Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 467629, 17 pages
doi:10.1155/2012/467629

Research Article

Stability of Teleoperation Systems for Time-Varying Delays by Neutral LMI Techniques

Emma Delgado,[1] Miguel Díaz-Cacho,[2] and Antonio Barreiro[1]

[1] *Higher Technical School of Industrial Engineering, University of Vigo, 36310 Vigo, Spain*
[2] *Higher Technical School of Computer Engineering, University of Vigo, 32004 Orense, Spain*

Correspondence should be addressed to Emma Delgado, emmad@uvigo.es

Received 26 June 2012; Revised 18 October 2012; Accepted 22 October 2012

Academic Editor: Alexander Pogromsky

This paper investigates the delay-dependent stability of a teleoperation system based on the transparent Generalized Four-Channel control (G-4C) scheme under time-varying communication delays. To address stability we choose here a primitive result providing a Linear Matrix Inequalities (LMIs) approach based on Lyapunov-Krasovskii functionals. Firstly, the scheme is modeled as the neutral-type differential-delayed equation; that is, the delay affects not only the state but also the state derivative. Secondly, we apply a less conservative stability criteria based on LMIs that are delay dependent and delay's time-derivative dependent. The reason is that, for better performance in the case of small delays, we must accept the possibility that stability is lost for large delays. The approach is applied to an example, and its advantages are discussed. As a result, we propose to modify the values of standard controllers in G-4C defining the γ-4C scheme, which introduces a tuning factor γ to increase in practical conditions the stable region fixing the desired bounds on time-varying delay, with the particularity of maintaining the tracking properties provided by this transparent control scheme. The simulation results justify the proposed control architecture and confirm robust stability and performance.

1. Introduction

A teleoperation system consists of master and slave mechanical systems where the master is directly manipulated by a human operator and the slave, operating in a remote environment, is designed to track the master closely. The main aspects in the analysis and synthesis of these systems are stability and transparency, meaning this last condition the grade of achievement of direct action ideal situation of the operator on the remote environment. In practice, there is a compromise between these two goals mainly due to the presence of time delays generated by the communication channel [1].

Under this compromise, many control schemes for teleoperation have been proposed in the last years. A first comparative study among them presented by Arcara and Melchiorri can be seen in [2]. Following this work, as stability is concerned, the teleoperation schemes are classified in intrinsically stable schemes (passivity based), and delay-dependent stable schemes. Therefore, in the initial works, concerning constant delay, the stability is addressed by means of frequency Laplace or passivity techniques, applied to linear time invariant master-slave two-port systems [1–5]. On the other side, related with transparency properties, the most successful control scheme in achieving a fully transparency under ideal conditions is the four-channel control one [1, 2, 4].

But also, the low cost and availability offered by Internet have opened a new line of research to establish Internet-based teleoperation [6–9], which requires transmitting the control signal through the network, exposing the system control loop to the varying time delay of a packet-switched network. These new difficulties were already present in the historical survey presented by Hokayem and Spong in [10] about the control theoretic approaches for bilateral teleoperation.

Furthermore, Chopra et al. in [11] affirm that the bilateral teleoperators designed within the passivity framework using concepts of scattering and two-port network theory provide robust stability against constant delay in the network and velocity tracking, but cannot guarantee position tracking in general. For these reasons, many recent results try to extend the passivity-based architecture to solve these problems: see [12, 13] and references therein. Concerning these techniques, the tutorial [14] revisits several of the most recent passivity-based controllers, which include scattering-based, damping injection and adaptive controllers, with guaranteed stability properties.

Nowadays, several efforts are being made in the development of delay-dependent stability tools. In [15], the authors present an approach based on small-gain-type theorems in the input-to-state stability framework. This promising approach can be applied for a wider class of systems under very mild assumptions on time-varying delays and can be used even when passivity is lost. Its application to teleoperators was used in [16].

More specifically, focusing on the stability of the delay-dependent teleoperation control schemes, in [17], we develop a powerful generic approach to model a teleoperation setup, as a negative single feedback loop containing a linear time invariant block and an uncertain time-varying delay. The main added value of the proposed approach is the possibility of deriving frequency-domain conditions, based on the structured singular value by combining input-output stability criteria and μ-analysis and synthesis techniques, for robust stability in presence of time-varying delays and parametric uncertainties.

This paper focuses on the delay-dependent stability of the teleoperation systems that can be described by neutral differential equations. The particularity of the neutral systems is that the delay affects not only the state but also the state derivative. This makes the problem more complex, and there are many less theoretical results for this kind of systems [18].

The other issue of our approach is that we use delay-dependent stability tools. The rationale behind this is that, for better performance, as, for example, zero tracking error, in the case of small network delays (the typical operating condition), we must accept the pertinent possibility that stability is lost for large delays. It is more natural then to use LMI tools that check this fact and provide estimations of the maximum allowable delays.

Some of the relevant theoretical results obtained on delay-dependent stability analysis for neutral systems can be found in [18–20] for constant delay and [21] for time-varying delay. Although the theory developed for this kind of systems provides a systematic method to analyze the delay-dependent, neutral stability, there are limited applications of the theory in teleoperation. We can observe in [18] an outline of the basic guidelines to proceed, with only a description of a scalar teleoperation case and constant delay. The limits of the direct application of the stability conditions to these systems are due to the fact that several pertinent aspects have to be taken into account. First, the teleoperation system has to be remodeled in a state-space model and second, the bounds' values on the delay must be studied: some stability results in the literature require an upper bound on $\tau(t)$, and some others may require upper or lower bounds on $\dot{\tau}(t)$.

Following this research line, in previous works we have analyzed the stability by delay-dependent LMI techniques for constant delays and neutral control schemes belonging to two-channel architecture in [22] and for varying delays and retarded (not neutral) control scheme (Position-Error control) belonging to two-channel framework in [6, 7]. In [7] we also studied the time-varying nature of the communication delay. We obtained actual bounds for the delay magnitude and its derivative, appearing with UDP (user datagram protocol) protocol for different Internet locations. This logic allows us to apply stability theorems with delay-dependent conditions following, for example, Zhao et al. [21], which reduce the conservativeness.

Hence, in the present work, we apply this stability tool [21] to the Generalized Four-Channel control scheme (G-4C) as the one that accomplishes perfect transparency under ideal conditions. As a result of the application of this analysis technique, we propose incorporating a tuning factor γ in the G-4C standard controllers obtaining the new γ-4C scheme, to increase in practical conditions the stable region fixing the desired bounds on time-varying delay, with the particularity of maintaining the tracking properties provided by this transparent control scheme.

The paper is organized as follows. In Section 2 an overall description of teleoperation systems is introduced in order to define from the Generalized Four-Channel (G-4C) control scheme the new γ-4C one and its model as a neutral system established in Section 3. This Section also describes the robust stability condition under time-varying delay for neutral systems and its implementation. The analysis and simulation results are presented in Section 4. Finally, conclusions are discussed in Section 5.

2. Teleoperation System Description

A bilateral teleoperation system consists of master and slave mechanical systems, with separated control loops closed around them. The system may be described by means of the block scheme shown in Figure 1, where F_h and F_e are the forces imposed by the human operator and by the environment, respectively, and F_{mc}, F_{sc} are the forces computed by the local K_m, K_s master/slave control algorithms. So the forces F_m applied to the master and F_s applied to the slave can be defined as

$$\begin{aligned} F_m &= F_h - F_{mc}, \\ F_s &= F_e + F_{sc}. \end{aligned} \tag{2.1}$$

The blocks labeled *Master, Slave* represent the dynamics of the master and slave manipulators. A linear one degree-of-freedom dynamic model for master and slave systems

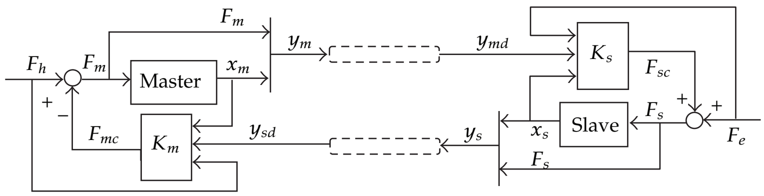


Figure 1: Control scheme in teleoperation.

has been considered. In the time domain with independent variable t and in Laplace domain with $x_m(0) = \dot{x}_m(0) = 0$ and independent variable s, we have the following expressions:

$$\begin{aligned} f_m(t) &= M_m \ddot{x}_m(t), \qquad F_m(s) = P_m(s)x_m(s) = M_m s^2 x_m(s), \\ f_s(t) &= M_s \ddot{x}_s(t), \qquad F_s(s) = P_s(s)x_s(s) = M_s s^2 x_s(s), \end{aligned} \tag{2.2}$$

where M_i is the manipulator's inertia coefficient, while F_i and x_i are the force and the position. The subscripts $i = m$ and $i = s$ indicate the master or slave manipulator, respectively.

In case of interaction with the environment at the slave side, in applications like robot arms with tools interacting with it, the force imposed by the environment can be described as

$$f_e(t) = -k_e x_s(t) - B_e \dot{x}_s(t), \qquad F_e(s) = -k_e x_s(s) - B_e s x_s(s). \tag{2.3}$$

This scheme also includes the signals' flow y_m, y_s through the communication channel characterized by a transmission delay y_{md}, y_{sd}. It is important to define how the signals sent from the master reach the slave and vice versa. In order to establish teleoperation via Internet, the results in [8] conclude that UDP (user datagram protocol) is more adequate for real-time control than TCP (transport control protocol), basically due to the time penalty of the connection-oriented characteristic of the TCP stream. Another relevant property of time-varying delay $\tau(t)$ is the behavior of the delay derivatives. For a discussion on varying delays and derivative bounds in teleoperation systems, see [6], in which we present a different explanation of derivative bounds based on the fact that there are two ways of interpreting the time-varying law $\tau(t)$, as perceived by the emitter $\tau_e(t)$, and by the receiver $\tau_r(t)$. Regarding the teleoperation systems like the one in Figure 1, the local controllers at each side, given by K_m, K_s, implement their dynamics as a function of their own states (master/slave) and of delayed information on the state at the other side (slave/master). The delay should be interpreted then as perceived by the receiver $\tau(t) = \tau_r(t)$. In this way the bound on delay derivatives discarding past samples (the protocol discards a sample if, when it reaches, the receiver has a more recent value) verifies $\dot{\tau}(t) < 1$. Thus, the delay parameter is assumed to be an unknown time-varying function that satisfies for all $t \geq 0$:

$$\begin{gathered} 0 \leq \tau(t) = h + \eta(t) \leq h_{\max}, \quad |\eta(t)| \leq \kappa \leq h, \\ |\dot{\tau}(t)| \leq d < 1. \end{gathered} \tag{2.4}$$

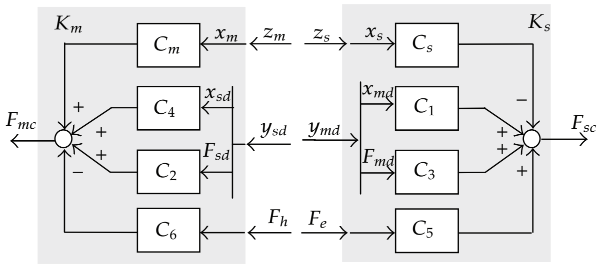


Figure 2: Local controllers in G-4C control scheme.

Hence, we assume that the main effect of the communication channel is to introduce a time-varying delay:

$$y_{id}(t) =: y_i(t - \tau(t)), \quad i = m, s. \tag{2.5}$$

The study focuses on the Generalized Four-Channel-transparency optimized controller. In this scheme (G-4C), the master and slave exchange four signals through the communication channel, the velocities or positions and the forces in both directions. The local controllers are defined as a function to the applied external force, the owner velocity/position (master/slave), and the delayed velocity/position and force (slave/master) from the other side. In the classical notation [1, 2, 4], the controllers, when the system exchanges positions and forces (see Figures 1 and 2), are usually defined as

$$\begin{aligned} F_{mc} &= C_m x_m + C_2 F_{sd} + C_4 x_{sd} - C_6 F_h, \\ F_{sc} &= -C_s x_s + C_3 F_{md} + C_1 x_{md} + C_5 F_e. \end{aligned} \tag{2.6}$$

We select the previous architecture as the most general and representative of delay-dependent ones. Values of the controllers in (2.6) can be found in [1, 2, 4] for seven nonpassive architectures (Generalized Four-Channel G-4C, Four-Channel 4C, three-channel architectures: Environment-Force-Compensated EFC and Operator-Force-Compensated OFC, two-channel architectures: Position-Error PE, Force-Reflection FR, Force-Force FF).

In the well-known design of the Four-channel Control scheme under ideal transparent conditions for zero delay [1, 2, 4], the teleoperation system stability critically depends on the exact implementation of control laws in order to achieve perfect cancellation of the dynamics of the master and slave; that is, to say, in practice there is a compromise between the two goals: stability and transparency. In this design, the values of the controllers are $C_m(s) = B_m s + k_m$, $C_s(s) = B_s s + k_s$, $C_4(s) = -(P_m(s) + C_m(s))$, $C_1(s) = P_s(s) + C_s(s)$, and C_2, C_3, C_5, and C_6 are fixed constant values.

Now, we state one of the main results of this research.

Definition 2.1. A G-4C control scheme, as described in Figures 1 and 2 and given by (2.1)–(2.3), (2.5), (2.6), is said to be a γ-4C control scheme, if the controllers in (2.6) are defined in the following form:

$$C_m(s) = B_m s + k_m, \qquad C_s(s) = B_s s + k_s,$$
$$C_4(s) = -\frac{P_m(s) + C_m(s)}{\gamma}, \qquad C_1(s) = P_s(s) + C_s(s), \tag{2.7}$$
$$C_2 = \frac{1 + C_6}{\gamma}, \qquad C_3 = C_5 - 1, \qquad C_5 = -1, \qquad C_6 = k_m \cdot \left(1 - \frac{1}{\gamma}\right) - 1.$$

Remark 2.2. The controllers in γ-4C scheme incorporate a constant tuning factor $\gamma > 1$, named from now the stability factor, that will increase in practical conditions the stability margin while maintaining the tracking properties of the system, as we will explain and justify in the next sections.

3. Delay-Dependent Stability for Neutral γ-4C-Based Teleoperation

In this paper we adopt some time-domain approach in order to obtain delay-dependent conditions for robust stability of the teleoperated system. The sufficient conditions for stability developed in Zhao et al. [21] are given in terms of the existence of solutions of some linear matrix inequalities, based on Lyapunov functionals. This method formulates computable criteria to check the stability for time-varying delays in the general case of neutral-type systems:

$$\dot{z}(t) = Az(t) + A_d z(t - \tau(t)) + C\dot{z}(t - \tau_d). \tag{3.1}$$

With the initial condition

$$z(t) = \phi(t), \quad t \in [-\max\{\tau_d, h_{\max}\}, 0]. \tag{3.2}$$

Consider the operator $\mathfrak{D} : \mathcal{C}([-\tau_d, 0], \Re^n) \to \Re^n$ defined in [23] as $\mathfrak{D}(z_t) = z(t) - Cz(t - \tau_d)$.

Definition 3.1 (see [23]). The operator $\mathfrak{D}$ is said to be stable if the zero solution of the homogeneous difference equation is uniformly asymptotically stable:

$$\mathfrak{D}(z_t) = 0, \quad t \geq 0, \qquad z_0 = \varphi \in \{\phi \in \mathcal{C}[-\tau_d, 0] : \mathfrak{D}\phi = 0\}. \tag{3.3}$$

The main theorem in Zhao et al. [21] says that given the scalars τ_d, $h_{\max}$, and d, the system (3.1), (3.2) is asymptotically stable, if the delay operator $\mathfrak{D}(z_t) = z(t) - Cz(t - \tau_d)$ as defined in [23] is stable and there exist $W = W^T > 0$, $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1 \cdots 2)$, $R = R^T > 0$, such that the following matrix inequality holds:

$$\begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & 0_{n\times n} & \phi_{15} \\ \phi_{12}^T & \phi_{22} & \phi_{23} & 0_{n\times n} & \phi_{25} \\ \phi_{13}^T & \phi_{23}^T & -Q_2 & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & -W & \phi_{45} \\ \phi_{15}^T & \phi_{25}^T & 0_{n\times n} & \phi_{45}^T & -S \end{pmatrix} < 0. \tag{3.4}$$

With

$$\begin{aligned} \phi_{11} &= A^T P + PA + Q_1 + Q_2 - R, \\ \phi_{12} &= PA_d + R, \\ \phi_{13} &= -A^T PC, \\ \phi_{15} &= A^T S, \\ \phi_{22} &= (d-1)Q_1 - R, \\ \phi_{23} &= -A_d^T PC, \\ \phi_{25} &= A_d^T S, \quad \phi_{45} = C^T S, \quad S = W + h_{\max}^2 R. \end{aligned} \tag{3.5}$$

Another result presented in Lemma 2.1 in [19] established that *A necessary and sufficient condition for the stability of the operator* $\mathfrak{D}(z_t) = z(t) - Cz(t - \tau_d)$ *as defined in [23] is the Schur-Cohn stability of the matrix C (the spectral radius* $\rho(C)$ *verifies* $\rho(C) < 1$*). Furthermore, the stability is ensured for all positive values of the delay* $\tau_d < \infty$.

The main theoretical results of this paper can now be formulated as follows.

Proposition 3.2. *Consider a* γ*-4C-based teleoperation system in Figures 1 and 2, by* (2.1)–(2.7)*. Consider the state-space vector* $z(t) \in \Re^n$ *as*

$$z(t) = \big(x_m(t) \;\; v_m(t) \;\; x_s(t) \;\; v_s\big)(t)^T, \tag{3.6}$$

where x_m is the master position, x_s is the slave position, v_m is the master velocity, and v_s is the slave velocity. A state-space model of the closed-loop system is described with the following, neutral differential equation, expression:

$$\begin{pmatrix} \dot{x}_m(t) \\ \dot{v}_m(t) \\ \dot{x}_s(t) \\ \dot{v}_s(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_m}{M_m} & -\dfrac{B_m}{M_m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\dfrac{k_s}{M_s} - \dfrac{k_e(1+C_5)}{M_s} & -\dfrac{B_s}{M_s} - \dfrac{B_e(1+C_5)}{M_s} \end{pmatrix} \cdot \begin{pmatrix} x_m(t) \\ v_m(t) \\ x_s(t) \\ v_s(t) \end{pmatrix}$$
$$+ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{k_m}{\gamma \cdot M_m} & \dfrac{B_m}{\gamma \cdot M_m} \\ 0 & 0 & 0 & 0 \\ \dfrac{k_s}{M_s} & \dfrac{B_s}{M_s} & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_m(t-\tau(t)) \\ v_m(t-\tau(t)) \\ x_s(t-\tau(t)) \\ v_s(t-\tau(t)) \end{pmatrix}$$
$$+ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{\gamma} - \dfrac{C_2 \cdot M_s}{M_m} \\ 0 & 0 & 0 & 0 \\ 0 & 1 + \dfrac{C_3 \cdot M_m}{M_s} & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \dot{x}_m(t-\tau_d) \\ \dot{v}_m(t-\tau_d) \\ \dot{x}_s(t-\tau_d) \\ \dot{v}_s(t-\tau_d) \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{1+C_6}{M_m} \\ 0 \\ 0 \end{pmatrix} \cdot f_h(t). \tag{3.7a}$$

In a compact form,

$$\dot{z}(t) = Az(t) + A_d z(t-\tau(t)) + C\dot{z}(t-\tau_d) + B \cdot f_h(t) \tag{3.7b}$$

and for nominal conditions ($f_h = 0$),

$$\dot{z}(t) = Az(t) + A_d z(t-\tau(t)) + C\dot{z}(t-\tau_d). \tag{3.7c}$$

Proof. The proof of this proposition is presented in the appendix. □

Theorem 3.3. *Consider a γ-4C-based teleoperation system in Figures 1 and 2, given by* (2.1)–(2.7) *and modeled in state space by* (3.7a) *and* (3.7b) *with state vector defined in* (3.6)*. Given the scalars $h_{\max}$ and d, the system* (3.7a), (3.7b), *and* (3.7c) *is asymptotically stable if matrix C is Schur-Cohn stable, and there exist $P = P^T > 0$, $Q_i = Q_i^T > 0$ $(i = 1 \ldots 2)$, $R = R^T > 0$, $W = W^T > 0$ such that the* (3.4), (3.5) *matrix inequality holds.*

Remark 3.4. A necessary condition for stability is that the system (3.7a), (3.7b), and (3.7c) with $n = 4$ state variables (3.6) must be Hurwitz stable for zero delay; that is, all the eigenvalues λ_i of $\Lambda = (I_{n\times n} - C)^{-1}(A + A_d)$ with an adequate selection of parameters must verify

$$\mathbf{real}(\lambda_i) < 0 \quad \forall i,\ i = 1 \cdots n, \tag{3.8}$$

where rank $(\Lambda) = n$.

Remark 3.5. In the case of identical master and slave systems with controllers given by (2.7), if $\gamma = 1$ the system for zero delay $\Lambda = (I_{n\times n} - C)^{-1}(A + A_d)$ does not present maximum rank in order to verify (3.8), a necessary stability condition, and two of the eigenvalues are zero. This problem is solved selecting $\gamma > 1$ in (2.7).

Based on the state-space model we can also predict the tracking capabilities related with the transparency properties. Let us consider zero delay $\tau = 0$, then the dynamics (3.7a), (3.7b) with $\Lambda = (I_{n\times n} - C)^{-1}(A + A_d)$ and $B = (I_{n\times n} - C)^{-1}B$, under force input f_h in case of interaction with the environment $f_e(t) = -k_e x_s(t) - B_e \dot{x}_s(t)$, are

$$\dot{z}(t) = \Lambda z(t) + B f_h(t). \tag{3.9}$$

Then the transfer function is

$$G(s) = (sI_{n\times n} - \Lambda)^{-1}B. \tag{3.10}$$

So the steady-state expected behavior is

$$G_\infty = \lim_{s\to 0} G(s) = \frac{1}{\Delta_\infty}\begin{pmatrix} \dfrac{1 + C_6}{C_{m_\infty}} \\ 0 \\ \dfrac{C_{1_\infty}}{(C_{s_\infty} + (1 + C_5)k_e)}\dfrac{(1 + C_6)}{C_{m_\infty}} \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{1 + C_6}{k_m} \\ 0 \\ \dfrac{k_s}{k_s}\dfrac{(1 + C_6)}{k_m} \\ 0 \end{pmatrix},$$
$$\Delta_\infty = 1 + \frac{C_{1_\infty}C_{4_\infty}}{(C_{s_\infty} + (1 + C_5)k_e)C_{m_\infty}} = 1 + \frac{-k_s(k_m/\gamma)}{k_s k_m} = 1 - \frac{1}{\gamma}. \tag{3.11}$$

That is, in the steady-state, with the values proposed in (2.7) the master and slave positions will be $(1 + C_6)/k_m$, meaning that the position tracking error between master and slave will be zero. The master and slave velocity will also be zero.

Furthermore, selecting $C_6 = (k_m \cdot \Delta_\infty - 1)$ we can also obtain the master and slave positions that follow the f_h reference with a steady-state zero error.

Remark 3.6. The tuning factor γ proposed in Definition 2.1 should not only be considered as an additional parameter to make the stability conditions less conservative. The proposed form in which this factor is included in the controllers in (2.7) increases in practical conditions the stable region fixing the desired bounds on time-varying delay, with the particularity of maintaining the tracking properties provided by this transparent control scheme.

4. Analysis and Simulation Results

The robust stability tool under time-varying delay described in Section 3 provides an estimate of the maximum allowable delays $h_{\max}$ that guarantees stability, depending on the bound on the delay variation d (a communication channel characteristic), and the local controllers applied to the system (values in A, A_d, and C).

We define first a simulation case for using with *Simulink* and *Matlab,* considering dynamic models for the master and slave with numerical values given by $M_m = M_s = 1$. We select the controllers as stated in (2.7) with $B_m = B_s = 12$, $k_m = k_s = 20$. The idea is to maintain the ideal tracking properties obtained by the standard controllers in G-4C control scheme increasing, through γ factor, the stability margin in practical conditions, that is, when there is time-varying delay in the signals transmission. In [7] we conclude, based on some experimental analysis, that the results of the worst cases found for Internet Teleoperation confirm delay fluctuations ranging from a minimum value of 120 ms to a maximum of 265 ms. and that d, also in case of poor communication conditions, belongs to the interval (0.5,1).

The stability analysis is assessed applying Theorem 3.3 using *Matlab* and the Linear Matrix Inequalities *LMI-toolbox* of this software. In all cases we have proved firstly, that the system (3.7a), (3.7b), and (3.7c) is Hurwitz stable for zero delay. Furthermore, matrix C is Schur-Cohn stable.

We study the performance through simulations when the human operator applies step inputs as reference to follow by the slave, that it is in free motion until $t = 18$ s in which the slave begins the contact with a hard environment modeled with $k_e = 3 \times 10^5$ to $t = 25$ s. In these simulations we will show the reference given by f_h and the master and slave positions, the position tracking error x_m-x_s, and the force tracking error f_m-f_s.

First, for illustrating the effect of the γ factor in the stability, we apply Theorem 3.3 for the simulation case already described, fixing the following bounds on the delay: $h = 3$ s, $|\eta(t)| \leq 0.1$ s, and $d = 0.7$. The (3.4), (3.5) matrix inequality holds for $\gamma > 22.6$.

Then, we show in the established simulation conditions, the theoretical and simulation results for $\gamma = 22.7$ (stable from Theorem 3.3) and $\gamma = 22.6$ (unstable from Theorem 3.3).

Case 1 ($\gamma = 22.7$). From Theorem 3.3, there exist:

$$P = 1 \times 10^8 \begin{pmatrix} 2.4736 & 0.1707 & -0.4519 & -0.0364 \\ 0.1707 & 0.0304 & 0.0880 & 0.0097 \\ -0.4519 & 0.0880 & 1.3797 & 0.1158 \\ -0.0364 & 0.0097 & 0.1158 & 0.0138 \end{pmatrix} > 0,$$

$$Q_1 = 1 \times 10^8 \begin{pmatrix} 1.9416 & 0.0190 & -0.4585 & -0.0596 \\ 0.0190 & 0.0042 & 0.0495 & -0.0019 \\ -0.4585 & 0.0495 & 0.8503 & -0.0040 \\ -0.0596 & -0.0019 & -0.0040 & 0.0023 \end{pmatrix} > 0,$$

$$Q_2 = 1 \times 10^8 \begin{pmatrix} 1.5564 & 0.0063 & -0.4297 & -0.0464 \\ 0.0063 & 0.0654 & 0.0699 & -0.0021 \\ -0.4297 & 0.0699 & 1.1095 & -0.0110 \\ -0.0464 & -0.0021 & -0.0110 & 0.0499 \end{pmatrix} > 0,$$

$$R = 1\times 10^8 \begin{pmatrix} 5.6382 & 2.8504 & 0.2428 & 0.0942 \\ 2.8504 & 3.4776 & 0.1193 & 0.0550 \\ 0.2428 & 0.1193 & 2.0754 & 0.0589 \\ 0.0942 & 0.0550 & 0.0589 & 2.0816 \end{pmatrix} > 0,$$

$$W = 1\times 10^8 \begin{pmatrix} 2.1039 & 0 & 0 & 0 \\ 0 & 2.1024 & 0 & -0.0001 \\ 0 & 0 & 2.1039 & 0 \\ 0 & -0.0001 & 0 & 2.1038 \end{pmatrix} > 0. \tag{4.1}$$

Such that the inequality (3.4) is feasible. Hence, the system (3.7a) and (3.7b) is asymptotically stable. See in Figure 3 the simulation results: the reference given by f_h and the master and slave positions.

Case 2 ($\gamma = 22.6$). However, selecting $\gamma = 22.6$ with the same delay conditions as described before, the LMI constraints were found infeasible. We can also see, through simulations presented in Figure 4, the unstable results when Theorem 3.3 is not accomplished.

Now, applying the theorem, we obtain $h_{\max}(d)$, shown in Figure 5. The bound on delay magnitude is calculated as a function of the bound d of the delay variation for the two values of γ.

Remark 4.1. It has been detected by simulation that stability is preserved until a value is similar to the theoretical one, as we can see comparing the simulation results in both cases. We can conclude that although the theorem conditions are sufficient, the results are not conservative.

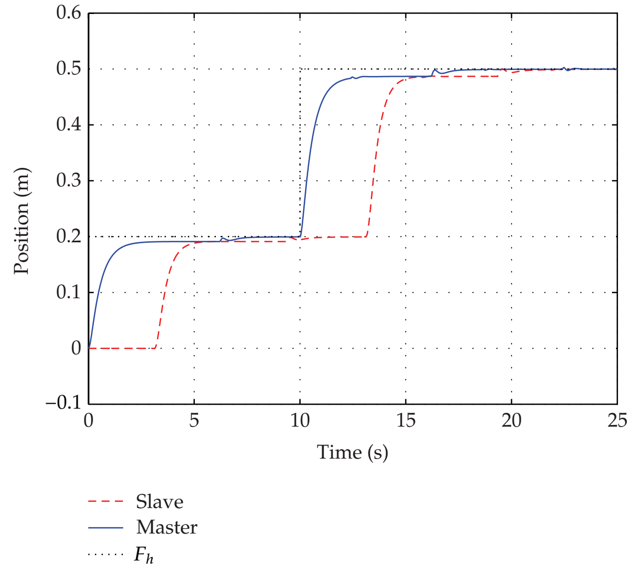
Furthermore, it is very interesting for the control design procedure, to see how γ affects the changes in the bound on the delay magnitude $h_{\max}(\gamma)$, for fixed values of the bound on the delay variation d. Viewing in Figure 6 the theoretical results from Theorem 3.3, we conclude that we can adjust the γ factor in order to ensure the stability of the system given the desired delay characteristics, and also, for high values in γ, the system is practically delay-independent. But then the following question is how these high values affect to the performance and transparency.

We try to answer this question selecting another simulation case fixing bounds on the delay: $h = 3\,\mathrm{s}$, $|\eta(t)| \le 0.1\,\mathrm{s}$, $d = 0.99$ and a high value in γ.

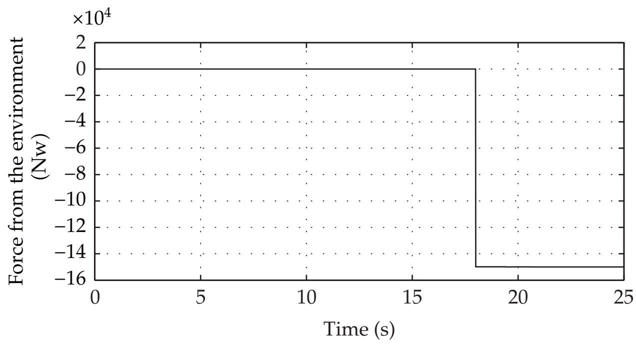
Case 3 ($\gamma = 500$). From Theorem 3.3, there exist

$$P = 1\times 10^5 \begin{pmatrix} 2.3056 & 0.0333 & -0.2301 & -0.0292 \\ 0.0333 & 0.1088 & 0.0377 & 0.0087 \\ -0.2301 & 0.0377 & 0.6221 & 0.0396 \\ -0.0292 & 0.0087 & 0.0396 & 0.0244 \end{pmatrix} > 0,$$

$$Q_1 = 1\times 10^4 \begin{pmatrix} 7.5672 & 1.9034 & -0.6723 & -0.6274 \\ 1.9034 & 7.2717 & 1.4226 & -0.6165 \\ -0.6723 & 1.4226 & 3.1138 & -0.0926 \\ -0.6274 & -0.6165 & -0.0926 & 0.8054 \end{pmatrix} > 0,$$



(a)



(b)

Figure 3: (a) Case 1. Reference and master and slave positions. (b) Force imposed by the environment.

$$Q_2 = 1 \times 10^4 \begin{pmatrix} 6.9191 & 1.1173 & -0.6417 & -0.5087 \\ 1.1173 & 8.5898 & 0.7219 & -0.3822 \\ -0.6417 & 0.7219 & 4.0713 & -0.0003 \\ -0.5087 & -0.3822 & -0.0003 & 1.3680 \end{pmatrix} > 0,$$
$$R = 1 \times 10^5 \begin{pmatrix} 1.2419 & 0.4566 & 0.0144 & 0.0225 \\ 0.4566 & 0.7594 & -0.0034 & 0.0041 \\ 0.0144 & -0.0034 & 0.4822 & -0.0228 \\ 0.0225 & 0.0041 & -0.0228 & 0.4471 \end{pmatrix} > 0,$$
$$W = 1 \times 10^4 \begin{pmatrix} 7.7108 & 0 & 0 & 0 \\ 0 & 7.7109 & 0 & 0 \\ 0 & 0 & 7.7108 & 0 \\ 0 & 0 & 0 & 7.7108 \end{pmatrix} > 0. \tag{4.2}$$

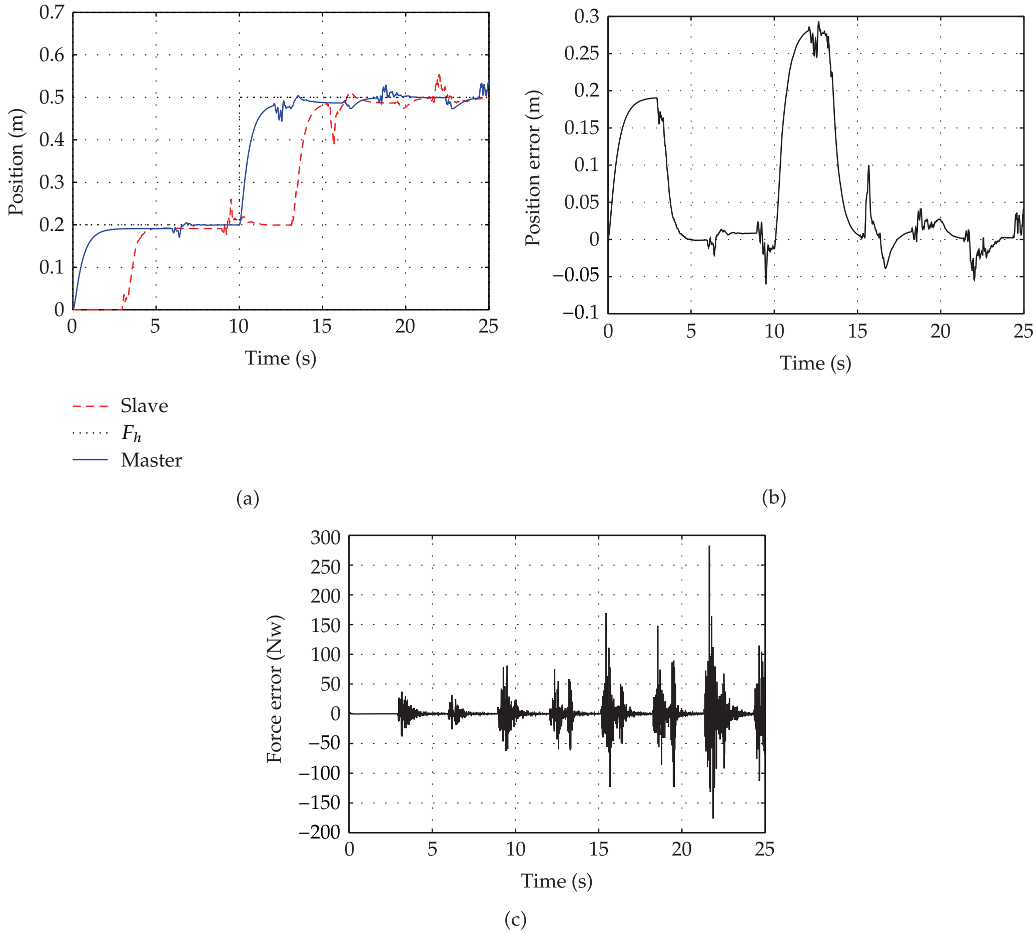


Figure 4: Case 2. (a) Reference and master and slave positions. (b) The position tracking error x_m-x_s. (c) The force tracking error f_m-f_s.

Such that the inequality (3.4) is feasible. The results in Figure 7 show the robust performance of the system, without overshoot, with small settling time, and maintaining the null error in force and position tracking.

5. Conclusions

This work has addressed a study on the stability of a certain teleoperation system, based on recent results on Lyapunov-Krasovskii functionals for time-varying delays that also take into account bounds on delay derivatives, to reduce conservativeness. The teleoperation scheme chosen is the γ-4C based on G-4C one, applied to a *1 DoF* master and slave manipulators, for free motion and for slave in contact with the environment.

This scheme is modeled by means of *4*-dimensional state-space equation, that results in a differential-delayed equation, which is of the neutral type that is, the delay affects not only the state but also the state derivative.

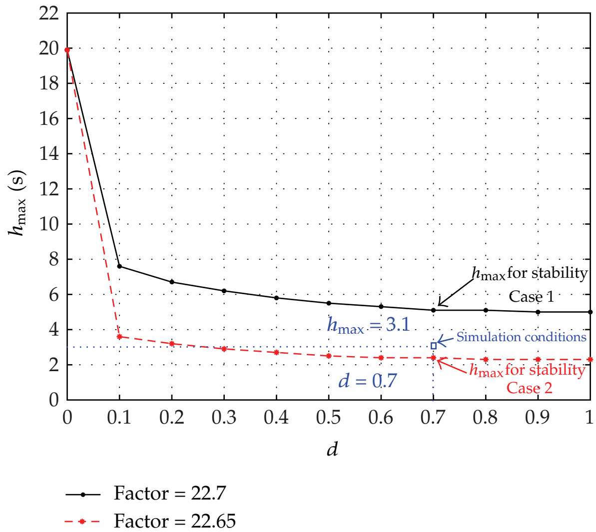


Figure 5: Estimation of the maximum allowable delays $h_{max}(d)$ that guarantee stability for Case 1 ($\gamma = 22.7$) and Case 2 ($\gamma = 22.6$).

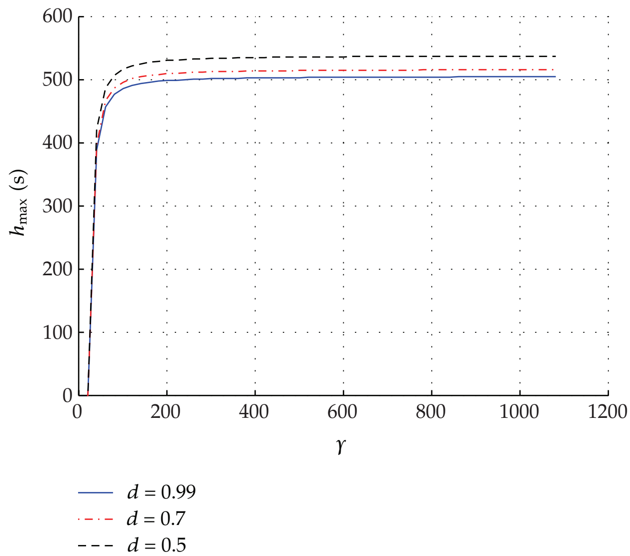


Figure 6: Estimation of the maximum allowable delays $h_{max}(\gamma)$ that guarantee stability for bounds $d = \{0.5, 0.7, 0.99\}$ on the delay variation.

Furthermore, as a result of the application of this analysis technique in teleoperation, with actual bounds for the delay magnitude and its derivative appearing with UDP protocol for different Internet locations, we propose new values for the standard controllers, incorporating a tuning factor to increase in practical conditions the stable region fixing the desired bounds on time-varying delay, with the particularity of maintaining the tracking properties

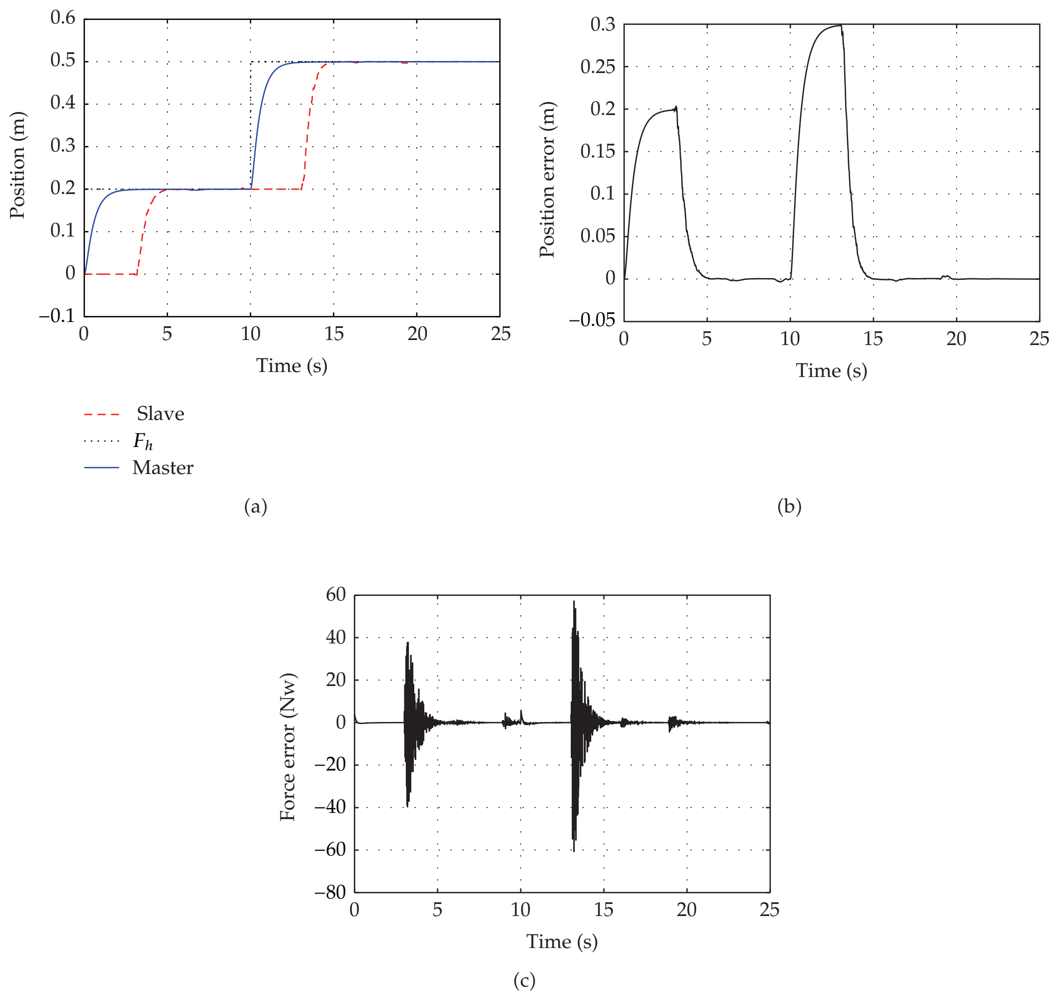


Figure 7: Case 3. (a) Reference and master and slave positions. (b) The position tracking error x_m-x_s. (c) The force tracking error f_m-f_s.

provided by this transparent control scheme. Simulations confirm the robust stability under time-varying delay besides of a good tracking behavior.

Appendix

Proof of Proposition 3.2. The state-space model in (3.7a) and (3.7b), with state-space vector $z(t) \in \Re^n$ given by (3.6), is obtained from (2.1), (2.2), (2.3), (2.5), (2.6), and (2.7) as follows.

The relationship between positions and velocities $\dot{x}_m(t) = v_m(t)$, $\dot{x}_s(t) = v_s(t)$ provides the first and third model equations that we will after rewrite in a compact form in (3.7a) and (3.7b).

For the second and fourth ones, we substitute first (2.5) and (2.7) into (2.6) and then the result in (2.1) to finally find from (2.2) $\dot{v}_m = f_m/M_m$, $\dot{v}_s = f_s/M_s$.

Here for simplicity, we avoid the time dependence (unless strictly necessary) in the equations and we show the steps only for the second equation (master velocity dynamic). The fourth one can be obtained in a parallel form.

Knowing that from (2.5) $y_{sd}(t) =: y_s(t-\tau)$, $y = f, x$, and from (2.7) $C_m(s) = B_m s + k_m$, $C_4(s) = -(P_m(s) + C_m(s))/\gamma$, $C_2 = (1 + C_6)/\gamma$, (2.6) $f_{mc} = C_m x_m + C_2 f_{sd} + C_4 x_{sd} - C_6 F_h$ results is

$$\begin{gathered} C_m x_m = B_m v_m + k_m x_m, \\ C_2 f_{sd} = C_2 f_s(t-\tau) = C_2 M_s \dot{v}_s(t-\tau), \\ C_4 x_{sd} = C_4 x_s(t-\tau) = -\frac{M_m}{\gamma}\dot{v}_s(t-\tau) - \frac{B_m}{\gamma}\dot{x}_s(t-\tau) - \frac{k_m}{\gamma}x_s(t-\tau). \end{gathered} \tag{A.1}$$

Then, from (2.1),

$$\begin{aligned} f_m = f_h - f_{mc} = (1 + C_6) f_h - k_m x_m - B_m v_m + \frac{k_m}{\gamma} x_s(t-\tau) \\ + \frac{B_m}{\gamma}\dot{x}_s(t-\tau) + \left[\frac{M_m}{\gamma} - C_2 M_s\right]\dot{v}_s(t-\tau), \end{aligned} \tag{A.2}$$

and $\dot{v}_m = f_m / M_m$ gives the second equation. □

Acknowledgment

This work is supported by MCINN, Spain, under Project DPI10-20466-C02-C01.

References

[1] D. A. Lawrence, "Stability and transparency in bilateral teleoperation," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 5, pp. 624–637, 1993.
[2] P. Arcara and C. Melchiorri, "Control schemes for teleoperation with time delay: a comparative study," *Robotics and Autonomous Systems*, vol. 38, no. 1, pp. 49–64, 2002.
[3] R. J. Anderson and M. W. Spong, "Bilateral control of teleoperators with time delay," *IEEE Transactions on Automatic Control*, vol. 34, no. 5, pp. 494–501, 1989.
[4] K. Hashtrudi-Zaad and S. E. Salcudean, "Transparency in time-delayed systems and the effect of local force feedback for transparent teleoperation," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 1, pp. 108–114, 2002.
[5] G. Niemeyer and J. J. E. Slotine, "Stable adaptive teleoperation," *IEEE Journal of Oceanic Engineering*, vol. 16, no. 1, pp. 152–162, 1991.
[6] E. Delgado and A. Barreiro, "Stability of teleoperation systems for time-varying delays by LMI techniques," in *Proceedings of the European Control Conference (ECC '07)*, pp. 4214–4221, Kos, Greece, July 2007.
[7] E. Delgado, M. Díaz-Cacho, and A. Barreiro, "Stability of teleoperation systems under time-varying delays by using Lyapunov-Krasovskii techniques," in *Proceedings of the 18th International Federation of Automatic Control World Congress*, vol. 18, part I, Milan, Italy, August 2011.
[8] M. Díaz-Cacho, A. Fernández, M. García, and A. Barreiro, "Internet emulation system for UDP-based teleoperation," in *Proceedings of the Mediterranean Conference on Control and Automation (MCA '08)*, pp. 1417–1422, Corsica, France, June 2008.

[9] D. Tian, D. Yashiro, and K. Ohnishi, “Haptic transmission by weighting control under time-varying communication delay,” *IET Control Theory & Applications*, vol. 6, no. 3, pp. 420–429, 2012.
[10] P. F. Hokayem and M. W. Spong, “Bilateral teleoperation: an historical survey,” *Automatica*, vol. 42, no. 12, pp. 2035–2057, 2006.
[11] N. Chopra, M. W. Spong, and R. Lozano, “Synchronization of bilateral teleoperators with time delay,” *Automatica*, vol. 44, no. 8, pp. 2142–2148, 2008.
[12] A. F. Villaverde, A. B. Blas, J. Carrasco, and A. B. Torrico, “Reset control for passive bilateral teleoperation,” *IEEE Transactions on Industrial Electronics*, vol. 58, no. 7, pp. 3037–3045, 2011.
[13] J. Ware and Y. J. Pan, “Realisation of a bilaterally teleoperated robotic vehicle platform with passivity control,” *IET Control Theory and Applications*, vol. 5, no. 8, pp. 952–962, 2011.
[14] E. Nuño, L. Basañez, and R. Ortega, “Passivity-based control for bilateral teleoperation: a tutorial,” *Automatica*, vol. 47, no. 3, pp. 485–495, 2011.
[15] S. Dashkovskiy, M. Kosmykov, and F. R. Wirth, “A small-gain condition for interconnections of ISS systems with mixed ISS characterizations,” *IEEE Transactions on Automatic Control*, vol. 56, no. 6, pp. 1247–1258, 2011.
[16] I. G. Polushin and S. N. Dashkovskiy, “A small-gain framework for networked cooperative teleoperation,” in *Proceedings of the 8th IFAC Symposium on Nonlinear Control Systems (NOLCOS ’10)*, pp. 90–95, Bologna, Italy, September 2010.
[17] E. Delgado, M. Díaz-Cacho, D. Bustelo, and A. Barreiro, “Generic approach to stability under time-varying delayin teleoperation. Application to the position-error control of a gantry crane,” *IEEE/ASME Transactions on Mechatronics*, vol. PP, no. 99, pp. 1–11, 2012.
[18] S.-I. Niculescu, *Delay Effects on Stability*, Lecture Notes in Control and Information Sciences, Springer, New York, NY, USA, 2001.
[19] D. Ivănescu, S.-I. Niculescu, L. Dugard, J.-M. Dion, and E. I. Verriest, “On delay-dependent stability for linear neutral systems,” *Automatica*, vol. 39, no. 2, pp. 255–261, 2003.
[20] Z. H. Wang, “Numerical stability test of neutral delay differential equations,” *Mathematical Problems in Engineering*, vol. 2008, Article ID 698043, 10 pages, 2008.
[21] Z. Zhao, W. Wang, and B. Yang, “Delay and its time-derivative dependent robust stability of neutral control system,” *Applied Mathematics and Computation*, vol. 187, no. 2, pp. 1326–1332, 2007.
[22] A. Barreiro and E. Delgado, “Stability of teleoperation systems by delay-dependent neutral LMI techniques,” in *Proceedings of the 32nd Annual Conference on IEEE Industrial Electronics (IECON ’06)*, pp. 342–347, November 2006.
[23] J. K. Hale and S. M. Verduyn Lunel, “Introduction to functional differential equations,” in *Applied Mathematical Sciences*, vol. 99, Springer, New York, NY, USA, 1993.