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Letter to the Editor

An Aproximation to Solution of Space and Time Fractional Telegraph Equations by the Variational Iteration Method

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Sevimlican suggested an effective algorithm for space and time fractional telegraph equations by the variational iteration method. This paper shows that algorithm can be updated by either variational iteration algorithm-II or the fractional variational iteration method.

As early as 1998, the variational iteration method was shown to be an effective tool for factional calculus [1]; afterwards, the method has been routinely used to solve various fractional differential equations for many years, see the review article in [2] for a detailed summarization. Sevimlican [3] also followed the solution given in [1]; however, the algorithm can be further improved.

Sevimlican considered the following one-dimensional space fractional telegraph equation:

$$\frac{\partial^{\alpha} u}{\partial x^{\alpha}} = \frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + u, \quad 1 < \alpha < 2, \tag{1}$$

and obtained the following iteration formulation:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^x (s-x) \left\{ \frac{\partial^{\alpha} u_n(s,t)}{\partial s^{\alpha}} - \frac{\partial^2 u_n(s,t)}{\partial s^2} - \frac{\partial u_n(s,t)}{\partial s} - u_n(s,t) \right\} ds. \tag{2}$$

We can also construct a correction functional in the form

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^x \lambda \left\{ \frac{\partial^\alpha \tilde{u}_n(s,t)}{\partial s^\alpha} - \frac{\partial^2 u_n(s,t)}{\partial s^2} - \frac{\partial \tilde{u}_n(s,t)}{\partial s} - \tilde{u}_n(s,t) \right\} ds. \quad (3)$$

If the multiplier λ can be exactly identified, then one iteration results in the exact solution; however, the exact identification of the multiplier is impossible for most problems, and an approximate identification is always followed. To this end, $\tilde{u}_n$ in (3) is assumed to be a known function, and it is generally called a restricted variable [4]. After identification of the multiplier, we obtain the following variational iteration Algorithm-II [5]:

$$u_{n+1}(x,t) = u_0(x,t) - \int_0^x (s-x) \left\{ \frac{\partial^\alpha u_n(s,t)}{\partial s^\alpha} - \frac{\partial u_n(s,t)}{\partial s} - u_n(s,t) \right\} ds \quad (4)$$

If we begin with $u_0(x,t) = u_0(0,t) + x u_x(0,t)$, (4) leads to the same result as given in [3].

The fractional variational iteration method is also suitable for the present problem, see the solution process in [5–7].

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References

[1] J.-H. He, "Approximate analytical solution for seepage flow with fractional derivatives in porous media," *Computer Methods in Applied Mechanics and Engineering*, vol. 167, no. 1-2, pp. 57–68, 1998.

[2] J. H. He, "Asymptotic methods for solitary solutions and compactons," *Abstract and Applied Analysis*. In press.

[3] A. Sevimlican, "An approximation to solution of space and time fractional telegraph equations by He's variational iteration method," *Mathematical Problems in Engineering*, vol. 2010, Article ID 290631, 10 pages, 2010.

[4] J.-H. He, "Some asymptotic methods for strongly nonlinear equations," *International Journal of Modern Physics B*, vol. 20, no. 10, pp. 1141–1199, 2006.

[5] J.-H. He, "A short remark on fractional variational iteration method," *Physics Letters A*, vol. 375, no. 38, pp. 3362–3364, 2011.

[6] G.-C. Wu, "A fractional variational iteration method for solving fractional nonlinear differential equations," *Computers & Mathematics with Applications*, vol. 61, no. 8, pp. 2186–2190, 2011.

[7] G.-C. Wu and E. W. M. Lee, "Fractional variational iteration method and its application," *Physics Letters A*, vol. 374, no. 25, pp. 2506–2509, 2010.