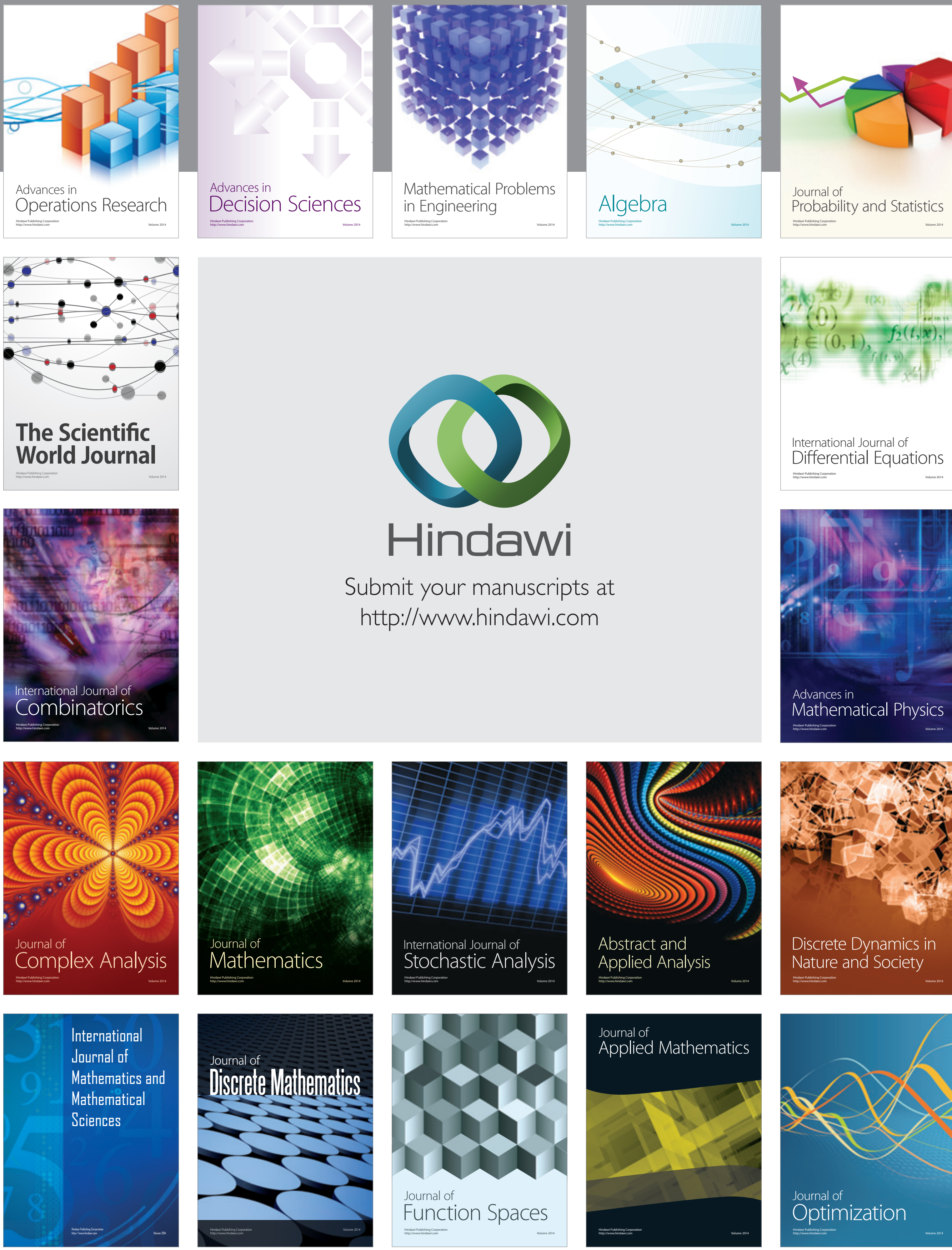
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