

## Research Article

# On Interval-Valued Supra-Fuzzy Syntopogenous Structure

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A new definition of interval-valued supra-fuzzy syntopogenous (resp., supra-fuzzy proximity) space is given. We show that for any interval-valued fuzzy syntopogenous structure  $S$ , there is another interval-valued fuzzy syntopogenous structure  $S^c$  called the conjugate of  $S$ . This leads to introducing the concept of interval-valued bifuzzy syntopogenous space. Finally, we show every interval-valued bifuzzy syntopogenous space induces an interval-valued fuzzy supra-syntopogenous space. Throughout this paper, the family of all fuzzy sets on nonempty set  $X$  will be denoted by  $I^X$  and the family of all interval-valued fuzzy sets on a nonempty set  $X$  will be denoted by  $\pi^X$ .

## 1. Introduction

In [1], Csaszar introduced the concept of a syntopogenous structure to develop a unified approach to the three main structures of set-theoretic-topology: topologies, uniformities, and proximities. This enables him to evolve a theory including the foundations of the three classical theories of topological spaces, uniform spaces, and proximity spaces. In the case of the fuzzy structures, there are at least three notions of fuzzy syntopogenous structures. The first notion worked out in [2–4] presents a unified approach to the theories of Chang fuzzy topological spaces [5], Hutton fuzzy uniform spaces [6], and Liu fuzzy proximity spaces [7]. The second notion worked out in [8] agrees very well with Lowen fuzzy topological spaces [9], Lowen-Hohle fuzzy uniform spaces [10], and Artico-Moresco fuzzy proximity spaces [11]. The third notion worked out in [12] agrees with the framework of a fuzzifying topology [13]. In [14], Kandil et al. introduced the concepts of a supra-fuzzy topological space and supra-fuzzy proximity space. In [15], Ghanim et al. introduced the concepts of supra-fuzzy syntopogenous space as a generalization of the concepts of supra-fuzzy proximity and supra-fuzzy topology.

Interval-valued fuzzy sets were introduced independently by Zadeh [16], Grattan-Guinness [17], Jahn [18], and Sambuc [19], in the seventies, in the same year. An interval-valued fuzzy set (IVF) is defined by an interval-valued membership function. As a generalization of fuzzy, the concept of intuitionistic fuzzy sets was introduced by Atanassov [20]. In [21], it is shown that the concepts of interval-valued fuzzy sets and intuitionistic fuzzy sets are equivalent and they are extensions of fuzzy sets.

Interval-valued fuzzy sets can be viewed as a generalization of fuzzy sets (Zadeh [22]) that may better model imperfect information which is omnipresent in any conscious decision making [23]. It can be considered as a tool for a more human consistent reasoning under imperfectly defined facts and imprecise knowledge, with an application in supporting medical diagnosis [24, 25]. In [26] new notions of interval-valued supra-fuzzy topology (resp., supra-fuzzy proximity) were introduced by using the notions of interval-valued fuzzy sets.

In this paper, we generalize the concept of supra-fuzzy syntopogenous space by using the notion of interval-valued set. Topology and its generalization proximity and syntopogenous are branches of mathematics which have many real life applications. We believe that the generalized topological structure suggested in this paper will be important base for modification of medical diagnosis, decision making, and knowledge discovery. The other parts of this paper are arranged as follows. In Section 2, We recall and develop some notions and notations concerning supra-fuzzy topological space, interval-valued fuzzy set, interval-valued fuzzy topology, interval-valued supra-fuzzy topology, and interval-valued supra-fuzzy proximity. In Section 3, we introduce the concept of interval-valued supra-fuzzy syntopogenous (resp., supra-fuzzy proximity) space. We show that there is one-to-one correspondence between family of all interval-valued supra-fuzzy topological spaces and the family of all perfect interval-valued supra-fuzzy topogenous spaces. Also, we prove that there is one-to-one correspondence between the family of all interval-valued supra-fuzzy proximity spaces and the family of all symmetrical interval-valued supra-fuzzy topogenous spaces. We show that for any interval-valued fuzzy topogenous order  $R$  on  $\pi^X$  there is another interval-valued fuzzy topogenous structure  $R^c$  in Section 3. In Section 4, we introduce the concept of interval-valued bifuzzy syntopogenous structure by using two interval-valued fuzzy syntopogenous spaces. In the last section we show that with every interval-valued bifuzzy syntopogenous space there is an interval-valued supra-fuzzy syntopogenous associated with this space.

## 2. Preliminaries

The concept of a supra-fuzzy topological space has been introduced as follows.

*Definition 2.1* (see [27]). A collection  $\tau^* \subset I^X$  is a supra-fuzzy topology on  $X$  if  $0, 1 \in \tau^*$  and  $\tau^*$  is closed under arbitrary supremum. The pair  $(X, \tau^*)$  is a supra-fuzzy topological space.

*Definition 2.2* (see [20]). An interval-valued fuzzy set (IVF set for short) is a set  $\underline{A} = (A_1, A_2) \in I^X \times I^X$  such that  $A_1 \leq A_2$ . The family of all interval-valued fuzzy sets on a given nonempty set  $X$  will be denoted by  $\pi^X$ .

The IVF set  $\underline{1} = (1, 1)$  is called the universal IVF set and the IVF set  $\underline{0} = (0, 0)$  is called the empty IVF set.

**Proposition 2.3** (see [20]). *The operations on  $\pi^X$  are given by the following: Let  $\underline{A}, \underline{B} \in \pi^X$ :*

- (1)  $\underline{A} = \underline{B} \Leftrightarrow A_1 = B_1, A_2 = B_2,$
- (2)  $\underline{A} \leq \underline{B} \Leftrightarrow A_1 \leq B_1, A_2 \leq B_2,$

- (3)  $\underline{A} \vee \underline{B} = (A_1 \vee B_1, A_2 \vee B_2)$ ,
- (4)  $\underline{A} \wedge \underline{B} = (A_1 \wedge B_1, A_2 \wedge B_2)$ ,
- (5)  $\underline{A}^c = (A_2^c, A_1^c)$ .

*Definition 2.4* (see [26]). The family  $\eta \subseteq \pi^X$  is called an interval-valued fuzzy topology (IVF topology for short) on  $X$  if and only if  $\eta$  contains  $\underline{1}, \underline{0}$ , and it is closed under finite intersection and arbitrary union. The pair  $(X, \eta)$  is called an interval-valued fuzzy topological space (IVF top. space for short). Any IVF set  $\underline{A} \in \eta$  is called an open IVF set and the complement of  $\underline{A}$  denoted by  $\underline{A}^c$  is called a closed IVF set. The family of all closed interval-valued fuzzy sets is denoted by  $\eta^c$ .

*Definition 2.5* (see [26]). A non empty family  $\eta^* \subseteq \pi^X$  is called an interval-valued supra-fuzzy topology (IVSF top. for short) on  $X$  if it contains  $\underline{0}, \underline{1}$  and it is closed under arbitrary unions.

*Definition 2.6* (see [26]). A binary relation  $\delta \subseteq \pi^X \times \pi^X$  is called an interval-valued supra-fuzzy (IVSF) proximity on  $X$  if it satisfies the following conditions:

- (1)  $\underline{A}\delta\underline{B} \Rightarrow \underline{B}\delta\underline{A}$ ,
- (2)  $\underline{A}\delta\underline{B}$  or  $\underline{A}\delta\underline{C} \Rightarrow \underline{A}\delta(\underline{B} \vee \underline{C})$ ,
- (3)  $\underline{0} \not\delta \underline{1}$ ,
- (4)  $\underline{A} \not\delta \underline{B} \Rightarrow \underline{A} \leq 1 - \underline{B}$ ,
- (5)  $\underline{A} \not\delta \underline{B} \Rightarrow \exists C \in \pi^X$  s.t.  $\underline{A}\delta\underline{C}$  and  $(1 - \underline{C})\not\delta\underline{B}$ .

### 3. Interval-Valued Fuzzy Topogenous Order

In this article, we define a new concept of topogenous structure by using the concept of interval-valued fuzzy sets.

*Definition 3.1.* Let  $R$  be a binary relation on  $\pi^X$ . Consider the following axioms:

- (1)  $\underline{0}R\underline{0}$ ,
- (2)  $\underline{A}R\underline{B} \Rightarrow \underline{A} \subseteq \underline{B}$ ,
- (3)  $\underline{A}_1 \leq \underline{A}R\underline{B} \leq \underline{B}_1 \Rightarrow \underline{A}_1R\underline{B}_1$ ,
- (4)  $\underline{A}_1R\underline{B}_1$  and  $\underline{A}_2R\underline{B}_2 \Rightarrow (\underline{A}_1 \vee \underline{A}_2)R(\underline{B}_1 \vee \underline{B}_2)$  and  $(\underline{A}_1 \wedge \underline{A}_2)R(\underline{B}_1 \wedge \underline{B}_2)$ ,
- (5)  $\underline{A}_iR\underline{B}_i$  for all  $i \Rightarrow (\vee \underline{A}_i)R(\vee \underline{B}_i)$ ,
- (6)  $\underline{A}_iR\underline{B}_i$  for all  $i \Rightarrow (\wedge \underline{A}_i)R(\wedge \underline{B}_i)$ ,
- (7)  $\underline{A}R\underline{B} \Leftrightarrow (1 - \underline{B})R(1 - \underline{A})$ .

If  $R$  satisfies (1)–(4) then it is called an interval-valued fuzzy topogenous order (IVF topogenous order for short) on  $X$ .

If  $R$  satisfies (1)–(3) then it is called an interval-valued supra-fuzzy topogenous order (IVSF topogenous order for short) on  $X$ .

The interval-valued fuzzy topogenous order is called perfect (resp., biperfect, symmetrical) if it satisfies the condition (5) (resp., (6), (7)).

*Definition 3.2.* Let  $X \neq \emptyset$  and  $S$  be a nonempty family of IVF topogenous orders on  $X$  satisfy the following conditions:

$$(s_1) R_1, R_2 \in S \Rightarrow \exists R \in S \text{ s.t. } R_1 \leq R \text{ and } R_2 \leq R,$$

$$(s_2) R \in S \Rightarrow \exists L \in S \text{ s.t. } R \leq L \circ L.$$

Then  $S$  is said to be interval-valued fuzzy (IVF) syntopogenous structure on  $X$ . The pair  $(X, S)$  is called an interval-valued fuzzy syntopogenous (IVF syntopogenous for short) space. If  $S$  satisfy the single element, then  $S$  is called an IVF topogenous structure on  $X$  and the pair  $(X, S)$  is called an IVF topogenous space.

*Definition 3.3.* Let  $X \neq \emptyset$  and  $S^*$  be a nonempty family of IVSF topogenous orders on  $X$  satisfy the conditions  $(s_1)$  and  $(s_2)$ . Then  $S^*$  is called an IVSF syntopogenous space. If  $S^*$  consists of single element, then it is called an IVSF topogenous structure. If each element of  $S^*$  is perfect (resp., biperfect symmetrical), then it is called a perfect (resp., biperfect symmetrical) IVSF structure on  $X$ .

The following proposition investigates the relation between the family of all IVSF topological spaces and the family of all perfect IVSF topogenous spaces.

**Proposition 3.4.** *There is one-to-one correspondence between the family of all IVSF topological spaces and the family of all perfect IVSF topogenous spaces.*

*Proof.* Let  $(X, \tau^*)$  be an IVSF topological space. Define a relation  $R_{\tau^*}$  by  $\underline{A}R_{\tau^*}\underline{B} \Leftrightarrow \exists C \in \tau^* \text{ s.t. } \underline{A} \leq C \leq \underline{B}$ . Then  $R_{\tau^*}$  is a perfect IVSF topogenous order on  $X$ . In fact, since  $\underline{0}, \underline{1} \in \tau^*$ ,  $\underline{0} \leq \underline{0} \leq \underline{0}$ , and  $\underline{1} \leq \underline{1} \leq \underline{1}$ , then  $\underline{0}R_{\tau^*}\underline{0}$  and  $\underline{1}R_{\tau^*}\underline{1}$ . Let  $\underline{A}R_{\tau^*}\underline{B} \Rightarrow \exists C \in \tau^* \text{ s.t. } \underline{A} \leq C \leq \underline{B} \Rightarrow \underline{A} \leq \underline{B}$ . Assume that  $\underline{A}_1 \leq \underline{A}R_{\tau^*}\underline{B} \leq \underline{B}_1 \Rightarrow \exists E \in \tau^* \text{ s.t. } \underline{A}_1 \leq \underline{A} \leq \underline{E} \leq \underline{B} \leq \underline{B}_1 \Rightarrow \underline{A}_1R_{\tau^*}\underline{B}_1$ . Thus,  $R_{\tau^*}$  is an IVSF topogenous order on  $X$ . We show that  $R_{\tau^*}$  is perfect. Let  $\underline{A}_iR_{\tau^*}\underline{B}_i \Rightarrow \exists \underline{C}_i \in \tau^* \text{ s.t. } \underline{A}_i \leq \underline{C}_i \leq \underline{B}_i$ . Hence  $\forall \underline{C}_i \in \tau^*$  and  $\forall \underline{A}_i \leq \underline{C}_i \leq \underline{B}_i$ . Thus,  $(\forall \underline{A}_i)R_{\tau^*}(\forall \underline{B}_i)$  and consequently  $R_{\tau^*}$  is perfect. Thus  $R_{\tau^*}$  is an IVSF topogenous order. Consequently,  $S_{\tau^*} = \{R_{\tau^*}\}$  is a perfect IVSF topogenous structure.

Conversely, let  $(X, S^*)$  be a perfect IVSF topogenous space. Then  $S^* = \{R^*\}$ , where  $R^*$  is a perfect IVSF order. Define  $\tau_{R^*} \subseteq \pi^X$  by  $\underline{A} \in \tau_{R^*} \Leftrightarrow \underline{A}R^*\underline{A}$ . We show that  $\tau_{R^*}$  is an IVSF topology on  $X$ . Since  $\underline{0}R^*\underline{0}$  and  $\underline{1}R^*\underline{1}$ , then  $\underline{0}, \underline{1} \in \tau_{R^*}$ . Also,  $\underline{A}_i \in \tau_{R^*}$  for all  $i \Rightarrow \underline{A}_iR^*\underline{A}_i$  for all  $i \Rightarrow \forall \underline{A}_iR^*\underline{A}_i$ , because  $R^*$  is perfect. Hence  $\forall \underline{A}_i \in \tau_{R^*}$  and, consequently,  $\tau_{R^*}$  is an IVSF topology on  $X$ .  $\square$

*Remark 3.5.* One can easily show that  $R_{\tau_{R^*}} = R^*$  and  $\tau_{R_{\tau^*}} = \tau^*$ .

*Example 3.6.* This example is a small form of interval-valued information table of a file containing some patients  $X = \{Li, Wang, Zhang, Sun\}$ . We consider the set of symptoms  $S = \{Chest-pain, Cough, Stomach-pain, Headache, Temperature\}$ . Each symptom is described by an interval fuzzy sets on  $X$ . The symptoms are given in Table 1. Define a binary relation  $R$  on  $\pi^X$  by  $A, B \in \pi^X ARB$  if and only if  $x \leq y$  implies  $A(x) \leq B(y)$  for all  $x \in X$ . Therefore,  $R$  is biperfect IVSF topogenous order on  $X$ . The set of patients are ordered by  $Li \leq Wang \leq Zhang \leq Sun$ . So, we have chest-pain  $R$  Cough.

The following proposition shows the relation between the family of all IVSF proximity spaces and the family of all symmetrical IVSF topogenous spaces.

Table 1

	Chest-pain	Cough	Stomach-pain	Headache	Temperature
Li	[0.1, 0.2]	[0.1, 0.3]	[0, 0.2]	[0.3, 0.9]	[0.1, 0.9]
Wang	[0.1, 0.2]	[0.2, 0.3]	[0.3, 0.9]	[0.3, 0.6]	[0.2, 0.2]
Zhang	[0.5, 0.5]	[0.2, 0.3]	[0.4, 0.4]	[0.1, 0.9]	[0.1, 0.9]
Sun	[0.3, 0.6]	[0.7, 0.8]	[0.3, 0.6]	[0.1, 0.6]	[0.3, 0.4]

**Proposition 3.7.** *There is one-to-one correspondence between the family of all IVSF proximity spaces and the family of all symmetrical IVSF topogenous spaces.*

*Proof.* Let  $\delta^*$  be an IVSF proximity. Define  $\underline{A}R_{\delta^*}\underline{B} \Leftrightarrow \underline{A}\delta^*(1 - \underline{B})$ . Then  $R_{\delta^*}$  is a symmetrical IVSF topogenous order. In fact, since  $0\delta^*1$  and  $1\delta^*0$ , then  $0R_{\delta^*}0$  and  $1R_{\delta^*}1$ . Also  $\underline{A}R_{\delta^*}\underline{B} \Rightarrow \underline{A}\delta^*(1 - \underline{B}) \Rightarrow \underline{A} \leq \underline{B}$ . Finally,  $\underline{A}_1 \leq \underline{A}R_{\delta^*}\underline{B} \leq \underline{B}_1 \Rightarrow \underline{A}_1 \leq \underline{A}$ ,  $\underline{A}\delta^*(1 - \underline{B})$  and  $1 - \underline{B} \geq 1 - \underline{B}_1$ . Thus  $\underline{A}_1\delta^*(1 - \underline{B}_1)$  and therefore  $\underline{A}_1R_{\delta^*}\underline{B}_1$ . Consequently  $(X, S^*)$ , where  $S^* = \{R_{\delta^*}\}$  is IVSF topogenous space.

Conversely, let  $(X, S^*)$  a symmetrical IVSF topogenous space. Then  $S^* = \{R^*\}$ , where  $R^*$  is a symmetrical IVSF topogenous order on  $X$ . Define  $\delta_{R^*}$  by  $\underline{A}\delta_{R^*}\underline{B} \Leftrightarrow \underline{A}R^*(1 - \underline{B})$ . We show that  $\delta_{R^*}$  is an IVSF topogenous order on  $X$ . Assume that  $\underline{A}\delta_{R^*}\underline{B}$ , then  $\underline{A}R^*(1 - \underline{B})$ . Therefore  $\underline{B}R^*(1 - \underline{A})$  because  $R^*$  is symmetric. Hence  $\underline{B}\delta_{R^*}\underline{A}$ . Since  $0R^*0$  and  $1R^*1$ , then  $0\delta_{R^*}0$ . Assume that  $\underline{A}\delta_{R^*}\underline{B}$  or  $\underline{A}\delta_{R^*}\underline{C}$ . Then  $\underline{A}R^*(1 - \underline{B})$  or  $\underline{A}R^*(1 - \underline{C})$ . Suppose that  $\underline{A}\delta_{R^*}\underline{B} \vee \underline{C}$ , then  $\underline{A}R^*(1 - (\underline{B} \vee \underline{C}))$ . Thus  $\underline{A}R^*(1 - \underline{B}) \wedge (1 - \underline{C})$ . Consequently  $\underline{A}R^*(1 - \underline{B})$  and  $\underline{A}R^*(1 - \underline{C})$ . So,  $\underline{A}\delta_{R^*}\underline{B}$  and  $\underline{A}\delta_{R^*}\underline{C}$  a contradiction. Now  $\underline{A}\delta_{R^*}\underline{B}$  implies  $\underline{A}R^*(1 - \underline{B})$  and hence  $\underline{A} \leq (1 - \underline{B})$ .  $\square$

#### 4. Interval-Valued Bifuzzy Syntopogenous Space

**Proposition 4.1.** *Let  $R$  be an IVF topogenous order on  $X$ . Then the relation  $R^c$  on  $\pi^X$  defined by  $\underline{A}R^c\underline{B} \Leftrightarrow (1 - \underline{B})R(1 - \underline{A})$  is an IVF topogenous order on  $X$ .*

*Proof.* Since  $0R0$  and  $1R1$ , then  $1R^c1$  and  $0R^c0$ . Also,  $\underline{A}R^c\underline{B} \Rightarrow (1 - \underline{B})R(1 - \underline{A}) \Rightarrow 1 - \underline{B} \leq 1 - \underline{A} \Rightarrow \underline{A} \leq \underline{B}$ . Furthermore,  $\underline{A}_1 \leq \underline{A}R^c\underline{B} \leq \underline{B}_1 \Rightarrow (1 - \underline{B})R(1 - \underline{A})$ ,  $1 - \underline{B}_1 \leq 1 - \underline{B}$ ,  $1 - \underline{A} \leq 1 - \underline{A}_1 \Rightarrow (1 - \underline{B}_1)R(1 - \underline{A}_1) \Rightarrow \underline{A}_1R^c\underline{B}_1$ . Finally,  $\underline{A}_1R^c\underline{B}_1$  and  $\underline{A}_2R^c\underline{B}_2 \Rightarrow (1 - \underline{B}_1)R(1 - \underline{A}_1)$  and  $(1 - \underline{B}_2)R(1 - \underline{A}_2)$ . Since  $R$  is a topogenous order, then  $(1 - \underline{B}_1) \wedge (1 - \underline{B}_2)R(1 - \underline{A}_1) \wedge (1 - \underline{A}_2)$  and hence  $(1 - (\underline{B}_1 \vee \underline{B}_2))R(1 - (\underline{A}_1 \vee \underline{A}_2))$ . Consequently,  $(\underline{A}_1 \vee \underline{A}_2)R^c(\underline{B}_1 \vee \underline{B}_2)$ . Similarly, we can show that  $(\underline{A}_1 \wedge \underline{A}_2)R^c(\underline{B}_1 \wedge \underline{B}_2)$ . Thus  $R^c$  is an IVF topogenous order on  $X$ .  $\square$

*Remark 4.2.* (1) The relation  $R^c$  defined in the previous proposition is called the conjugate of  $R$ .

(2) If  $R$  is perfect or biperfect, then  $R^c$  is also.

**Proposition 4.3.** *Let  $R_1$  and  $R_2$  be two IVF topogenous orders on  $X$ . If  $R_1 \subseteq R_2$ , then  $R_1^c \subseteq R_2^c$  and  $(R_1 \circ R_2)^c = R_2^c \circ R_1^c$ .*

*Proof.* Assume that,  $R_1 \subseteq R_2$ . Then  $\underline{A}R_1^c\underline{B} \Rightarrow (1 - \underline{B})R_1(1 - \underline{A}) \Rightarrow (1 - \underline{B})R_2(1 - \underline{A}) \Rightarrow \underline{A}R_2^c\underline{B}$ . Hence,  $R_1^c \subseteq R_2^c$ . Also,  $\underline{A}(R_1 \circ R_2)^c\underline{B} \Rightarrow (1 - \underline{B})(R_1 \circ R_2)(1 - \underline{A}) \Rightarrow \exists \underline{C} \in \pi^X$  s.t.  $(1 - \underline{B})R_1\underline{C} \wedge \underline{C}R_2(1 - \underline{A}) \Rightarrow \exists \underline{C} \in \pi^X$  s.t.  $(1 - \underline{C})R_1^c\underline{B} \wedge \underline{A}R_2^c(1 - \underline{C}) \Rightarrow \exists 1 - \underline{C} \in \pi^X$  s.t.  $\underline{A}R_2^c(1 - \underline{C}) \wedge (1 - \underline{C})R_1^c\underline{B} \Rightarrow \underline{A}(R_2^c \circ R_1^c)\underline{B} \Rightarrow (R_1 \circ R_2)^c \leq R_2^c \circ R_1^c$ . Similarly, we can show that  $R_2^c \circ R_1^c \leq (R_1 \circ R_2)^c$ . Consequently,  $(R_1 \circ R_2)^c = R_2^c \circ R_1^c$ .  $\square$

**Proposition 4.4.** *Let  $S$  be an IVF syntopogenous structure on  $X$ . Then the family  $S^c = \{R^c : R \in S\}$  is also an IVF syntopogenous structure.*

*Proof.* Since  $S \neq \emptyset$ , then  $S^c \neq \emptyset$ . Let  $R_1^c, R_2^c \in S^c$ , then  $R_1, R_2 \in S$ . Hence  $\exists R \in S$  s.t.  $R_1 \leq R$  and  $R_2 \leq R$ . Therefore  $\exists R^c \in S^c$  s.t.  $R_1^c \leq R^c$  and  $R_2^c \leq R^c$ . Also,  $R^c \in S^c \Rightarrow R \in S \Rightarrow \exists R_1 \in S$  s.t.  $R \leq (R_1 \circ R_1) \Rightarrow R^c \leq (R_1 \circ R_1)^c = R_1^c \circ R_1^c$ . Thus  $\exists R_1^c \in S^c$  s.t.  $R^c \leq R_1^c \circ R_1^c$ . Consequently,  $S^c$  is also IVF syntopogenous structure on  $X$ .

Hence, we have the result that on a nonempty set  $X$  we have two interval-valued fuzzy syntopogenous structures.  $\square$

*Definition 4.5.* Let  $X$  a nonempty set. Let  $S_1, S_2$  be two IVF syntopogenous structures on  $X$ . The triple  $(X, S_1, S_2)$  is called an interval-valued bifuzzy syntopogenous space. If each  $S_1, S_2$  is perfect (resp., biperfect symmetrical), then the space  $(X, S_1, S_2)$  is perfect (resp. biperfect symmetrical).

The following proposition shows that two IVF topogenous orders on  $X$  can induce an IVSF topogenous order on  $X$ .

**Proposition 4.6.** *Let  $R_1, R_2$  be two interval-valued fuzzy topogenous orders on  $X$ . Then the order  $R = R_1 \vee R_2$  defined by*

$$ARB \iff AR_1B \quad \text{or} \quad AR_2B \quad (4.1)$$

*is IVSF topogenous order on  $\pi^X$ .*

*Proof.* Since,  $0R_i0$  and  $1R_i1$  for all  $i = 1, 2$ , then  $0R0$  and  $1R1$ . Also,  $\underline{A}_1 \leq \underline{ARB} \leq \underline{B}_1 \Rightarrow \underline{A}_1 \leq \underline{AR}_i\underline{B} \leq \underline{B}_1$  for some  $i = 1, 2 \Rightarrow \underline{A}_1 R_i \underline{B}_1$  for some  $i = 1, 2 \Rightarrow \underline{A}_1 R B_1$ . Finally,  $\underline{ARB} \Rightarrow \underline{AR}_i\underline{B}$  for some  $i = 1, 2 \Rightarrow \underline{A} \leq \underline{B}$ . Thus,  $R$  is an IVSF topogenous order on  $\pi^X$ .  $\square$

## 5. Interval-Valued Supra-Fuzzy Syntopogenous Space Associated with Interval-Valued Bifuzzy Syntopogenous Spaces

**Proposition 5.1.** *Let  $(X, S_1, S_2)$  be interval-valued bifuzzy syntopogenous space. Then the family  $S_{12}^* = \{R = R_1 \vee R_2 : R_1 \in S_1, R_2 \in S_2\}$  is an IVSF syntopogenous structure on  $X$ .*

*Proof.* (s<sub>1</sub>) Let  $R, L \in S_{12}^* \Rightarrow R = R_1 \vee R_2$  and  $L = L_1 \vee L_2 R_i$  and  $L_i \in S_i$  for some  $i = 1, 2 \Rightarrow \exists K_1 \in S_1$  s.t.  $R_1 \vee L_1 = K_1$  and  $\exists K_2 \in S_1$  s.t.  $R_2 \vee L_2 = K_2$ . So,  $\exists K = K_1 \vee K_2 \in S_{12}^*$  s.t.  $R \vee L \leq K_1 \vee K_2 = K$ .

(s<sub>2</sub>)  $R \in S_{12}^* \Rightarrow \exists R_1 \in S_1, R_2 \in S_2$  s.t.  $R = R_1 \vee R_2 \Rightarrow \exists L_1 \in S_1$  s.t.  $R_1 \leq L_1 \circ L_1$  and  $\exists L_2 \in S_2$  s.t.  $R_2 \leq L_2 \circ L_2$ . Since  $L_1, L_2 \leq L_1 \vee L_2$ , then  $L_1 \circ L_1 \leq L_1 \vee L_2 \circ L_1 \vee L_2$  and  $L_2 \circ L_2 \leq L_1 \vee L_2 \circ L_1 \vee L_2$ . Therefore  $R_1 \leq L_1 \circ L_1 \leq L_1 \vee L_2 \circ L_1 \vee L_2$  and  $R_2 \leq L_2 \circ L_2 \leq L_1 \vee L_2 \circ L_1 \vee L_2$ . So  $R_1 \vee R_2 \leq (L_1 \vee L_2) \circ (L_1 \vee L_2)$  and consequently  $S_{12}^*$  is an IVSF syntopogenous structure on  $X$ .  $\square$

*Remark 5.2.* The structure  $S_{12}^*$  is called the interval-valued supra-fuzzy syntopogenous associated with the space  $(X, S_1, S_2)$ .

**Proposition 5.3.** *Let  $(X, T_i) i = 1, 2$  be an IVF topological spaces. Then the family  $T_{12}^* = \{\underline{A} = \underline{A}_1 \vee \underline{A}_2 : \underline{A}_i \in T_i \quad i = 1, 2\}$  is an IVSF topology on  $X$ .*

*Proof.* Since  $\underline{0}, \underline{1} \in T_i$  for all  $i = 1, 2$ , then  $\underline{0}, \underline{1} \in T_{12}^*$ . Also,  $\underline{A}_i \in T_{12}^* \Rightarrow \underline{A}_i = \underline{B}_1^i \vee \underline{B}_2^i$ ,  $\underline{B}_1^i \in T_1, \underline{B}_2^i \in T_2 \Rightarrow \vee_i \underline{A}_i = \vee_i (\underline{B}_1^i \vee \underline{B}_2^i) = (\vee_i \underline{B}_1^i) \vee (\vee_i \underline{B}_2^i)$ . Since,  $T_i$   $i = 1, 2$  is an IVSF topology, then  $(\vee_i \underline{B}_1^i) \in T_1$  and  $(\vee_i \underline{B}_2^i) \in T_2$ . Therefore,  $\vee_i \underline{A}_i \in T_{12}^*$  and consequently,  $T_{12}^*$  is an IVSF topology on  $X$ .  $\square$

**Proposition 5.4.** *Let  $(X, S_1, S_2)$  be an interval-valued bifuzzy syntopogenous space. Assume that  $T_i$  is the interval-valued fuzzy topology associated with  $S_i$  for all  $i = 1, 2$  and  $T_{12}^*$  is interval-valued supra-fuzzy topology associated with the space  $(X, T_1, T_2)$ . Then  $T_{12}^*$  is the IVSF topology associated with the space  $(X, S_{12}^*)$ , that is,  $T_{12}^* = T_{S_{12}^*}$ .*

*Proof.* The proof is obvious.  $\square$

*Remark 5.5.* In crisp case, let  $R$  be a fuzzy topogenous order on  $I^X$ . Define a binary relation  $R^p$  as follows:

$$\mu R^p \rho \iff \exists \text{ family } \{\mu_i : i \in I\} \quad \text{s.t. } \mu = \vee_i \mu_i, \mu_i R \rho \quad \forall i. \quad (5.1)$$

Then,  $R^p$  is a perfect fuzzy topogenous order on  $X$  finer than  $R$  and coarser than any perfect fuzzy semitopogenous order on  $I^X$  which is finer than  $R$ .

Let  $S$  be a fuzzy syntopogenous structure on  $X$ . Define a relation  $R_S$  on  $I^X$  by

$$\mu R^S \rho \iff \mu R \rho \quad \text{for some } R \in S. \quad (5.2)$$

The binary relation  $R^S$  as defined above is a perfect topogenous order on  $I^X$ . Let  $R^p$  be the coarsest perfect topogenous order on  $I^X$  finer than each member of  $S$ .  $(X, R_S^p)$  is the topogenous space. Let  $R_o = R_S^p$ .

## 6. Conclusion

Computer scientists used the relation concepts in many areas of life such as artificial intelligence, knowledge discovery, decision making and medical diagnosis [28]. One of the important theories depend on relations is the rough set theory [29]. Many authors studied the topological properties of rough set [30, 31]. In this paper we generalized the topological concepts by introducing the notions of IVSF syntopogenous structures. We can use the relations obtained from an interval-valued information systems to generate an IVSF syntopogenous structure and use the mathematical properties of these structures to discover the knowledge in the information modelings.

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