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Research Article

Competition and Integration in Closed-Loop Supply Chain Network with Variational Inequality

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A closed-loop supply chain network involves the manufactured and remanufactured homogeneous products. It comprises operation links to represent business activities including manufacturing/remanufacturing activities, treatment activities for EOL products, transportation activities, and storage activities, which are performed by the firms. Among all closed-loop supply chain problems, the horizontal merger of oligopolistic firms is so important and attracting to both businessman and researchers. In this paper, the interaction of the competitive firms prior to horizontal merger is analyzed. Three networks including prior to horizontal merger, postpartial merger, and complete merger are studied. Simultaneously, three economical models for these networks on different conditions of mergers are established and discussed. The variational inequality formulations are used for these three models, whose solutions give out the production quantity of new products, and remanufactured products, the product flows for new products, remanufactured products and end-of-life products at every path, the demand quantity, the recovery quantity of end-of-life products and the equilibrium prices. Finally, numerical examples are tested and illustrated for the proposed models.

1. Introduction

In light of increasing environmental consciousness and stricter legislation, disposal of plentiful EOL (end-of-life) products becomes a critical problem. EOL product recovery aims to minimize the amount of waste sent to landfills by recovering materials and parts from old or outdated products by means of recycling and remanufacturing [1]. Nowadays, EOL product recovery has become an obligation to the environment and to the society itself. Hence, more and more countries extend manufacturers' responsibility and require them to take back the EOL products. This concern for the environment has motivated increased interest in reverse logistics arose from EOL product recovery, which has become the subject of growing attention over the last decade [2].

In an effort to curb pollution and achieve the goal of sustainable development, some measures have been proposed in China to promote the traded-in of the automobiles and home appliances covering five types of commodities: TV, refrigerator, washing machine, air conditioner, and computer. The trial points of traded-in comprise Beijing, Shanghai, Tianjin, Jiangsu, Zhejiang, Shandong, Guangdong, Fuzhou, and Changsha. The consumers who purchase new home appliances and meet the requirements of subsidy can be provided a subsidy worth 10 percent of the prices of the new products. The highest subsidies will be determined by their types. The transportation cost of the products to the recovery centers for treatment can be subsidized within a certain range. It shows the stronger background and important meaning to deal with the EOL products in China.

Manufacturers have faced increasing pressures from both governmental regulations and from consumers and become more environmentally responsible [3]. Under this background, manufacturers not only focus on their forward logistics, but also have to consider reverse logistics. This may lead to open systems if the recovered content of the original products leaves the original supply chains and is used by other manufacturers to manufacture products serving a different purpose, but it may also encourage manufacturers to have reverse flows implemented into their own supply chains, giving rise to a closed-loop supply chain [4].

The study of closed-loop supply chain is a relatively new field of research. More recently, a variety of closed-loop supply chain network optimization models have been developed. Schultmann et al. [5] proposed a hybrid approach to establish a closed-loop supply chain for spent batteries. It combines an optimization model for planning a reverse-supply network and a flow-sheeting process model that enables a simulation tailored to potential recycling options for spent batteries in the steelmaking industry. Easwaran and Üster [6] presented a mixed-integer linear programming to determine the optimal locations of the collection centers and remanufacturing facilities along with the integrated forward and reverse flows and devised two Tabu Search heuristics—sequential and random neighborhood search procedures to solve the problem. Wang and Hsu [7] considered the integration of forward and reverse logistics. A generalized closed-loop model for the logistics planning was proposed by formulating a cyclic logistics network problem into an integer linear programming model, and a revised spanning-tree-based genetic algorithm was developed. Moreover, concerning the uncertain factors, a stochastic programming model was developed by Chouinard et al. aiming at evaluating impacts of randomness related to recovery, processing, and demand volumes on the design decisions [8]. Francas and Minner presented a two-stage stochastic programming model with normally distributed demands and returns [9]. In addition, a multiperiod integer programming model [10] and a multiperiod and multiproduct network model [11] were proposed in recent years.

Among all closed-loop supply chain problems, the horizontal merger of oligopolistic firms with closed-loop supply chain networks is so important and attracting to both businessman and researchers. Mergers and acquisitions have become an inseparable part of international business and a major means for companies to grow in size, go international, or obtain knowhow [12]. Companies seek profit improvement by the rapid expansion of sales via mergers. In addition, a merger motive can be to build a conglomerate [13]. Every company pursues continuously profit growth and expects to take away more market share from its competitors. Thus, the pursuit of those merger advantages might explain recent horizontal mergers such as those of Kmart and Sears in the retail industry, the merger of

Interbrew and Ambev and that of SAB-Miller and Molson Coors as well as the merger of InBev and Anheuser-Busch in the beer industry, the merger of Gome and Yongle in retail industry in China.

Many researchers have focused their attention on the study of horizontal merger. Chakravarty [14] used the Debreu-Farrell-Färe measure of efficiency of merger to compare economic efficiencies of alternative merged entities in a homogeneous good industry. Nilssen and Sørgard [15] analyzed the interdependence of merger decisions over time. The Fudenberg-Tirole taxonomy of business strategies was used to discuss merger decisions made in sequence by disjoint groups of firms and a simple oligopoly model proposed to these strategies. Ivaldi and Verboven [16] used the parameter estimates to compute the postmerger equilibrium and the implied price and welfare changes under various alternative scenarios. Davidson and Mukherjee [17] analyzed horizontal mergers in a simple model in which free entry and exit and provided a general analysis of the impact of mergers on the long-run industry structure. In contrast to the aforementioned models, Nagurney [18] presented a new theoretical framework for the quantification of strategic advantages associated with horizontal mergers through the integration of supply chain networks.

In this paper, within the context of closed-loop supply chain network, the firms in the network include manufacturing/remanufacturing plants and distribution/recovery centers and serve the demand markets. Manufacturers/remanufacturers are involved in the production of a kind of homogeneous products from the raw materials and recovered EOL products. We consider that each firm as a network of business activities represented by forward links, and reverse links. The forward links consist of manufacturing links, remanufacturing links, forward transportation links and forward storage links, while reverse links include treatment links for EOL products, reverse transportation links, and reverse storage links. Three networks including prior to horizontal merger, post partial merger and complete merger are studied. Simultaneously, we propose three economical models for these networks on different conditions of mergers. Theory of variational inequalities is used to derive the variational inequality formulations of those models. Finally, several examples are studied to describe the effects of parameters on equilibrium results and analyze what impact the costs of horizontal merger will have on profits of the firms apter partial merger and complete merger.

This paper is organized as follows. In Section 2, we analyze the interaction of the competitive firms prior to horizontal merger and propose an equilibrium model for the network. In Section 3, the partial firms are merged into one. We develop an economical model for the network after partial merger. In Section 4, a nonlinear optimization model for the closed-loop supply chain network after all firms merged is given. In Section 5, we provide numerical examples to present the effects of parameters on the equilibrium results in the case of different merger scenarios and analyze what impact the costs of horizontal merger will have on profits of the firms after partial merger and complete merger. At last, the paper ends with conclusions in Section 6.

2. Prehorizontal Merger Economical Model

We present a closed-loop supply chain network prior to horizontal merger associated with a number of firms in the same industry, denoted by Firm $1, 2, \ldots, I$. Each Firm i has m_i manufacturing/remanufacturing plants, n_i distribution/recovery centers, and serves J demand markets. The plants produce the same homogeneous manufactured and remanufactured products. The distribution centers are responsible for transporting the manufactured and remanufactured products from manufacturing/remanufacturing plants, and storing these products, then transporting these products to the demand markets. The recovery centers are responsible for recovering and storing the EOL products and transporting these products to remanufacturing plants to be treated and to be ready for remanufacturing.

Let $G^1 = [N^1, L^1]$ denote the network, where N^1 denotes the set of nodes and L^1 denotes the set of operation links including forward and reverse links. The forward links are the links which are located on the forward supply chains, and the reverse links are the links which are located on the reverse supply chains. We assume that each firm is represented as a network of many operation links which represent business activities including manufacturing/remanufacturing activities, treatment activities for EOL products, transportation activities, and storage activities performed by the firms, as depicted in Figure 1.

Product flows on the forward links are classified into two types: the products remanufactured from recovered EOL products and the products manufactured from new materials. The product flows on the reverse links are the recovered EOL products. The forward links between the first layer nodes with the second layer nodes are the manufacturing/remanufacturing links representing manufacturing and remanufacturing operations, and the reverse links between them are the treatment links representing treatment operations for the EOL products. The links between second layer nodes with third layer nodes are transportation links representing transportation operations between the manufacturing plants with distribution centers. The next layer links are storage links representing storage operations for the manufactured/remanufactured products and EOL products. Moreover, there are transportation links connecting distribution center nodes with all demand market nodes. The majority of the required notation is given in Table 1.

Suppose that q_j^N denotes the demand for the products at demand market j. The quantity of products transacted for Firm i is equal to that of manufactured products and remanufactured products. Therefore, we have the following conservation of product flows:

$$q_j^N = \sum_{p \in P_j^{1N}} \left(x_p^1 + y_p^1 \right). \tag{2.1}$$

Let q_j^R denote the quantity of EOL products recovered from the demand market j. Then, we have the following conservation of product flow:

$$q_j^R = \sum_{p \in P_j^{1R}} z_p^1. \tag{2.2}$$

Let μ denote the fraction of the product that can be remanufactured from one unit of recovered EOL product. Therefore, we have:

$$\sum_{p \in P_i^{1N}} x_p^1 \leq \mu \sum_{p \in P_i^{1R}} z_p^1. \tag{2.3}$$

Let ρ_j^N denote the sale price of products at demand market j, and the sale price function is associated with the quantity of products at each demand market, and we assume that $\rho_j^N = \rho_j^N(q_j^N)$. In the reverse supply chains, demand markets act as a source of EOL

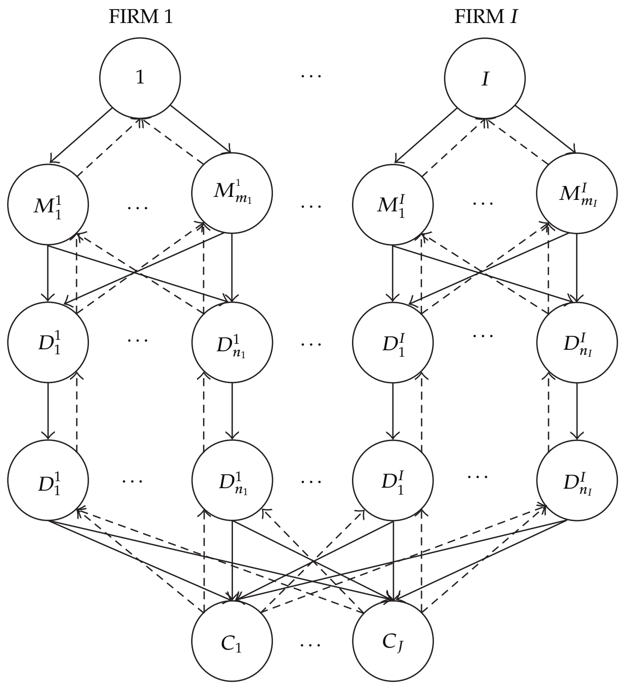

Figure 1: The closed-loop supply chain network prior to horizontal merger.

products. Increasing the recovery price, more consumers are persuaded to return products. Let ρ_j^R denote the recovery price of EOL products at demand market j, and the price function is associated with the supply quantity of EOL products, that is, $\rho_j^R = \rho_j^R(q_j^R)$. Let σ denote the ratio of EOL products recovered to total quantity sold at the demand market; we have

$$q_j^R \leq \sigma q_j^N. \tag{2.4}$$

For any forward link $a \in L^1$, we denote by h_a^1, the remanufactured product flow on link a. Let δ_{ap} if the link a participates in forward path p, and $\delta_{ap} = 0$, otherwise. Hence, the following conservation equation should be satisfied for the remanufactured product flow on forward paths and on operation links:

$$h_a^1 = \sum_{p \in P^{1N}} \delta_{ap} x_p^1. \tag{2.5}$$

For any forward link $b \in L^1$, we denote by h_b^1 the manufactured product flow on link b. Let $\delta_{bp} = 1$ if the link b participates in forward path p, and $\delta_{bp} = 0$, otherwise. Hence, the following conservation equation should be satisfied for the manufactured product flow on forward paths and on operation links:

$$h_b^1 = \sum_{p \in P^{1N}} \delta_{bp} y_p^1. \tag{2.6}$$

Table 1: Notation for the closed-loop supply chain network prior to horizontal merger.

Notation	Definition
L_i^{1N}	The set of all forward links for the Firm i, where $i = 1,\ldots,I$
L_i^{1R}	The set of all reverse links for the Firm i, where $i = 1,\ldots,I$
p	A path of the network
P_{ij}^{1N}	The set of all forward paths connecting the first layer node i with demand market j, where $i = 1,\ldots,I$, $j = 1,\ldots,J$
P_{ij}^{1R}	The set of all reverse paths connecting the node i with demand market j, where $i = 1,\ldots,I$, $j = 1,\ldots,J$
P_j^{1N}	The set of all forward paths between all I nodes with demand market j, where $j = 1,\ldots,J$
P_j^{1R}	The set of all reverse paths between all I nodes with demand market j, where $j = 1,\ldots,J$
P_i^{1N}	The set of all forward paths between the node i with all demand markets, where $i = 1,\ldots,I$
P_i^{1R}	The set of all reverse paths between the node i with all demand markets, where $i = 1,\ldots,I$
P^{1N}	The set of all forward paths
M^{1N}	The number of all forward paths
P^{1R}	The set of all reverse paths
M^{1R}	The number of all reverse paths
x_p^1	The nonnegative remanufactured product flow on the path p ($p \in P^{1N}$)
x^1	M^{1N}-dimensional vector of all remanufactured product flows on the paths in $p \in P^{1N}$
y_p^1	The nonnegative manufactured product flow on the path p ($p \in P^{1N}$)
y^1	M^{1N}-dimensional vector of all manufactured product flows on the paths in $p \in P^{1N}$
z_p^1	The nonnegative recovered EOL product flow on the path p ($p \in P^{1R}$)
z^1	M^{1R}-dimensional vector of all EOL product flows on the paths in $p \in P^{1R}$
X^1	The set of all product flows, $X^1 \equiv (x^1, y^1, z^1)$

For any reverse link $c \in L^1$, we denote by h_c^1 the product flow on link c. Let $\delta_{cp} = 1$ if the link c participates in forward path p, and $\delta_{cp} = 0$, otherwise. Similarly, we have

$$h_c^1 = \sum_{p \in P^{1R}} \delta_{cp} z_p^1. \tag{2.7}$$

We define the cost on each link in performing the corresponding task, and the cost function is a generic cost function of its product flow, that is, $e_a = e_a(h_a^1)$, $e_b = e_b(h_b^1)$, and $e_c = e_c(h_c^1)$.

Therefore, the profit of Firm i is

$$\begin{aligned}\mathrm{TP}_i = {} & \sum_j \left(\rho_j^N \left(q_j^N \right) \sum_{p \in P_{ij}^{1N}} \left(x_p^1 + y_p^1 \right) \right) - \sum_j \left(\rho_j^R \left(q_j^R \right) \sum_{p \in P_{ij}^{1R}} z_p^1 \right) \\ & - \sum_{a \in L_i^{1N}} e_a \left(h_a^1 \right) - \sum_{b \in L_i^{1N}} e_b \left(h_b^1 \right) - \sum_{c \in L_i^{1R}} e_c \left(h_c^1 \right).\end{aligned} \tag{2.8}$$

We assume that the above profit function is continuously differentiable and convex, all firms compete in a noncooperative manner, and each one is a profit maximizer. The optimality conditions for all firms simultaneously, under the above assumptions, coincide with the solution of the following variational inequality [19]: determine $X^{1*} = (x^{1*}, y^{1*}, z^{1*}) \in K^1$ satisfying

$$-\sum_i \left(\sum_{p \in P_i^{1N}} \frac{\partial \mathrm{TP}_i}{\partial x_p^1} \times \left(x_p^1 - x_p^{1*}\right) + \sum_{p \in P_i^{1N}} \frac{\partial \mathrm{TP}_i}{\partial y_p^1} \times \left(y_p^1 - y_p^{1*}\right) + \sum_{p \in P_i^{1R}} \frac{\partial \mathrm{TP}_i}{\partial z_p^1} \times \left(z_p^1 - z_p^{1*}\right) \right) \geq 0,$$
$$\forall X^1 \in K^1 \equiv \left\{ \left(x^1, y^1, z^1\right) \mid \left(x^1, y^1, z^1\right) \in R_+^{N_1} \text{ and } (2.1) - (2.7) \text{ hold} \right\}, \tag{2.9}$$

where $N_1 = 2M^{1N} + M^{1R}$. Upon using (2.1)–(2.7), variational inequality (2.9) may be re-expressed as [20]: determine $X^{1*} = (x^{1*}, y^{1*}, z^{1*}, \lambda_1^{1*}, \lambda_2^{1*}) \in R_+^{N_1+I+J}$, such that

$$\begin{aligned}
&\sum_i \sum_j \sum_{p \in P_{ij}^{1N}} \left[\sum_{a \in L_i^{1N}} \frac{\partial e_a\left(\sum_{p \in P^{1N}} \delta_{ap} x_p^{1*}\right)}{\partial h_a^1} \delta_{ap} - \rho_j^N \left(\sum_{p \in P_j^{1N}} \left(x_p^{1*} + y_p^{1*}\right) \right) \right. \\
&\qquad \left. - \frac{\partial \rho_j^N\left(\sum_{p \in P_j^{1N}} \left(x_p^{1*} + y_p^{1*}\right)\right)}{\partial q_j^N} \sum_{p \in P_{ij}^{1N}} \left(x_p^{1*} + y_p^{1*}\right) + \lambda_{1i}^{1*} - \sigma \lambda_{2j}^{1*} \right] \times \left[x_p^1 - x_p^{1*}\right] \\
&+ \sum_i \sum_j \sum_{p \in P_{ij}^{1N}} \left[\sum_{b \in L_i^{1N}} \frac{\partial e_b\left(\sum_{p \in P^{1N}} \delta_{bp} y_p^{1*}\right)}{\partial h_b^1} \delta_{bp} - \rho_j^N \left(\sum_{p \in P_j^{1N}} \left(x_p^{1*} + y_p^{1*}\right) \right) \right. \\
&\qquad \left. - \frac{\partial \rho_j^N\left(\sum_{p \in P_j^{1N}} \left(x_p^{1*} + y_p^{1*}\right)\right)}{\partial q_j^N} \sum_{p \in P_{ij}^{1N}} \left(x_p^{1*} + y_p^{1*}\right) - \sigma \lambda_{2j}^{1*} \right] \times \left[y_p^1 - y_p^{1*}\right] \\
&+ \sum_i \sum_j \sum_{p \in P_{ij}^{1R}} \left[\sum_{c \in L_i^{1R}} \frac{\partial e_c\left(\sum_{p \in P^{1N}} \delta_{cp} z_p^{1*}\right)}{\partial h_c^1} \delta_{cp} + \rho_j^R \left(\sum_{p \in P_j^{1R}} z_p^{1*} \right) \right. \\
&\qquad \left. + \frac{\partial \rho_j^R\left(\sum_{p \in P_j^{1R}} z_p^{1*}\right)}{\partial q_j^R} \sum_{p \in P_{ij}^{1R}} z_p^{1*} - \mu \lambda_{1i}^{1*} + \lambda_{2j}^{1*} \right] \times \left[z_p^1 - z_p^{1*}\right] \\
&+ \sum_i \left[\mu \sum_{p \in P_i^{1R}} z_p^{1*} - \sum_{p \in P_i^{1N}} x_p^{1*} \right] \times \left[\lambda_{1i}^1 - \lambda_{1i}^{1*}\right] \\
&+ \sum_j \left[\sigma \sum_{p \in P_j^{1N}} \left(x_p^{1*} + y_p^{1*}\right) - \sum_{p \in P_j^{1R}} z_p^{1*} \right] \times \left[\lambda_{2j}^1 - \lambda_{2j}^{1*}\right] \geq 0, \\
&\qquad \forall X^1 = \left(x^1, y^1, z^1, \lambda_1^1, \lambda_2^1\right) \in R_+^{N_1+I+J},
\end{aligned} \tag{2.10}$$

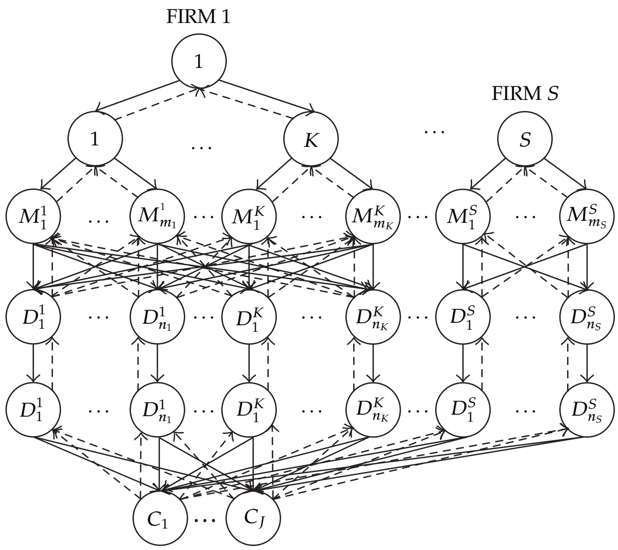

Figure 2: The closed-loop supply chain network after horizontal merger of partial firms.

where $\lambda_1^1 = (\lambda_{11}^1 \;\; \lambda_{12}^1 \;\; \cdots \;\; \lambda_{1I}^1)^T$ and $\lambda_2^1 = (\lambda_{21}^1 \;\; \lambda_{22}^1 \;\; \cdots \;\; \lambda_{2J}^1)^T$ are the vectors of Lagrange multipliers associated with constraints (2.3) and (2.4).

3. Postpartial Horizontal Merger Model

In this section, we consider the partial horizontal merger case, that is, K firms in I firms are horizontal merged into a new Firm 1 and $K + S - 1 = I$, as depicted in Figure 2. The new firm shares manufacturing/remanufacturing plants and distribution/recovery centers of original K firms, that is, the manufactured/remanufactured products at any of these plants can be distributed by any of these distribution centers, and the recovered EOL products at any of these recovery centers can be transported to any of these plants. We denote the new network by $G^2 = [N^2, L^2]$, where N^2 denotes the set of nodes and L^2 denotes the set of operation links in this network. The majority of the required notation for the network post horizontal merger of partial firms is given in Table 2.

For any forward link $a \in L^2$, we denote by h_a^2 the remanufactured product flow on link a. For any forward link $b \in L^2$, we denote by h_b^2 the manufactured product flow on link b. For any reverse link $c \in L^2$, we denote by h_c^2 the EOL product flow on link c. The cost functions are $e_a = e_a(h_a^2)$, $e_b = e_b(h_b^2)$, and $e_c = e_c(h_c^2)$. Since we have assumed that each firm is a profit maximizer, similarly, the optimization problem of Firm i can be expressed as follows.

Maximize

$$\mathrm{TP}_i = \sum_j \left(\rho_j^N\left(q_j^N\right) \sum_{p \in P_{ij}^{2N}} \left(x_p^2 + y_p^2\right) \right) - \sum_j \left(\rho_j^R\left(q_j^R\right) \sum_{p \in P_{ij}^{2R}} z_p^2 \right) - \sum_{a \in L_i^{2N}} e_a\left(h_a^2\right) - \sum_{b \in L_i^{2N}} e_b\left(h_b^2\right) - \sum_{c \in L_i^{2R}} e_c\left(h_c^2\right). \tag{3.1}$$

Table 2: Notation for the closed-loop supply chain network after horizontal merger of partial firms.

Notation	Definition
L_i^{2N}	The set of all forward links for the Firm i, where $i = 1, 2, \ldots, S$
L_i^{2R}	The set of all reverse links for the Firm i, where $i = 1, 2, \ldots, S$
P_{ij}^{2N}	The set of all forward paths connecting the first layer node i ($i = 1, 2, \ldots, S$) with demand market j, where $j = 1, 2, \ldots, J$
P_{ij}^{2R}	The set of all reverse paths connecting the demand market j with the first layer node i ($i = 1, 2, \ldots, S$)
P_j^{2N}	The set of all forward paths between all S nodes with demand market j
P_j^{2R}	The set of all reverse paths between all S nodes with demand market j
P_i^{2N}	The set of all forward paths between the node i ($i = 1, 2, \ldots, S$) with all demand markets
P_i^{2R}	The set of all reverse paths between the node i ($i = 1, 2, \ldots, S$) with all demand markets
P^{2N}	The set of all forward paths
M^{2N}	The number of all forward paths
P^{2R}	The set of all reverse paths
M^{2R}	The number of all reverse paths
x_p^2	The nonnegative remanufactured product flow on the path p ($p \in P^{2N}$)
x^2	M^{2N}-dimensional vector of all remanufactured product flows on the paths in $p \in P^{2N}$
y_p^2	The nonnegative manufactured product flow on the path p ($p \in P^{2N}$)
y^2	M^{2N}-dimensional vector of all manufactured product flows on the paths in $p \in P^{2N}$
z_p^2	The nonnegative recovered EOL product flow on the path p ($p \in P^{2R}$)
z^2	M^{2R}-dimensional vector of all EOL product flows on the paths in $p \in P^{2R}$
X^2	The set of all product flows, $X^2 \equiv (x^2, y^2, z^2)$

Subject to:

$$q_j^N = \sum_{p \in P_j^{2N}} \left(x_p^2 + y_p^2 \right), \quad \forall j, \tag{3.2}$$

$$q_j^R = \sum_{p \in P_j^{2R}} z_p^2, \quad \forall j, \tag{3.3}$$

$$\sum_{p \in P_i^{2N}} x_p^2 \le \mu \sum_{p \in P_i^{2R}} z_p^2, \tag{3.4}$$

$$q_j^R \le \sigma q_j^N, \quad \forall j, \tag{3.5}$$

$$h_a^2 = \sum_{p \in P^{2N}} \delta_{ap} x_p^2, \quad a \in L_i^{2N}, \tag{3.6}$$

$$h_b^2 = \sum_{p \in P^{2N}} \delta_{bp} y_p^2, \quad b \in L_i^{2N}, \tag{3.7}$$

$$h_c^2 = \sum_{p \in P^{2R}} \delta_{cp} z_p^2, \quad c \in L_i^{2R}, \tag{3.8}$$

$$x_p^2 \ge 0, \quad y_p^2 \ge 0, \quad \forall p \in P_i^{2N}; \qquad z_p^2 \ge 0, \quad \forall p \in P_i^{2R}. \tag{3.9}$$

We assume that the above profit function is continuously differentiable and convex, S firms compete in a noncooperative manner, and each one is a profit maximizer. The optimality conditions for all firms simultaneously, under the above assumptions, coincide with the solution of the following variational inequality: determine $X^{2*} = (x^{2*}, y^{2*}, z^{2*}, \lambda_1^{2*}, \lambda_2^{2*}) \in R_+^{N_2+S+J}$, such that

$$
\begin{aligned}
&\sum_i \sum_j \sum_{p\in P_{ij}^{2N}} \left[\sum_{a\in L_i^{2N}} \frac{\partial e_a\left(\sum_{p\in P^{2N}} \delta_{ap} x_p^{2*}\right)}{\partial h_a^2} \delta_{ap} - \rho_j^N \left(\sum_{p\in P_j^{2N}} \left(x_p^{2*} + y_p^{2*}\right) \right) \right. \\
&\qquad \left. - \frac{\partial \rho_j^N \left(\sum_{p\in P_j^{2N}} \left(x_p^{2*} + y_p^{2*}\right)\right)}{\partial q_j^N} \sum_{p\in P_{ij}^{2N}} \left(x_p^{2*} + y_p^{2*}\right) + \lambda_{1i}^{2*} - \sigma \lambda_{2j}^{2*} \right] \times \left[x_p^2 - x_p^{2*}\right] \\
&+ \sum_i \sum_j \sum_{p\in P_{ij}^{2N}} \left[\sum_{b\in L_i^{2N}} \frac{\partial e_b\left(\sum_{p\in P^{2N}} \delta_{bp} y_p^{2*}\right)}{\partial h_b^2} \delta_{bp} - \rho_j^N \left(\sum_{p\in P_j^{2N}} \left(x_p^{2*} + y_p^{2*}\right) \right) \right. \\
&\qquad \left. - \frac{\partial \rho_j^N \left(\sum_{p\in P_j^{2N}} \left(x_p^{2*} + y_p^{2*}\right)\right)}{\partial q_j^N} \sum_{p\in P_{ij}^{2N}} \left(x_p^{2*} + y_p^{2*}\right) - \sigma \lambda_{2j}^{2*} \right] \times \left[y_p^2 - y_p^{2*}\right] \\
&+ \sum_i \sum_j \sum_{p\in P_{ij}^{2R}} \left[\sum_{c\in L_i^{2R}} \frac{\partial e_c\left(\sum_{p\in P^{2N}} \delta_{cp} z_p^{2*}\right)}{\partial h_c^2} \delta_{cp} + \rho_j^R \left(\sum_{p\in P_j^{2R}} z_p^{2*} \right) \right. \\
&\qquad \left. + \frac{\partial \rho_j^R \left(\sum_{p\in P_j^{2R}} z_p^{2*}\right)}{\partial q_j^R} \sum_{p\in P_{ij}^{2R}} z_p^{2*} - \mu \lambda_{1i}^{2*} + \lambda_{2j}^{2*} \right] \times \left[z_p^2 - z_p^{2*}\right] \\
&+ \sum_i \left[\mu \sum_{p\in P_i^{2R}} z_p^{2*} - \sum_{p\in P_i^{2N}} x_p^{2*} \right] \times \left[\lambda_{1i}^2 - \lambda_{1i}^{2*}\right] \\
&+ \sum_j \left[\sigma \sum_{p\in P_j^{2N}} \left(x_p^{2*} + y_p^{2*}\right) - \sum_{p\in P_j^{2R}} z_p^{2*} \right] \times \left[\lambda_{2j}^2 - \lambda_{2j}^{2*}\right] \geq 0, \\
&\qquad\qquad\qquad \forall X^2 = \left(x^2, y^2, z^2, \lambda_1^2, \lambda_2^2\right) \in R_+^{N_2+S+J},
\end{aligned}
\tag{3.10}
$$

where $N_2 = 2M^{2N} + M^{2R}$, $\lambda_1^2 = (\lambda_{11}^2 \ \ \lambda_{12}^2 \ \ \cdots \ \ \lambda_{1S}^2)^T$ and $\lambda_2^1 = (\lambda_{21}^1 \lambda_{22}^1 \cdots \lambda_{2J}^1)^T$, are the vectors of Lagrange multipliers associated with constraints (3.4) and (3.5).

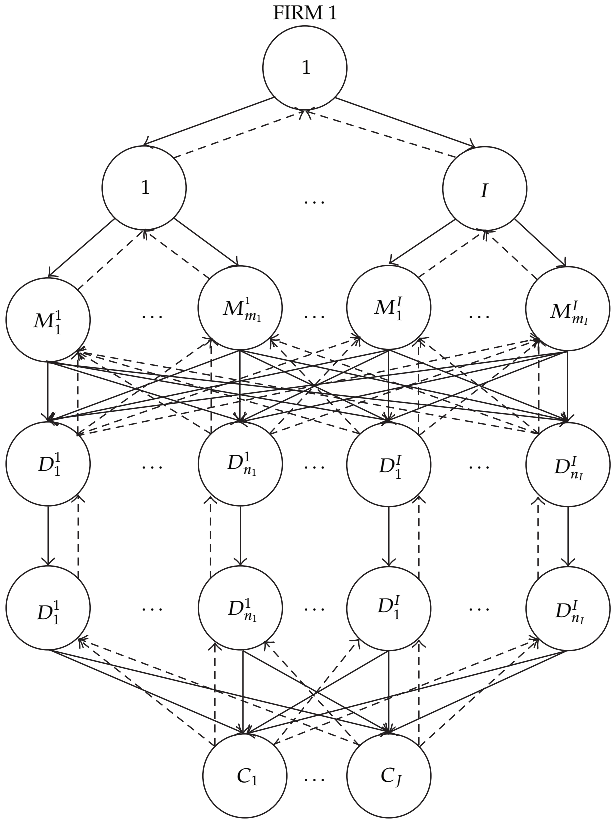

Figure 3: The integrated closed-loop supply chain network after complete horizontal merger of all the firms.

4. Postcomplete Horizontal Merger Integration Model

In this section, we develop a postcomplete merger model in the closed-loop supply chain network. I firms merge into a new firm, and the new one shares manufacturing/remanufacturing plants and distribution/recovery centers of original I firms. The closed-loop supply chain network prior to horizontal merger is integrated, as depicted in Figure 3. There is the first layer Node 1 which represents the integration of all firms in terms of their closed-loop supply chain network with additional operation links connecting Node 1 to the nodes of original firms. Moreover, we add the additional operation links connecting the manufacturing/remanufacturing plants of each firm with the distribution/recovery centers of the other firms. We denote the new network by $G^3 = [N^3, L^3]$, where $N^3 \equiv N^1 \cup$ Node 1 and $L^3 \equiv L^1 \cup$ the additional operation links. The majority of the required notation for the network after complete horizontal merger is given in Table 3.

For any forward link $a \in L^3$, we denote by h_a^3 the remanufactured product flow on link a. For any forward link $b \in L^3$, we denote by h_b^3 the manufactured product flow on link b. For any reverse link $c \in L^3$, we denote by h_c^3 the product flow on link c. We define that the cost functions are $e_a = e_a(h_a^3)$, $e_b = e_b(h_b^3)$, and $e_c = e_c(h_c^3)$.

Since all firms are merged into a firm, there is no longer competition. Hence, the profit of the new firm can be expressed as follows.

Table 3: Notation for the closed-loop supply chain network after complete horizontal merger.

Notation	Definition
L^{3N}	The set of all forward links
L^{3R}	The set of all reverse links
P_j^{3N}	The set of forward paths between the first layer node 1 with the demand market j, where $j = 1, \dots, J$
P_j^{3R}	The set of reverse paths between the demand market j with the first layer node 1
P^{3N}	The set of all forward paths
M^{3N}	The number of all forward paths
P^{3R}	The set of all reverse paths
M^{3R}	The number of all revere paths
x_p^3	The nonnegative remanufactured product flow on the path p ($p \in P^{3N}$)
x^3	M^{3N}-dimensional vector of all remanufactured product flows on the paths in $p \in P^{3N}$
y_p^3	The manufactured product flow on the path p ($p \in P^{3N}$)
y^3	M^{3N}-dimensional vector of all manufactured product flows on the paths in $p \in P^{3N}$
z_p^3	The EOL product flow on the path p ($p \in P^{3R}$)
z^3	M^{3R}-dimensional vector of all EOL product flows on the paths in $p \in P^{3R}$
X^3	The set of all product flows, $X^3 = (x^3, y^3, z^3)$

Maximize

$$\begin{aligned} \mathrm{TP} = {} & \sum_j \left(\rho_j^N \left(q_j^N \right) \sum_{p \in P_j^{3N}} \left(x_p^3 + y_p^3 \right) \right) - \sum_j \left(\rho_j^R \left(q_j^R \right) \sum_{p \in P_j^{3R}} z_p^3 \right) \\ & - \sum_{a \in L^{3N}} e_a \left(h_a^3 \right) - \sum_{b \in L^{3N}} e_b \left(h_b^3 \right) - \sum_{c \in L^{3R}} e_c \left(h_c^3 \right). \end{aligned} \tag{4.1}$$

Subject to:

$$q_j^N = \sum_{p \in P_j^{3N}} \left(x_p^3 + y_p^3 \right), \quad \forall j, \tag{4.2}$$

$$q_j^R = \sum_{p \in P_j^{3R}} z_p^3, \quad \forall j, \tag{4.3}$$

$$\sum_{p \in P^{3N}} x_p^3 \le \mu \sum_{p \in P^{3R}} z_p^3, \tag{4.4}$$

$$q_j^R \le \sigma q_j^N, \quad \forall j, \tag{4.5}$$

$$h_a^3 = \sum_{p \in P^{3N}} \delta_{ap} x_p^3, \quad a \in L^{3N}, \tag{4.6}$$

$$h_b^3 = \sum_{p \in P^{3N}} \delta_{bp} y_p^3, \quad b \in L^{3N}, \tag{4.7}$$

$$h_c^3 = \sum_{p \in P^{3R}} \delta_{cp} z_p^3, \quad c \in L^{3R}, \tag{4.8}$$

$$x_p^3 \ge 0, \quad y_p^3 \ge 0, \quad \forall p \in P^{3N}; \qquad z_p^3 \ge 0, \quad \forall p \in P^{3R}. \tag{4.9}$$

The objective of the model shown in (4.1) is to maximize the total profit of the firm after complete merger. Constraint (4.2) represents that the quantity of manufactured and remanufactured products shipped to demand market j is equal to the demand of the market. Constraint (4.3) represents that the quantity of recovered EOL products from demand market j is equal to the supply of them. Constraint (4.4) restricts that the quantity of remanufactured products come from the EOL products. Constraint (4.5) represents that the quantity of EOL products recovered must not exceed their supply at the demand market j. Constraint (4.6) represents that conservation equation for the remanufactured product flow on forward paths and on operation links. Constraint (4.7) represents that conservation equation for the manufactured product flow on forward paths and on operation links. Constraint (4.8) represents conservation equation for the EOL product flow on reverse paths and on operation links. Constraint (4.9) represents the nonnegative restriction on product flows.

Similarly, we assume that the above profit function is continuously differentiable and convex with respect to variable X^3. The optimality conditions for the firm, under the above assumptions, coincide with the solution of the following variational inequality: determine $X^{3*} = (x^{3*}, y^{3*}, z^{3*}, \lambda_1^{3*}, \lambda_2^{3*}) \in R_+^{N_3+J+1}$ satisfying

$$\begin{aligned}
&\sum_j \sum_{p\in P_j^{3N}} \left[\sum_{a\in L^{3N}} \frac{\partial e_a\left(\sum_{p\in P^{3N}} \delta_{ap} x_p^{3*}\right)}{\partial h_a^3} \delta_{ap} - \rho_j^N \left(\sum_{p\in P_j^{3N}} \left(x_p^{3*} + y_p^{3*}\right) \right) \right. \\
&\qquad \left. - \frac{\partial \rho_j^N \left(\sum_{p\in P_j^{3N}} \left(x_p^{3*} + y_p^{3*}\right)\right)}{\partial q_j^N} \sum_{p\in P_j^{3N}} \left(x_p^{3*} + y_p^{3*}\right) + \lambda_1^{3*} - \sigma \lambda_{2j}^{3*} \right] \times \left[x_p^3 - x_p^{3*}\right] \\
&\quad + \sum_j \sum_{p\in P_j^{3N}} \left[\sum_{b\in L^{3N}} \frac{\partial e_b\left(\sum_{p\in P^{3N}} \delta_{bp} y_p^{3*}\right)}{\partial h_b^3} \delta_{bp} - \rho_j^N \left(\sum_{p\in P_j^{3N}} \left(x_p^{3*} + y_p^{3*}\right) \right) \right. \\
&\qquad \left. - \frac{\partial \rho_j^N \left(\sum_{p\in P_j^{3N}} \left(x_p^{3*} + y_p^{3*}\right)\right)}{\partial q_j^N} \sum_{p\in P_j^{3N}} \left(x_p^{3*} + y_p^{3*}\right) - \sigma \lambda_{2j}^{3*} \right] \times \left[y_p^3 - y_p^{3*}\right] \\
&\quad + \sum_j \sum_{p\in P_j^{3R}} \left[\sum_{c\in L^{3R}} \frac{\partial e_c\left(\sum_{p\in P^{3R}} \delta_{cp} z_p^{3*}\right)}{\partial h_c^3} \delta_{cp} + \rho_j^R \left(\sum_{p\in P_j^{3R}} z_p^{3*} \right) \right. \\
&\qquad \left. + \frac{\partial \rho_j^R \left(\sum_{p\in P_j^{3R}} z_p^{3*}\right)}{\partial q_j^R} \sum_{p\in P_j^{3R}} z_p^{3*} - \mu \lambda_1^{3*} + \lambda_{2j}^{3*} \right] \times \left[z_p^3 - z_p^{3*}\right] \\
&\quad + \left[\mu \sum_{p\in P^{3R}} z_p^{3*} - \sum_{p\in P^{3N}} x_p^{3*} \right] \times \left[\lambda_1^3 - \lambda_1^{3*}\right] \\
&\quad + \sum_j \left[\sigma \sum_{p\in P_j^{3N}} \left(x_p^{3*} + y_p^{3*}\right) - \sum_{p\in P_j^{3R}} z_p^{3*} \right] \times \left[\lambda_{2j}^3 - \lambda_{2j}^{3*}\right] \geq 0, \\
&\qquad\qquad\qquad\qquad \forall X^3 = \left(x^3, y^3, z^3, \lambda_1^3, \lambda_2^3\right) \in R_+^{N_3+J+1},
\end{aligned} \tag{4.10}$$

where $N_3 = 2M^{3N} + M^{3R}$, λ_1^2 and $\lambda_2^2 = (\lambda_{21}^2 \lambda_{22}^2 \cdots \lambda_{2J}^2)^T$ are the vectors of Lagrange multipliers associated with constraints (4.4) and (4.5).

5. Discussion and Numerical Analysis

In this section, four sets of numerical examples are tested and discussed on the proposed models. The models are computed by using modified projection method [21] and implemented in MATLAB. The convergence criterion used is that the absolute value of the product flows between two successive iterations differed by no more than 10^{-5}. The constant step length within the modified projection method is set to 0.01. All solutions obtained satisfy the optimality conditions with good accuracy.

Example Set 1. We consider that a closed-loop supply chain network prior to horizontal merger consists of three firms, each with its manufacturing/remanufacturing plants, distribution/recovery centers, and a demand market. Then, Firm 1 and Firm 2 merge into a new firm. Finally, the three firms merge completely. The networks prior to merger, postpartial merger and complete merger, are depicted in Figure 4.

The cost function on the links for the closed-loop supply chain network prior to merger is as follow. The cost functions on the first layer links are $e_a(h_a^1) = 1.5(h_a^1)^2 + h_a^1$, $e_b(h_b^1) = 2(h_b^1)^2 + 1.5h_b^1$, and $e_c(h_c^1) = 0.2(h_c^1)^2$. The cost functions on the second layer links are $e_a(h_a^1) = 0.5(h_a^1)^2 + h_a^1$, $e_b(h_b^1) = 0.5(h_b^1)^2 + h_b^1$, and $e_c(h_c^1) = 0.3(h_c^1)^2 + 0.5h_c^1$. The cost functions on the third layer links are $e_a(h_a^1) = 0.5(h_a^1)^2$, $e_b(h_b^1) = 0.5(h_b^1)^2$, and $e_c(h_c^1) = 0.3(h_c^1)^2$. The cost functions on the fourth layer links are $e_a(h_a^1) = 0.1(h_a^1)^2$, $e_b(h_b^1) = 0.1(h_b^1)$, and $e_c(h_c^1) = 0.1(h_c^1)^2$. The sale price function is $\rho_1^N(q_1^N) = 160 - q_1^N$. The recovery price of EOL products at demand market is $\rho_1^R = 0.5q_1^R$.

When the first two firms merge, we set the cost on the new links connecting the first layer Node 1 with the second layer Nodes 1 and 2, equal to zero, and the cost functions on the new links connecting manufacturing/remanufacturing plants with distribution/recovery centers are identical to the cost functions on the links of the network prior to merger. When all firms merge, we assume that the cost on the new links connecting the first layer Node 1 with the second layer Nodes 1, 2, and 3, equal to zero, the cost functions of other new links are identical to the original functions.

Firstly, we set $\mu = 0.6$ and $\sigma = 0$. There are three forward paths and three reverse paths in the network prior to merger. The equilibrium results prior to merger can be obtained by variational inequality (2.10), and $x_p^1 = 0$ for all $p \in P^{1N}$, $y_p^1 = 15.44$ for all $p \in P^{1N}$, $z_p^1 = 0$ for all $p \in P^{1R}$. The revenue for each firm is 1755.30. The cost for each firm is 777.70, and the profit for each firm is 977.64. There are five forward paths and five reverse paths in the network after partial merger. The equilibrium results after partial merger can be obtained by variational inequality (3.10), and $x_p^2 = 0$ for all $p \in P^{2N}$, $z_p^2 = 0$ for all $p \in P^{2R}$. However, y_p^2 for all $p \in P^{2N}$ has changed, the manufactured product flow on each path p connecting Firm 2 with the demand market is 15.64, and the product flows on the other paths are 7.31. The revenue of the new Firm 1 is 3366.00, and the revenue of the new Firm 2 is 1800.40. The cost of the new Firm 1 is 1292.20, and the cost of the new Firm 2 is 797.40. The profit of the new Firm 1 is 2074.60, and the profit of the new Firm 2 is 1003.00. There are nine forward paths and nine reverse paths in the network after complete merger. The equilibrium results post complete merger can be obtained by variational inequality (4.10), and $x_p^3 = 0.00$ for all

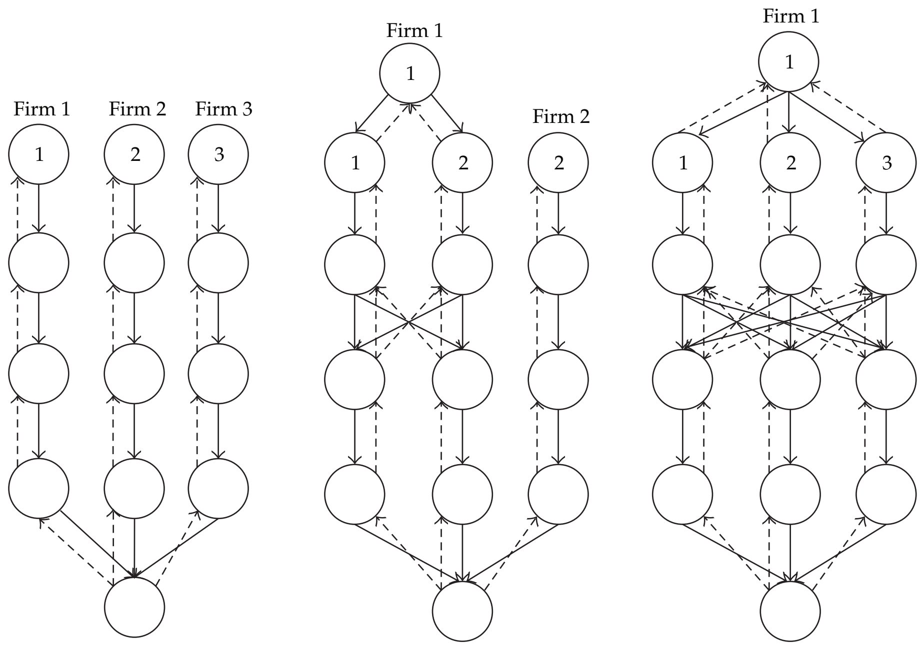

Figure 4: The networks prior to merger, postpartial merger, and complete merger.

$p \in P^{3N}$, $y_p^3 = 4.55$ for all $p \in P^{3N}$, $z_p^3 = 0.00$ for all $p \in P^{3R}$. The total revenue is 4876.60. The total cost is 1650.20. The total profit is 3226.30.

Secondly, we set $\mu = 0.6$, $\sigma = 0.2$. The equilibrium results for the network prior to merger are $x_p^1 = 2.09$ for all $p \in P^{1N}$, $y_p^1 = 15.35$ for all $p \in P^{1N}$, $z_p^1 = 3.49$ for all $p \in P^{1R}$. The revenue for each firm is 1878.00. The cost for each firm is 815.10. The profit for each firm is 1062.90. The equilibrium results for the network post partial merger are obtained. The remanufactured product flow on each forward path p connecting the new Firm 2 with the demand market is 2.80, and the product flows on the other paths are 0.81, respectively. The manufactured product flow on each forward path p connecting the new Firm 2 with the demand market is 15.42, and the product flows on the other path are 7.18. The EOL product flow on each reverse path p connecting the new Firm 2 with the demand market is 4.67, and the product flows on the other paths are 1.34. The revenue of the new Firm 1 is 3509.30, and the revenue of the new Firm 2 is 2001.60. The cost of the new Firm 1 is 1307.50, and the cost of the new Firm 2 is 847.40. The profit of the new Firm 1 is 2201.80, and the profit of the new Firm 2 is 1154.20. The equilibrium results for the network after complete merger are $x_p^3 = 0.60$ for all $p \in P^{3N}$, $y_p^3 = 4.36$ for all $p \in P^{3N}$ and $z_p^3 = 0.99$ for all $p \in P^{3R}$. The total revenue is 5149.20. The total cost is 1616.10. The total profit is 3533.10.

Finally, we set $\mu = 0.6$, $\sigma = 0.7$. The equilibrium results for the network prior to merger are $x_p^1 = 5.25$ for all $p \in P^{1N}$, $y_p^1 = 13.38$ for all $p \in P^{1N}$, $z_p^1 = 8.74$ for all $p \in P^{1R}$. The revenue for each firm is 1939.60. The cost for each firm is 858.59. The profit for each firm is 1081.00. The equilibrium results for the network post partial merger are obtained. The remanufactured product flow on each forward path p connecting the new Firm 2 with the demand market is 5.61, and the product flows on the other paths are 2.18. The manufactured product flow on each forward path p connecting the new Firm 2 with the demand market is 13.74, and the product flows on the other paths are 6.22. The EOL product flow on each reverse path p connecting the new Firm 2 with the demand market is 9.34, and the product flows on the other paths are 3.63. The revenue of the new Firm 1 is 3597.20, and the revenue of the new Firm 2 is 2071.20. The cost of the new Firm 1 is 1311.40, and the cost of the new Firm 2 is 907.40. The profit of the new Firm 1 is 2285.90, and the profit of the new Firm 2 is 1163.80. The equilibrium results for the network after complete merger are $x_p^3 = 1.34$ for all $p \in P^{3N}$, $y_p^3 = 3.85$ for all $p \in P^{3N}$, and $z_p^3 = 2.24$ for all $p \in P^{3R}$. The total revenue is 5296.00. The total cost is 1638.60. The total profit is 3657.50.

The other equilibrium results including the demand quantity, the quantity of recovered EOL products, the prices, the total revenues, the total costs, and the total profits for the networks prior to merger, postpartial merger, and complete merger from three different initial parameter setting are presented in Table 4.

It is important to note that, in each of parameter setting, all firms gained from the partial merger. The merged Firm 1 has higher total profit than the sum of those two original firms' profits, and the unmerged Firm 2 (original Firm 3) obtains also more profit. It is also worth noting that the total revenues and the total costs are lower in these partial merger examples. Moreover, the new firm after complete merger obtains more profit than the sum of those three original firms' profits, also more than the sum of the two firms' profits in partial merger case. In addition, in every example for each parameter setting, the demand for remanufactured and manufactured products, the quantity of recovered EOL products, and the recovery price decrease slightly; however, the demand price at demand market increases.

Example Set 2. The second set of examples uses the same network and cost functions as in Example Set 1 but assumes that different μ and σ are faced. We assume that μ increases from 0.1 to 0.5 and increases to 0.8. The ratio σ is set to 0.6.

When $\mu = 0.1$, the revenue for each firm prior to merger is 1764.90, the cost for each firm is 783.74, and the profit for each firm is 981.17. The revenue of the new Firm 1 after partial merger is 3377.30, and the revenue of the new Firm 2 is 1814.70. The cost of the new Firm 1 after partial merger is 1295.60, and the cost of the new Firm 2 is 805.00. The profit of the new Firm 1 after partial merger is 2081.80, and the profit of the new Firm 2 is 1009.70. The revenue for the firm after complete merger is 4894.30, its cost is 1651.10, and its profit is 3243.30.

When $\mu = 0.5$, the revenue for each firm prior to merger is 1906.80, the cost for each firm is 850.09, and the profit for each firm is 1056.70. The revenue of the new Firm 1 after partial merger is 3551.70, and the revenue of the new Firm 2 is 2023.20. The cost of the new Firm 1 after partial merger is 1315.50, and the cost of the new Firm 2 is 893.50. The profit of the new Firm 1 after partial merger is 2236.20, and the profit of the new Firm 2 is 1129.70. The revenue for the firm after complete merger is 5195.10, its cost is 1630.90, and its profit is 3564.20.

When $\mu = 0.8$, the revenue for each firm prior to merger is 1988.80, the cost for each firm is 863.70, and the profit for each firm is 1125.00. The revenue of the new Firm 1 after partial merger is 3670.40, and the revenue of the new Firm 2 is 2143.90. The cost of the new Firm 1 post partial merger is 1294.40, and the cost of the new Firm 2 is 921.20. The profit of the new Firm 1 postpartial merger is 2376.00, and the profit of the new Firm 2 is 1222.60. The revenue for the firm after complete merger is 5412.00, its cost is 1588.30, and its profit is 3823.80.

The other equilibrium results including the demand quantity, the quantity of recovered EOL products, the equilibrium prices, the total revenues, the total costs, and the total profits for the networks prior to merger, postpartial merger, and complete merger from three different initial parameter settings are presented in Table 5.

Table 4: Results of Example Set 1.

Results	$\mu = 0.6$ $\sigma = 0$			$\mu = 0.6$ $\sigma = 0.2$			$\mu = 0.6$ $\sigma = 0.7$		
	Prior merger	Partial merger	Complete merger	Prior merger	Partial merger	Complete merger	Prior merger	Partial merger	Complete merger
q_1^N	46.33	44.89	40.97	52.32	50.18	44.63	55.89	52.95	46.77
q_1^R	0	0	0	10.47	10.04	8.93	26.23	23.86	20.17
ρ_1^N	113.67	115.11	119.03	107.68	109.82	115.37	104.11	107.05	113.23
ρ_1^R	0	0	0	5.23	5.02	4.46	13.11	11.93	10.08
Total revenue	5265.90	5167.30	4876.60	5634.00	5510.90	5149.20	5818.80	5668.40	5296.00
Total cost	2333.10	2089.60	1650.20	2445.29	2154.90	1616.10	2575.77	2218.80	1638.60
Total profit	2932.92	3077.60	3226.30	3188.70	3356.00	3533.10	3243.00	3449.70	3657.50

Table 5: Results of Example Set 2.

Results	$\mu = 0.1$ $\sigma = 0.6$			$\mu = 0.5$ $\sigma = 0.6$			$\mu = 0.8$ $\sigma = 0.6$		
	Prior merger	Partial merger	Complete merger	Prior merger	Partial merger	Complete merger	Prior merger	Partial merger	Complete merger
q_1^N	46.75	45.24	40.97	53.93	51.27	45.44	59.17	55.80	48.98
q_1^R	7.06	6.27	0	25.03	22.66	18.62	26.42	24.23	20.87
ρ_1^N	113.25	114.76	119.03	106.07	108.73	114.56	100.83	104.20	111.02
ρ_1^R	3.53	3.13	0	12.51	11.33	9.31	13.21	12.12	10.44
Total revenue	5294.70	5192.00	4876.60	5720.40	5574.90	5205.30	5966.40	5814.30	5437.70
Total cost	2351.23	2100.60	1650.20	2550.27	2209.00	1641.50	2591.11	2215.60	1615.70
Total profit	2943.50	3091.50	3226.30	3170.10	3365.90	3563.80	3375.00	3598.60	3822.00

Due to the increased μ, all firms choose to recover more products. As a result, market share increases, and the total profits for the networks prior to merger, postpartial merger, and postcomplete merger are increased.

Example Set 3. The example uses the same network and cost functions as in Example Set 1, $\mu = 0.5$, $\sigma = 0.6$, but assumes that the cost functions of the new links connecting the first layer Node 1 with the second layer Nodes 1 and 2 are $e_a(h_a^2) = gh_a^2$, $e_b(h_b^2) = gh_b^2$, $e_c(h_c^2) = gh_c^2$, which can be regarded as the cost of horizontal merger including direct costs and indirect costs associated with merger. In order to analyze what impact the costs of horizontal merger will have on profits of the new Firm 1 and Firm 2 after partial merger, it is feasible to vary the parameter g from 0 to 10, then to 20, and so on, which means different from the cost of horizontal merger. The results from different parameter settings are presented in Figure 5. We find that the parameter is set to 140 then the profit of the new Firm 1 after partial merger is zero and it makes no economic sense for such a merger, but the profit of the new Firm 2 reach the maximum 1905.20. With further experimentation, note that the parameter is equal to 2, then the profit of the new Firm 1 after partial merger is 2138.90 that is greater than the sum of the profits for the Firm 1 and Firm 2 prior to merger (cf. Example Set 2). Thus, the parameter is equal to or smaller than 2 then, the merged firms can obtain gain from such a merger. However, after the parameter is set to 3, the profit of the new Firm 1 after partial merger is 2091.70, which is smaller than the sum of the profits for the Firm 1 and Firm 2 prior to merger (cf. Example Set 2). Therefore, the parameter is equal to or greater than 3, then the merged firms cannot obtain profit growth from such a merger.

Example Set 4. The example uses the same network and cost functions as in Example Set 1, $\mu = 0.8$, $\sigma = 0.6$, but assumes that the cost functions of the new links connecting the first layer Node 0 with the second lager Nodes 1, 2, and 3 are $e_a(h_a^2) = gh_a^2$, $e_b(h_b^2) = gh_b^2$, $e_c(h_c^2) = gh_c^2$. Further, we analyze what impact the costs of horizontal merger will have on profits of the new firm after complete merger. Similarly, we vary the parameter g from 0 to 10, then to 20, and so on. The results from different parameter settings are presented in Figure 6. We find that the parameter is set to 160 then the profit of the new firm after complete merger is zero and the merger has no economic sense. Further, after the parameter is set to 6, the profit of the new firm after complete merger is 3429.10, which is still greater than the sum of the profits for all firms prior to merger (cf. Example Set 2). Thus, the parameter is equal to or smaller than 6, then, the merged firms can obtain gain from such a complete merger. However, after the parameter is set to 7, the profit of the new firm is 3366.60, which is smaller than the sum of the profits for all firms prior to merger (cf. Example Set 2). Therefore, the parameter is equal to or greater than 7, then the merged firms cannot obtain profit growth from this complete merger.

6. Conclusion

This paper has studied the closed-loop supply chain network including a number of firms in the same industry. The network is represented by forward and reverse operation links including manufacturing/remanufacturing links, treatment links for EOL products, transportation links, and storage links. The economical models for the networks prior to merger, postpartial merger, and complete merger are developed. The research work has presented the variational inequality formulations of these models, whose solutions yield the production quantity of new products and remanufactured products, the product flows

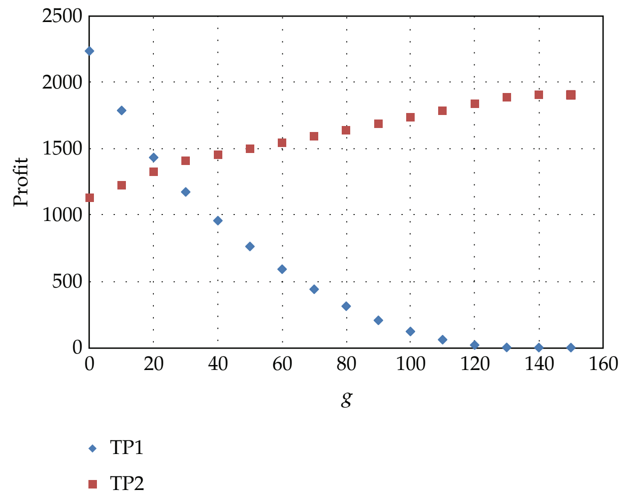

Figure 5: The total profits of the Firm 1 and Firm 2 after partial merger from different parameter settings.

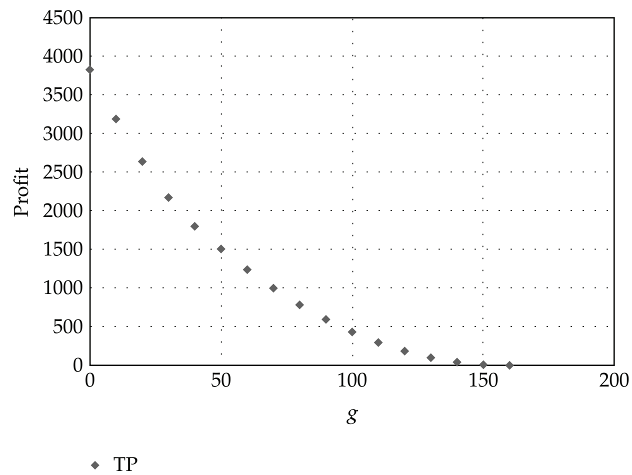

Figure 6: The profit of the Firm 1 after complete merger from different parameter settings.

for new products, remanufactured products and EOL products at every path, the demand quantity, the quantity of recovered EOL products, and the equilibrium prices. The modified projection method is applied for all numerical examples tested.

In addition, this paper can be extended in several directions, such as more merger forms and the models for multiproduct closed-loop supply chain network.

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