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Research Article

Adaptive Control of Chaos in Chua's Circuit

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A feedback control method and an adaptive feedback control method are proposed for Chua's circuit chaos system, which is a simple 3D autonomous system. The asymptotical stability is proven with Lyapunov theory for both of the proposed methods, and the system can be dragged to one of its three unstable equilibrium points respectively. Simulation results show that the proposed methods are valid, and control performance is improved through introducing adaptive technology.

1. Introduction

In the last several decades, much effort has been devoted to the study of nonlinear chaotic systems. As more and more knowledge is gained about the nature of chaos, recent interests are now focused on controlling a chaotic system, that is, bringing the chaotic state to an equilibrium point or a small limit cycle.

After the pioneering work on controlling chaos of Ott et al. [1], there have been many other attempts to control chaotic systems. These attempts can be classified into two main streams: the first is parameters' perturbation that is introduced in [2] and the references therein. The second is feedback control on an original chaotic system [3–8].

Chua's circuit has been studied extensively as a prototypical electronic system [7–11]. Chen and Dong [4] applied the linear feedback control for guiding the chaotic trajectory of the circuit system to a limit cycle. Hwang et al. [5] proposed a feedback control on a modified Chua's circuit to drag the chaotic trajectory to its fixed points. He et al. [10] proposed an adaptive tracking control for a class of Chua's chaotic systems. At the same time, the adaptive control technology for chaos systems has undergone rapid developments (see [6, 7, 10] and the references therein) in the past decade.

The aim of this paper is to introduce a simple, smooth, and adaptive controller for resolving the control problems of Chua's circuit system. It is assumed that one state

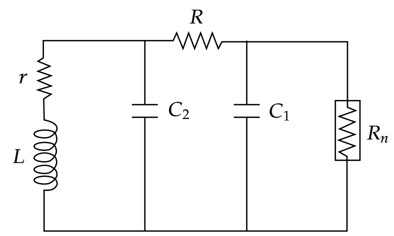

Figure 1: Chua's Circuit Model.

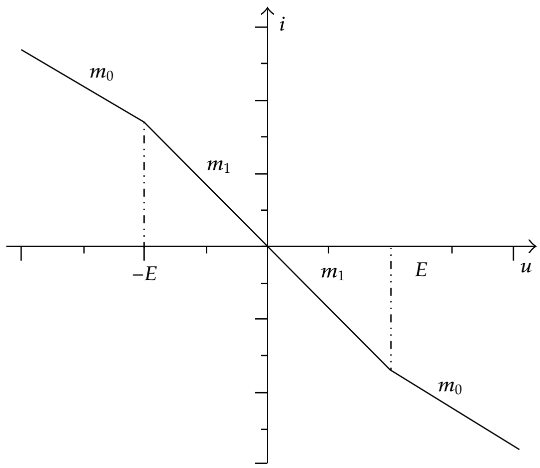

Figure 2: Nonlinear representation of R_n.

variable is available for implementing the feedback controller. In Section 2, Chua's circuit system model is built and its three equilibrium points are analyzed. In Section 3, a feedback control approach and an adaptive feedback control approach are proposed. In Section 4, the numerical simulations are presented for two proposed control approaches. Section 5 is the conclusion.

2. Chua's Circuit Modeling

Chua's circuit is a well-known electronic system, which displays very rich and typical bifurcation and chaos phenomena such as double scroll, dual double scroll, and double hook, and so forth. The Chua's circuit is illustrated Figure 1. In the circuit, there are one inductor (L,r is its inner resistor), two capacitors (C_1 and C_2), one linear resistor (R), and one piece-linear resistor (R_n), which has the following volt-ampere characteristic:

$$G = \frac{i}{u} = \begin{cases} m_1, & |u| < E, \\ m_0, & |u| > E, \end{cases} \tag{2.1}$$

where, u and i, respectively, are the voltage across R_n and the current through R_n and E is a positive constant. The characteristic of R_n is illustrated in Figure 2.

According to the circuit theory, the dynamics of Chua's circuit systems can be obtained:

$$C_1 \frac{d}{dt}(v_{C1}) = \frac{(v_{C2} \quad v_{C1})}{R} - i(v_{C1}),$$
$$C_2 \frac{d}{dt}(v_{C2}) = \frac{(v_{C1} - v_{C2})}{R} + i_L, \quad (2.2)$$
$$L\frac{di_L}{dt} = -v_{C2},$$

where v_{C1} and v_{C2} are the voltages across C_1 and C_2, respectively, i_L is the current through the inductor L, and

$$i(v_{C1}) = \begin{cases} m_0 v_{C1} + E(m_1 - m_0), & v_{C1} > E, \\ m_1 v_{C1}, & |v_{C1}| \leq E, \\ m_0 v_{C1} - E(m_1 - m_0), & v_{C1} < -E. \end{cases} \quad (2.3)$$

For Chua's circuit system described by (2.2) and (2.3), let $x_1 = v_{C1}$, $x_2 = v_{C2}$, $x_3 = Ri_L$, $\tau = t/RC_2$, $a = m_1 R$, $b = m_0 R$, $p = C_2/C_1$, and $q = R^2 C_2/L$, where x_1, x_2, and x_3 are system states. We can obtain the system model of Chua's circuit:

$$\frac{dx_1}{d\tau} = p\big(x_2 - x_1 - f(x_1)\big),$$
$$\frac{dx_2}{d\tau} = x_1 - x_2 + x_3, \quad (2.4)$$
$$\frac{dx_3}{d\tau} = -qx_2,$$

where the differential is with respect to variable τ and $f(x_1)$ is a piece-linear function as:

$$f(x_1) = \begin{cases} bx_1 + E(a - b), & x_1 > E, \\ ax_1, & |x_1| \leq E, \\ bx_1 - E(a - b), & x_1 < -E. \end{cases} \quad (2.5)$$

Normally due to the piece-linear function, the system described by (2.4) has three equilibrium points, which are denoted by $E(x_{1r}, x_{2r}, x_{3r})$ $(r = 1, 2, 3)$. For Chua's circuit system described by (2.4), the following conclusion holds.

Theorem 2.1 (see [9]). *For Chua's circuit system described by* (2.4) *and* (2.5)*, its first Lyapunov exponent is positive real number, that is, the system trajectory has some chaotic behaviors.*

3. Chaos Control in Chua's Circuit

When the feedback control is added to the system (2.4), the controlled closed-loop Chua's circuit system can be written as

$$\dot{x}_1 = p\big(x_2 - x_1 - f(x_1)\big) - u_1,$$
$$\dot{x}_2 = x_1 - x_2 + x_3 - u_2, \tag{3.1}$$
$$\dot{x}_3 = -qx_2 - u_3,$$

where u_1, u_2, and u_3 are external control inputs that calculated according to system states. It is desired that the control inputs can drag the chaotic trajectory of Chua's circuit system (2.4) to one of its three unstable equilibrium points. That is to say, the inputs can change three unstable equilibrium points of the open-loop system (2.4) to stable equilibrium points of the closed-loop Chua's circuit system (3.1).

3.1. Feedback Control

Let the control law take the following form:

$$u_1 = k_1(x_1 - x_{1r}), \qquad u_2 = u_3 = 0, \tag{3.2}$$

where k_1 is a positive feedback gain and x_{1r} is the goal of the available state x_1.

Thus, the closed-loop Chua's circuit model can be written as

$$\dot{x}_1 = p\big(x_2 - x_1 - f(x_1)\big) - k_1(x_1 - x_{1r}),$$
$$\dot{x}_2 = x_1 - x_2 + x_3, \tag{3.3}$$
$$\dot{x}_3 = -qx_2.$$

For closed-loop system (3.3), the following conclusion can be drawn.

Theorem 3.1. *When the feedback gain k_1 satisfies*

$$k_1 > k_{1r} = -pc, \tag{3.4}$$

where both p and c are the system parameters and k_{1r} is the minimal stable feedback gain. The equilibrium points $E(x_{1r}, x_{2r}, x_{3r})$ of closed-loop Chua's circuit system (3.3) *are asymptotically stable.*

Proof. In the neighbourhood of the equilibrium points $E(x_{1r}, x_{2r}, x_{3r})$, the jacobian matrix of feedback Chua's circuit system is

$$J = \begin{bmatrix} -p(1+c) - k_1 & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}, \tag{3.5}$$

where $c = b$ or a.

Let us define the state errors

$$\begin{aligned} e_1 &= x_1 - x_{1r}, \\ e_2 &= x_2 - x_{2r}, \\ e_3 &= x_3 - x_{3r}. \end{aligned} \tag{3.6}$$

The error linearized equation of controlled system in the neighborhood of the equilibrium points is

$$\begin{aligned} \dot{e}_1 &= p(e_2 - (1 + c)e_1) - k_1 e_1, \\ \dot{e}_2 &= e_1 - e_2 + e_3, \\ \dot{e}_3 &= -q e_2. \end{aligned} \tag{3.7}$$

The Lyapunov function is defined as

$$V = \frac{1}{2}\left(\frac{q}{p}e_1^2 + q e_2^2 + e_3^2\right). \tag{3.8}$$

The time derivative of V in the neighborhood of the equilibrium point is

$$\begin{aligned} \dot{V} &= \frac{q}{p}e_1\dot{e}_1 + q e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= \frac{q}{p}e_1(p(e_2 - (1 + c)e_1) - k_1 e_1) + q e_2(e_1 - e_2 + e_3) - q e_3 e_2 \\ &= -q(e_1 - e_2)^2 - q\left(c + \frac{k_1}{p}\right)e_1^2. \end{aligned} \tag{3.9}$$

It is clear that for the system parameters p, q, and c, if we choose

$$k_1 > k_{1r} = -pc, \tag{3.10}$$

then $\dot{V}$ is negative definite and the Lyapunov function V is positive definite. From Lyapunov stability theorem it follows that the equilibrium point of the system (3.3) is asymptotically stable. □

3.2. Adaptive Control

In the feedback control of Section 3.1,

$$u_1 = k_1(x_1 - x_{1r}), \qquad u_2 = u_3 = 0 \tag{3.11}$$

the feedback gain k_1 is a constant. In this section, k_1 will be adjusted by an adaptive algorithm with respect to system state error $x_1 - x_{1r}$, that is to say, the feedback gain k_1 automatically changes according to the system state x_1. For closed-loop system (3.3), the following adaptive algorithm is designed:

$$\dot{k}_1 = \gamma(x_1 - x_{1r})^2, \tag{3.12}$$

where γ is the adaptive gain. For Chua's circuit adaptive control system, the following conclusion holds.

Theorem 3.2. *For $\gamma > 0$ and $\dot{k}_1 = \gamma(x_1 - x_{1r})^2$, the equilibrium point $E(x_{1r}, x_{2r}, x_{3r})$ of the closed-loop Chua's circuit adaptive control system is asymptotically stable.*

Proof. In the neighborhood of the equilibrium points $E(x_{1r}, x_{2r}, x_{3r})$, the jacobian matrix of adaptive feedback Chua's circuit system is

$$J = \begin{bmatrix} -p(1+c) - k_1 & p & 0 \\ 1 & -1 & 1 \\ 0 & -q & 0 \end{bmatrix}, \tag{3.13}$$

where, $c = b$ or a.

Let us define the state errors

$$\begin{aligned} \eta_1 &= x_1 - x_{1r}, \\ \eta_2 &= x_2 - x_{2r}, \\ \eta_3 &= x_3 - x_{3r}. \end{aligned} \tag{3.14}$$

The error linearized equation of controlled system in the neighbourhood of the equilibrium points is

$$\begin{aligned} \dot{\eta}_1 &= p(\eta_2 - (1+c)\eta_1) - k_1\eta_1, \\ \dot{\eta}_2 &= \eta_1 - \eta_2 + \eta_3, \\ \dot{\eta}_3 &= -q\eta_2. \end{aligned} \tag{3.15}$$

The Lyapunov function is defined for the closed-loop Chua's circuit adaptive control system as

$$V = \frac{1}{2}\left(\frac{q}{p}(x_1 - x_{1r})^2 + q(x_2 - x_{2r})^2 + (x_3 - x_{3r})^2 + \frac{q}{p\gamma}(k_1 - k_{1r})^2\right). \tag{3.16}$$

The time derivative of V in the neighbourhood of the equilibrium point is

$$\begin{aligned}
\dot{V} &= \frac{q}{p}(x_1 - x_{1r})\dot{x}_1 + q(x_2 - x_{2r})\dot{x}_2 + (x_3 - x_{3r})\dot{x}_3 + \frac{q}{p\gamma}(k_1 - k_{1r})\dot{k}_1 \\
&= \frac{q}{p}\eta_1\dot{x}_1 + q\eta_2\dot{x}_2 + \eta_3\dot{x}_3 + \frac{q}{p\gamma}(k_1 - k_{1r})\dot{k}_1 \\
&= q\eta_1\eta_2 - q(1+c)\eta_1^2 - \frac{q}{p}k_1\eta_1^2 + q\eta_1\eta_2 - q\eta_2^2 + q\eta_2\eta_3 - q\eta_2\eta_3 + \frac{q}{p}k_1\eta_1^2 - \frac{q}{p}k_{1r}\eta_1^2 \\
&= -q{\eta_1}^2 + 2q\eta_1\eta_2 - q\eta_2^2 - qc{\eta_1}^2 - \frac{q}{p}k_{1r}\eta_1^2 \\
&= -q(\eta_1 - \eta_2)^2 - q\left(c + \frac{k_{1r}}{p}\right)\eta_1^2.
\end{aligned} \tag{3.17}$$

As we know, $k_{1r} = -pc$, $c = b$ or a, and the system parameter $q > 0$. From Lyapunov stability theorem, $V > 0$ and $\dot{V} < 0$, it follows that the equilibrium point of Chua's circuit adaptive control system is asymptotically stable. □

4. Simulation Studies

In the simulations, the following system parameters are used: $p = 10$, $q = 100/7$, $a = -8/7$, and $b = -5/7$. The fourth-order Runge-Kutta algorithm is applied to calculate the number integral. The system initial state is $x_{10} = 0.15264$, $x_{20} = -0.02281$, and $x_{30} = 0.38127$. In order to illustrate the effectiveness of the proposed control methods, the control is added at the 40th second. According to the given system parameters, the minimal stable feedback gains for different equilibrium points can be calculated: $k_{1r} = 50/7$ for equilibrium point $E(1.5, 0-1.5)$, $k_{1r} = 80/7$ for equilibrium point $E(0,0,0)$, and $k_{1r} = 50/7$ for equilibrium point $E(-1.5, 0, 1.5)$.

4.1. Feedback Control

The simulation results for the proposed feedback control of Chua's circuit system are illustrated in Figures 2–7, where Figures 3 and 4, respectively, are the simulation results to drag Chua's circuit system to one equilibrium point $E(1.5, 0-1.5)$ with the feedback gain $k = 15$. Figure 3 illustrates the time response of system state x_1 and Figure 4 is the phase plane portrait of system states x_1 and x_2. Figures 5 and 6, respectively, are the simulation results to drag the Chua's circuit system to one equilibrium point $E(0,0,0)$ with the feedback gain $k = 15$. Figure 5 illustrates the time response of system state x_1 and Figure 6 is the phase plane portrait of system states x_1 and x_2. Figures 7 and 8, respectively, are the simulation results to drag the Chua's circuit system to one equilibrium point $E(-1.5, 0, 1.5)$ with the feedback gain $k = 15$. Figure 7 illustrates the time response of system state x_1 and, Figure 8 is the phase plane portrait of system states x_1 and x_2.

The simulation results show that feedback control with suitable feedback gain can drag Chua's circuit system to one of its three equilibrium points. Moreover, the system initial state, feedback gain, and the position of system equilibrium point have some effects on the time responses of system states.

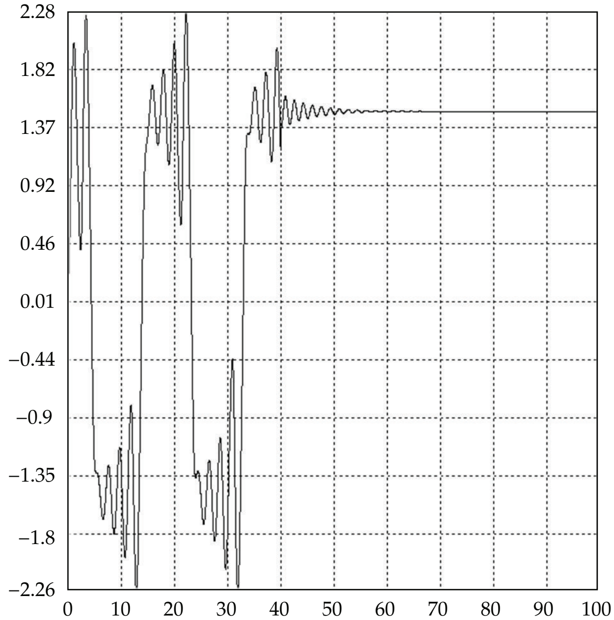

Figure 3: x_1 time response of $E(1.5, 0, -1.5)$ with feedback control.

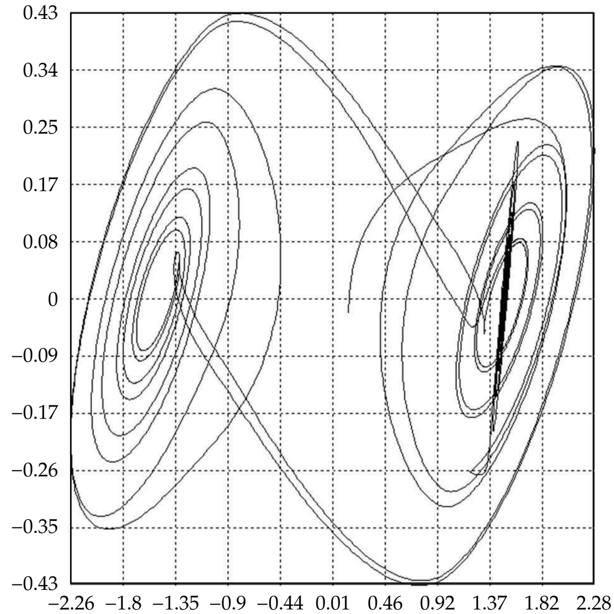

Figure 4: x_1-x_2 phase plane of $E(1.5, 0, -1.5)$ with feedback control.

4.2. Adaptive Control

Next is the simulation of adaptive control with adaptive gain $\gamma = 15$ to drag the Chua's circuit system to one of its three equilibrium points. The simulation results are illustrated in

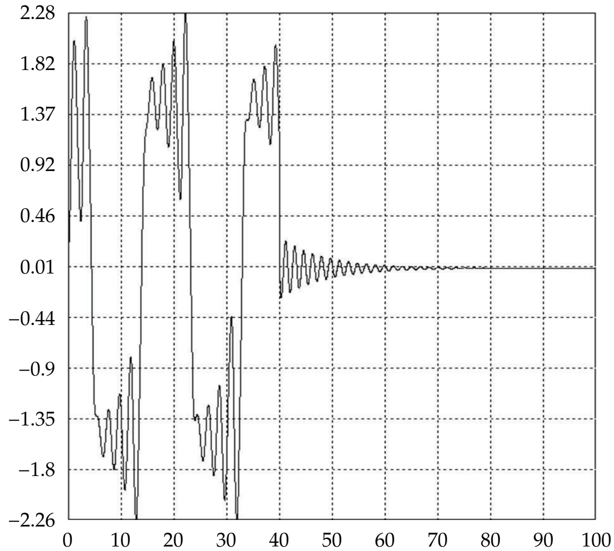

Figure 5: x_1 time response of $E(0,0,0)$ with feedback control.

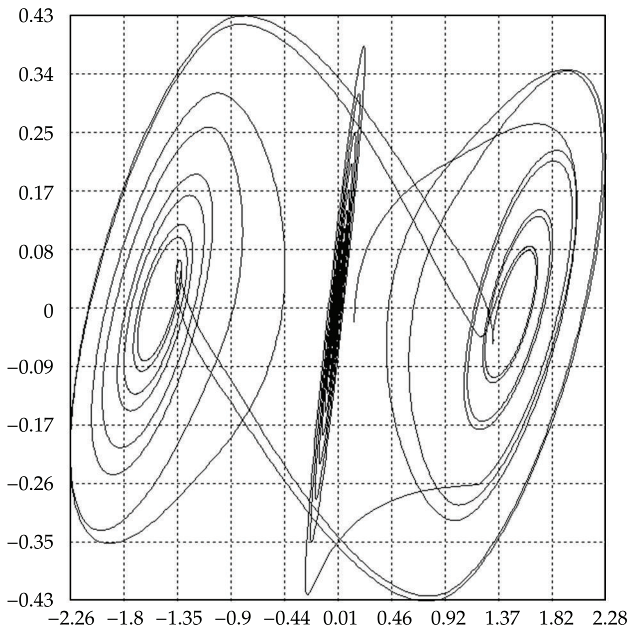

Figure 6: x_1-x_2 phase plane of $E(0,0,0)$ with feedback control.

Figures 8–13, where Figures 9, 10, 11, 12, 13, and 14 are the time responses and phase plane portrait corresponding with Figures 3, 4, 5, 6, 7, and 8.

The simulation results show that adaptive control with suitable adaptive gain also can drag the Chua's circuit system to one of its three equilibrium points. Moreover the adaptive

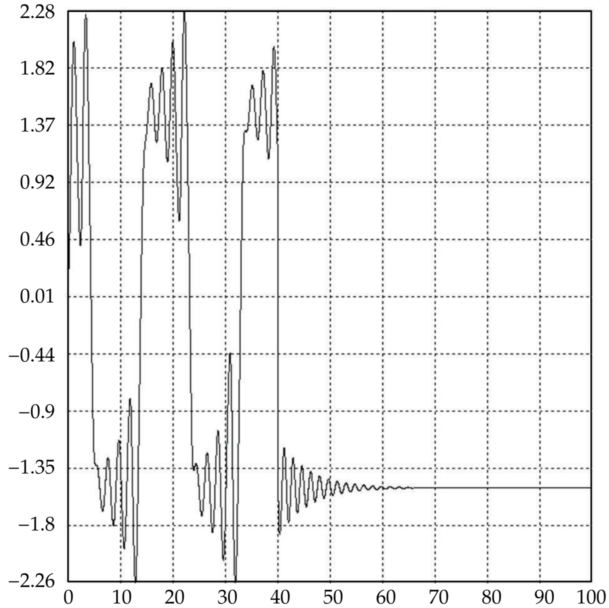

Figure 7: x_1 time response of $E(1.5, 0, -1.5)$ with feedback control.

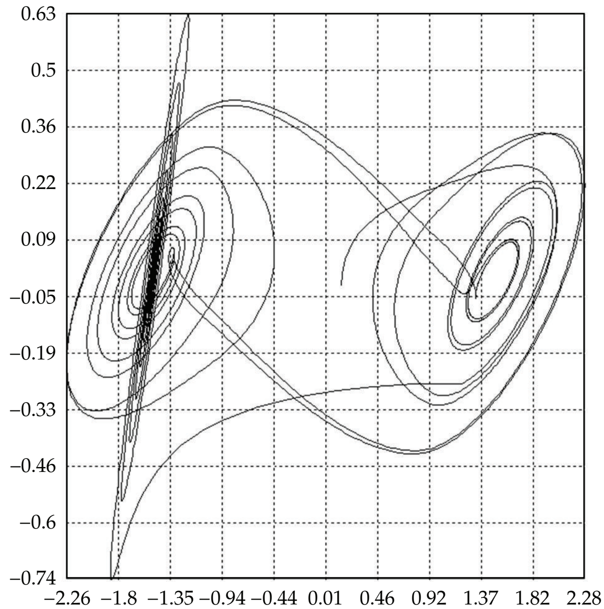

Figure 8: x_1-x_2 phase plane of $E(1.5, 0, -1.5)$ with feedback control.

control has the following advantages over feedback control: shorter time to drag the system state to its equilibrium point, more satisfactory dynamic response and wider application scope.

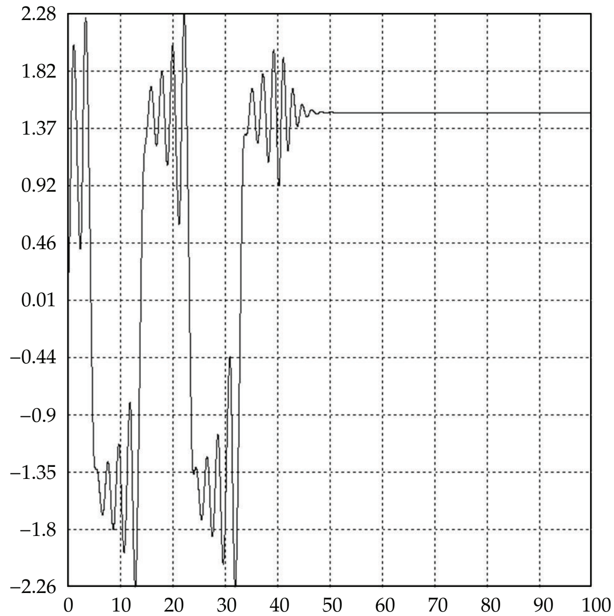

Figure 9: x_1 time response of $E(1.5, 0, -1.5)$ with adaptive control.

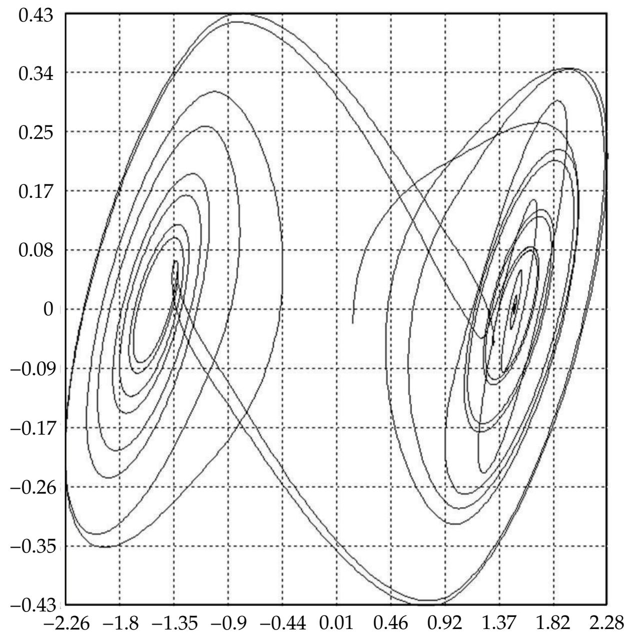

Figure 10: x_1-x_2 phase plane of $E(1.5, 0, -1.5)$ with adaptive control.

5. Conclusions

The advantage of using feedback control is that one can bring the system state away from chaotic motion and into any desired equilibrium point. In this paper, we have discussed a feedback control approach and an adaptive control approach for controlling the chaos in

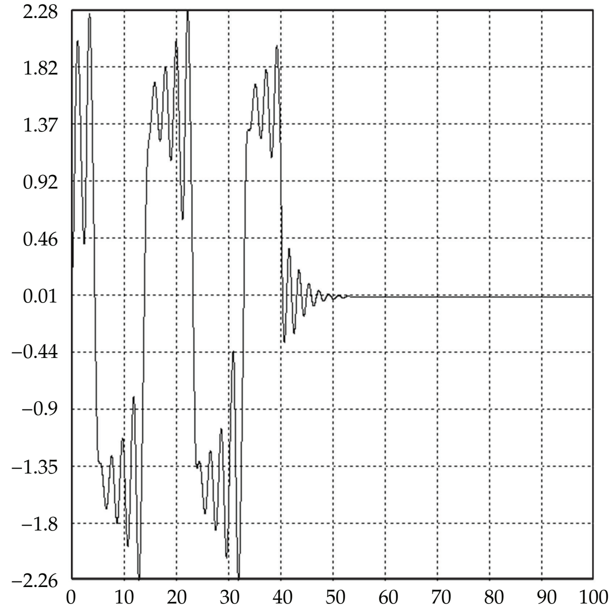

Figure 11: x_1 time response of $E(0, 0, 0)$ with adaptive control.

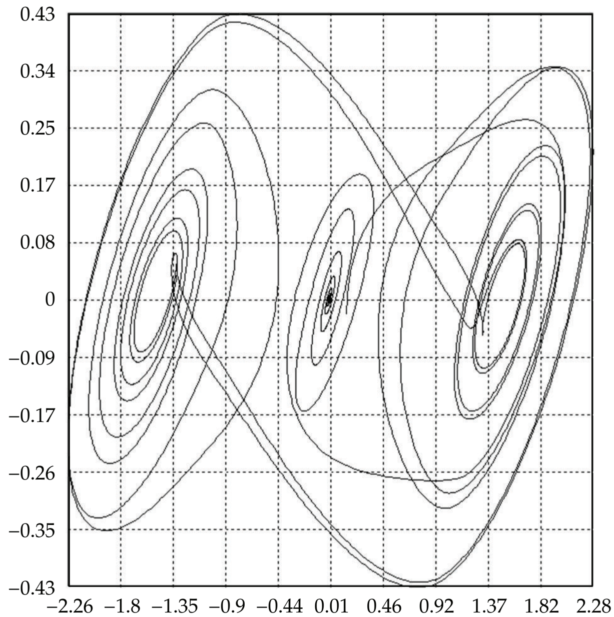

Figure 12: x_1-x_2 phase plane of $E(0, 0, 0)$ with adaptive control.

Chua's circuit. The control schemes of the two proposed approaches are given and their stabilities are proven in detail. Simulation results show that, in the closed-loop system, system state asymptotically converges to the desired equilibrium point and the adaptive feedback control approach has some advantages over feedback control approach.

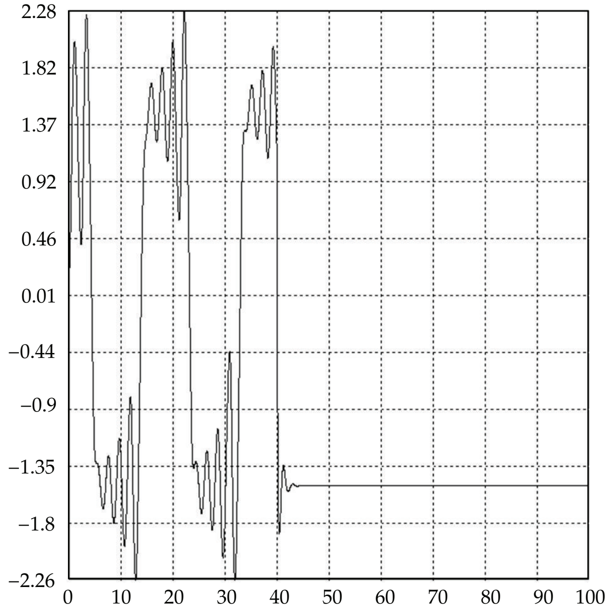

Figure 13: x_1 time response of $E(1.5, 0, -1.5)$ with adaptive control.

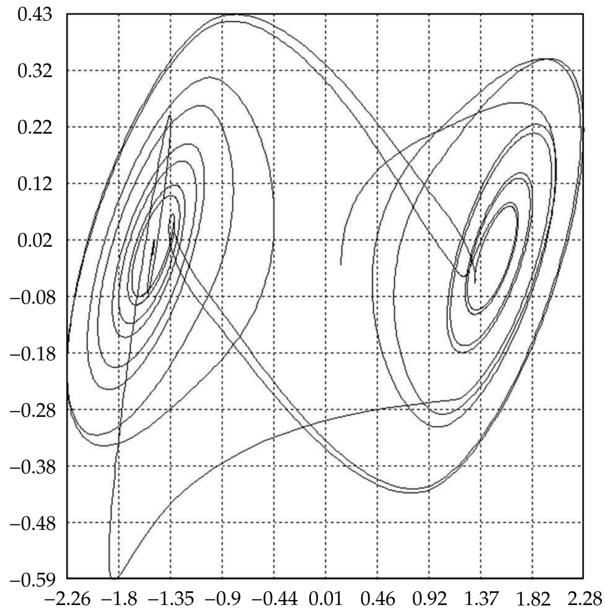

Figure 14: x_1-x_2 phase plane of $E(1.5, 0, -1.5)$ with adaptive control.

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