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THREE DIMENSIONAL GREEN'S FUNCTION FOR SHIP MOTION AT FORWARD SPEED

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ABSTRACT. The Green's function formulation for ship motion at forward speed contains double integrals with singularities in the path of integrations with respect to the wave number. In this study, the double integrals have been replaced by single integrals with the use of complex exponential integrals. It has been found that this analysis provides an efficient way of computing the wave resistance for three dimensional potential problem of ship motion with forward speed.

KEY WORDS AND PHRASES. Ship motion, Green's function, Hydrodynamics, Wave Resistance and Wave Responses.

1. INTRODUCTION.

In ship hydrodynamics, Green's functions play a very important role in predicting the wave resistance, wave induced responses at zero forward speed, and the motions of a vessel advancing in waves. The Green's function formulation for ship motions at forward speed is the most difficult part of the problem partly because it contains double integrals and partly because of the presence of the singularities in the path of integrations with respect to the wave numbers. Nowadays, considerable interest has been paid to evaluate the three dimensional Green's function for ship motions at forward speed.

Many authors including Haskind [1] and Havelock [2] have expressed the Green's function having a constant forward speed as a double integral. This form of Green's function is not suitable for numerical analysis because the detailed computation of the double integral is very expensive. Therefore, in the present study, we have replaced the double integral by a single integral (see Wu and Taylor [3]) involving a complex exponential integral, and it is found that it is more efficient to calculate the Green's function numerically.

2. A FORM OF THE GREEN'S FUNCTION.

Consider the coordinate system oxyz which is moving at constant forward speed U along the x axis and z measured positive upwards from the mean free surface (see Figure 1).

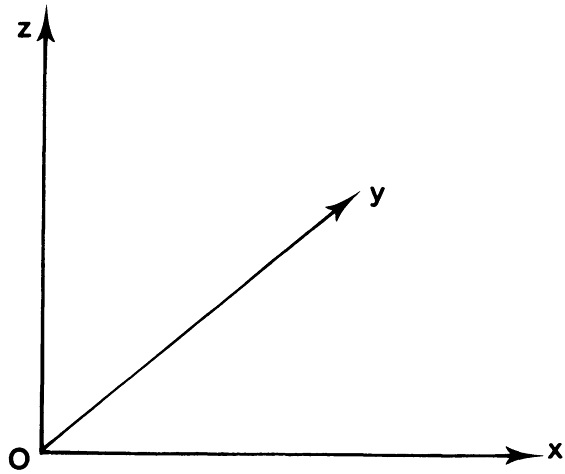


Figure 1: The Coodinate System

It is assumed that a ship is travelling at a constant forward speed U along the Ox direction and oscillating at a frequency ω as in the form of $e^{i\omega t}$. Wehausen and Laitone [4] in 1960 have shown that the Green's function which satisfies the exact free surface condition can be written as

$$G(x,y,z;a,b,c) = \frac{1}{R} - \frac{1}{R_1} + \frac{2g}{\pi}\int_o^{\gamma} d\theta \int_o^{\infty} F(\theta,k)dk + \frac{2g}{\pi}\int_{\gamma}^{\pi/2} d\theta \int_{L_1} F(\theta,k)dk$$

$$+ \frac{2g}{\pi}\int_{\pi/2}^{\pi} d\theta \int_{L_2} F(\theta,k)dk \tag{2.1}$$

where

$R = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$ Rankine source located at (a,b,c)

$R_1 = \sqrt{(x-a)^2 + (y-b)^2 + (z+c)^2}$ Image about the mean free surface at (a,b,-c)

g = acceleration due to gravity

$$F(\theta,k) = \frac{k\, e^{k[(z+c) + i(x-a)\cos\theta]}\cos[k(y-b)\sin\theta]}{gk-(\omega+kU\cos\theta)^2} \tag{2.2}$$

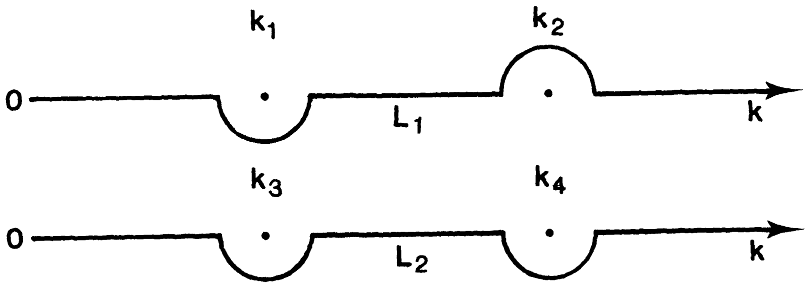
(x,y,z) is the field point and (a,b,c) is the source distribution point. The other parameters in Equation (2.1) are defined by

$$\gamma = o \qquad \text{if } \tau < \frac{1}{4} \tag{2.3}$$

$$\cos^{-1}\left(\frac{1}{t\tau}\right) \qquad \text{if } t > \frac{1}{4} \tag{2.4}$$

where $\tau = \frac{\omega U}{g}$ is called the Strouhal number

The contours L_1 and L_2 are defined as follows:



There are two singular points in L_1 and two singular points in the L_2 integral of Equation (2.1). These singular points can be obtained as follows:

$$\sqrt{gk_1},\ \sqrt{gk_3} = \frac{1 - \sqrt{1-4\tau\cos\theta}}{2\tau\cos\theta}\,\omega \tag{2.5}$$

$$\sqrt{gk_2},\ \sqrt{gk_4} = \frac{1 + \sqrt{1-4\tau\cos\theta}}{2\tau\cos\theta}\,\omega \tag{2.6}$$

The alternative forms of these singularities k_1, k_2, k_3 and k_4 can be written as

$$k_2,\ k_1 = \frac{(1 - 2\tau\cos\theta) \pm \sqrt{1-4\tau\cos\theta}}{2\tau^2\cos^2\theta}\,v \tag{2.8}$$

for $\pi/2 < \theta < \pi$; and where $v = \frac{\omega^2}{g}$.

It should be noted here that $k_1 = k_3$ and $k_2 = k_4$. These singularities are real in the ranges indicated. It is, however, worth mentioning here that in the range $0 < \theta < \gamma$, the singularities k_1 and k_2 become complex quantities and are either given by

$$\sqrt{gk_2},\ \sqrt{gk_1} = \frac{1 \pm i\sqrt{4\tau\cos\theta - 1}}{2\tau\cos\theta}\,\omega \tag{2.9}$$

or

$$k_2, k_1 = \frac{1 - 2\tau\cos\theta \pm i\sqrt{4\tau\cos\theta - 1}}{2\tau^2\cos^2\theta}\,v \tag{2.10}$$

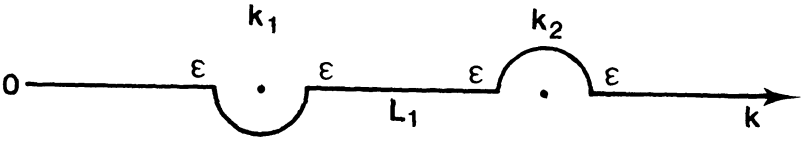
Thus the integrand in the integral

$$\int_0^{\gamma} \left[\int_0^{\infty} F(\theta,k)\,dk \right] d\theta$$

contains no real singular points in the path of integration from 0 to ∞.

3. EVALUATION OF INTEGRALS.

The Green's function given in equation (2.1) is difficult to integrate numerically because as we have seen in the previous section, the contours L_1 and L_2 have singularities at k_1, k_2, k_3 and k_4. This difficulty can be overcome by introducing the Cauchy Principal Value (PV) integrals.



The first contour integral along the path L_1 can be rewritten as

$$G_{L_1} = \frac{2g}{\pi} \int_{\gamma}^{\pi/2} d\theta \int_{L_1} F(\theta,k)dk$$

$$= \frac{2g}{\pi} \int_{\gamma}^{\pi/2} d\theta \, [\int_{o}^{k_1-\varepsilon} + \int_{\curvearrowleft} + \int_{k_1+\varepsilon}^{k_2-\varepsilon} \int_{\curvearrowright} + \int_{k_2+\varepsilon}^{\infty} F(\theta,k)dk \qquad (3.1)$$

When $\varepsilon \to 0$, equation (3.1) can be written as

$$G_{L_1} = \frac{2g}{\pi} \int_{\gamma}^{\pi/2} d\theta \, (P.V.) \int_{o}^{\infty} F(\theta,k)dk + \frac{2g}{\pi} \int_{\gamma}^{\pi/2} d\theta \, \{ \int_{\curvearrowleft} + \int_{\curvearrowright} \} F(\theta,k)dk \qquad (3.2)$$

where

$$(P.V.) \int_{o}^{\infty} F(\theta,k)dk = \text{Lim} \, \{ \int_{0}^{k_1-\varepsilon} + \int_{k_1+\varepsilon}^{k_2-\varepsilon} + \int_{k_2+\varepsilon}^{\infty} \} F(\theta,k)dk \quad \varepsilon \to 0 \qquad (3.3)$$

To evaluate the integral along the deformations $\curvearrowleft$ and $\curvearrowright$, we decompose the integral $F(\theta,k)$ in terms of its singularities. We write

$$F(\theta,k) = \frac{1}{g\sqrt{1-4\tau\cos\theta}} [\frac{k_1}{k-k_1} - \frac{k_2}{k-k_2}] \quad \exp\{k[(z+c) + i(x-a)\cos\theta]\} \cos(k(y-b)\sin\theta) \qquad (3.4)$$

which can be put in the following compact form

$$F(\theta,k) = \frac{1}{2g\sqrt{1-4\tau\cos\theta}} [\frac{k_1}{k-k_1} - \frac{k_2}{k-k_2}] \quad [\exp(kx_1) + \exp(kx_2)] \qquad (3.5)$$

where

$$x_1 = (z+c) + iw_+, \quad x_2 = (z+c) + i\, w.$$

$$w_+ = (x-a)\cos\theta + (y-b)\sin\theta, \quad w_- = (x-a)\cos\theta - (y-b)\sin\theta \qquad (3.6)$$

We know that $(z+c)<0$ so we can redefine x_1 and x_2 as follows

$$x_1 = - \{(z+c) - iw_+\}, \quad x_2 = - \{(z+c) - iw_-\}. \qquad (3.7)$$

Thus, equation (3.5) can be rewritten as

$$F(\theta,k) = \frac{1}{2g\sqrt{1-4\tau\cos\theta}} [\frac{k_1\{\exp(-kx_1) + \exp(-kx_2)\}}{k-k_1} - \frac{k_2\{\exp(-kx_1) + \exp(-kx_2)\}}{k-k_2}] \qquad (3.8)$$

The integration along the deformations $\curvearrowleft$ and $\curvearrowright$ in equation (3.2) can be obtained according to the residue theorem. Thus

$$\{ \int_{\curvearrowleft} + \int_{\curvearrowright} \} F(\theta,k)dk$$

$$= \frac{\pi i}{2g\sqrt{1-4\tau\cos\theta}} \{k_1(\exp(-k_1x_1) + \exp(-k_1x_2)) + k_2(\exp(-k_2x_1) + \exp(-k_2x_2))\} \qquad (3.9)$$

Thus, equation (3.9) reduces to

$$G_{L_1} = \frac{2g}{\pi} \int_{\gamma}^{\pi/2} d\theta \; (\text{P.V.}) \int_0^{\infty} F(\theta,k)dk$$
$$+ \frac{i}{\sqrt{1-4\tau\cos\theta}} \int_{\gamma}^{\pi/2} \{k_1(e^{-k_1 x_1} + e^{-k_1 x_2}) + k_2(e^{-k_2 x_1} + e^{-k_2 x_2})\} \, d\theta \tag{3.10}$$

In a similar manner the second contour integral along path L_2 equation (2.1) can be obtained

$$G_{L_2} = \frac{2g}{\pi} \int_{\pi/2}^{\pi} d\theta \; (\text{P.V.}) \int_0^{\infty} F(\theta,k)dk$$

$$+ \frac{i}{\sqrt{1-4\tau\cos\theta}} \int_{\pi/2}^{\pi} \{k_3(e^{-k_3 x_1} + e^{-k_3 x_2}) - k_4(e^{-k_4 x_1} + e^{-k_4 x_2})\} \tag{3.11}$$

Therefore the Greens function in equation (2.1) can be rewritten as follows:

$$G_1(x,y,z;\ a,b,c) = \frac{1}{R} - \frac{1}{R_1} + \frac{2g}{\pi} \int_0^{\gamma} d\theta \int_0^{\infty} F(\theta,k)dk$$
$$+ \frac{2g}{\pi} \left\{ \int_{\gamma}^{\pi/2} + \int_{\pi/2}^{\pi} \right\} d\theta \; (\text{P.V.}) \int_0^{\infty} F(\theta,k)dk$$

$$+ \frac{i}{\sqrt{1-4\tau\cos\theta}} \left\{ \int_{\gamma}^{\pi/2} \{k_1(e^{-k_1 x_1} + e^{-k_1 x_2}) + k_2(e^{-k_2 x_1} + e^{-k_2 x_2})\} \, d\theta \right.$$

$$\left. + \int_{\pi/2}^{\pi} \{k_3(e^{-k_3 x_1} + e^{-k_3 x_2}) - k_4(e^{-k_4 x_1} + e^{-k_4 x_2})\} \, d\theta \right\} \tag{3.12}$$

or,

$$G = G_1 + G_2 + G_3 + G_4 + iG_5 \tag{3.13}$$

where

$$G_1 = \frac{1}{R}, \quad G_2 = -\frac{1}{R_1}$$

$$G_3 = \frac{2g}{\pi} \int_0^{\gamma} d\theta \int_0^{\infty} F(\theta,k)dk \qquad G_4 = \frac{2g}{\pi} \left\{ \int_{\gamma}^{\pi/2} + \int_{\pi/2}^{\pi} \right\} d\theta \; (\text{P.V.}) \int_0^{\infty} F(\theta,k)dk$$

$$G_5 = \int_{\gamma}^{\pi/2} \frac{1}{\sqrt{1-4\tau\cos\theta}} \{k_1(e^{-k_1 x_1} + e^{-k_1 x_2}) + k_2(e^{-k_2 x_1} + e^{-k_2 x_2})\} \, d\theta$$

$$+ \int_{\pi/2}^{\pi} \frac{1}{\sqrt{1-4\tau\cos\theta}} \{k_3(e^{-k_3 x_1} + e^{-k_3 x_2}) - k_4(e^{-k_4 x_1} + e^{-k_4 x_2})\} \, d\theta \tag{3.14}$$

There are two cases to be considered.

Case I

$$\gamma = 0 \qquad \text{if } \tau < \frac{1}{4}$$

and in this case $G_3 = 0$.

Therefore, equation (3.13) becomes

$$G = G_1 + G_2 + G_4 + iG_5 \tag{3.15}$$

Case II

$$\gamma = \cos^{-1}\frac{1}{4\tau} \qquad \text{if } \tau > \frac{1}{4}$$

and in this case equation (3.13) becomes

$$G = G_1 + G_2 + G_3 + G_4 + iG_5 \tag{3.16}$$

The double integrals in G_3 and G_4 are highly oscillatory at large values of k because of the imaginary argument of the exponential function. In order to calculate them numerically, at minimum computer cost, these integrals must be reduced to single integrals as suggested by Shen and Farell [5], and Inglis and Price [6]. We shall treat Case I first and evaluate the Cauchy Principal Value (P.V.) integral in G_4.

The term G_4 of the Green function can be written as

$$G_4 = \frac{1}{\pi}\int_{\gamma}^{\pi/2}\frac{d\theta}{\sqrt{1-4\tau\cos\theta}}(I_1+I_2-I_3-I_4) \qquad + \frac{1}{\pi}\int_{\pi/2}^{\pi}\frac{d\theta}{\sqrt{1-4\tau\cos\theta}}(I_5+I_6-I_7-I_8) \tag{3.17}$$

where

$$I_1 = (\text{P.V.})\int_0^\infty \frac{k_1\exp(-kx_1)}{k-k_1}dk, \quad I_2 = (\text{P.V.})\int_0^\infty \frac{k_1\exp(-kx_2)}{k-k_1}dk \tag{3.18}$$

$$I_3 = (\text{P.V.})\int_0^\infty \frac{k_2\exp(-kx_1)}{k-k_2}dk, \qquad I_4 = (\text{P.V.})\int_0^\infty \frac{k_2\exp(-kx_2)}{k-k_2}dk$$

for $\gamma < \theta < \pi/2$

$$I_5 = (\text{P.V.})\int_0^\infty \frac{k_2\exp(-kx_1)}{k-k_3}dk, \qquad I_6 = (\text{P.V.})\int_0^\infty \frac{k_3\exp(-kx_1)}{k-k_3}dk \tag{3.19}$$

$$I_7 = (\text{P.V.})\int_0^\infty \frac{k_4\exp(-kx_1)}{k-k_4}dk \qquad I_8 = (\text{P.V.})\int_0^\infty \frac{k_4\exp(-kx_2)}{k-k_4}dk$$

for $\pi/2 < \theta < \pi$.

To obtain analytic expressions of these integrals, we consider a contour in the $K = k + ik'$ plane as suggested by Smith et al [7] (see Figure 2) and later used by Chen et al [8].

We impose the condition that

$$I_m[-Kx_1] = 0$$

on the integration path 5 which makes an angle α with the real axis, so that the argument of the exponential can be made real along the ray.

Therefore, we get

$$I_m[-(k + ik')(|z+c|-iw_+)] = 0$$

which simplifies to yield

$$\frac{k'}{k} = \frac{w_+}{|z+c|}$$

and

$$\alpha = \tan^{-1}\frac{k'}{k} = \tan^{-1}\frac{w_+}{|z+c|}$$

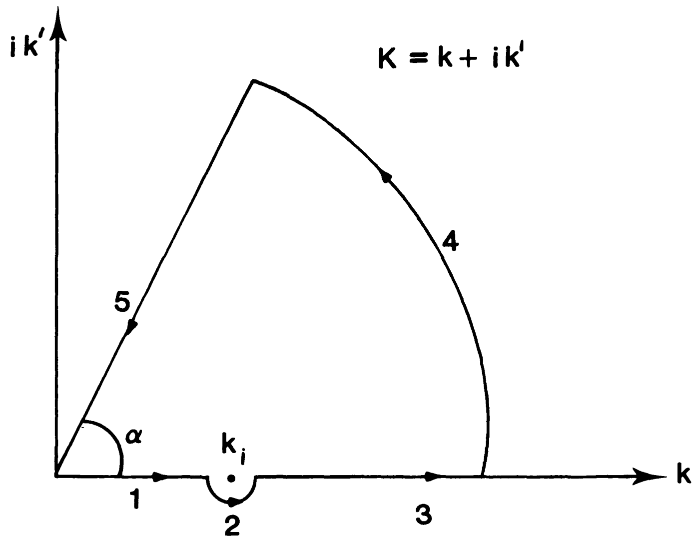


Figure 2: A closed contour for $w_+ > 0$

Thus with this value of α,

$$R_e[-(k + ik')(|z+c| - iw_+)] = -k\frac{|z+c|^2 + w_+^2}{|z+c|} = -kV < 0$$

Also, we have

$$K = k + ik' = \frac{kV}{|z+c| - iw_+}$$

Integrating along the contour shown in Figure 2,

$$I_1 = 2\pi i\ \{k_1\exp(-k_1x_1)\} - \pi i\ k_1\exp(-k_1x_1) - \int_5 \frac{k_1\exp(-kx)}{k-k_1}\,dk$$

$$= (\pi i)\ k_1\exp(-k_1x_1) - \int_5 \frac{k_1\exp(-kx_1)}{k-k_1}\,dk$$

Along the path 5

$$\int_5 = k_1 \int_\infty^0 \frac{\exp(-kV)}{(kV) - k_1x_1}\,d(kV) = -k_1 \int_0^\infty \frac{\exp(-u)}{u - k_1x_1}\,du$$

$$= -k_1\ \exp(-k_1x_1)\ E_1(-k_1x_1)$$

where

$$E_1(-z) = \int_{-z}^\infty \frac{e^{-t}}{t}\,dt,\ |\arg(z)| < \pi = \text{exponential integral.}$$

Therefore, for $w_+ > 0$,

$$I_1 = k_1\ \exp(-k_1x_1)\ \{E_1-(k_1x_1) + \pi i\} \tag{3.20}$$

It is to be noted here that for $w_+ < 0$ the contour will be as follows:

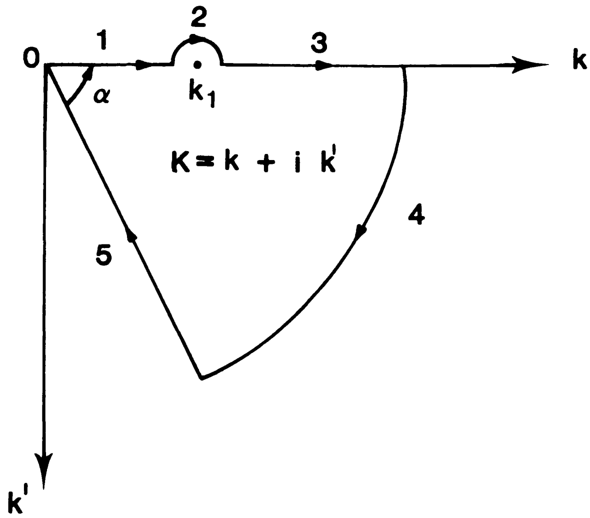


Figure 3: Closed contour for $w_+ < 0$

Thus

$$I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) - \pi i\} \tag{3.21}$$

Also, for $w_+ = 0$,

$$I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) + \pi i\} \tag{3.22}$$

which is obtained using the following definitions (see Abramowitz and Stegun [9] 1965, p. 228).

$$E_1(-x+i0) = (\text{P.V.}) \int_{-x}^{\infty} \frac{e^{-t}}{t} dt - \pi i, \quad E_1(-x-i0) = (\text{P.V.}) \int_{-x}^{\infty} \frac{e^{-t}}{t} dt + \pi i$$

such that for $w_+ = 0$

$$I_1 = k_1 \exp(-k_2 x_1) \{E_1(-k_1 x_1) + \pi i] \tag{3.23}$$

Similarly, we can calculate the other integrals. Thus summing up the situation, we get for $w_+ > 0$ or $I_m(-k_1 x_1) > 0$

$$I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) + \pi i\}$$

and for $w_+ < 0$ or $I_m(-k_1 x_1) < 0$

$$I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) - \pi i)$$

Similarly,

$$I_2 = k_1 \exp(-k_1 x_2) [E_1(-k_1 x_2) + \pi i], \text{ for } I_m(-k_1 x_2) > 0$$
$$k_1 \exp(-k_1 x_2) [E_1(-k_1 x_2) - \pi i], \text{ for } I_m(-k_1 x_2) < 0$$
$$I_3 = k_2 \exp(-k_2 x_1) [E_1(-k_2 x_1) - \pi i], \text{ for } I_m(-k_2 x_1) > 0$$
$$k_2 \exp(-k_2 x_1) [E_1(-k_2 x_1) + \pi i], \text{ for } I_m(-k_2 x_1) < 0$$

$$
\begin{aligned}
I_4 &= k_2\exp(-k_2x_2)[E_1(-k_2x_2) - \pi i], \text{ for } I_m(-k_2x_2) > 0\\
&\quad k_2\exp(-k_2x_2)\ [E_1(-k_2x_2) + \pi i], \text{ for } I_m(-k_2x_2) < 0\\
I_5 &= k_3\exp(-k_3x_1)\ [E_1(-k_2x_1) + \pi i], \text{ for } I_m(-k_3x_1) > 0\\
&\quad k_3\exp(-k_3x_1)\ [E_1(-k_3x_1) - \pi i], \text{ for } I_m(-k_3x_1) < 0\\
I_6 &= k_3\exp(-k_3x_2)\ [E_1(-k_3x_2) + \pi i], \text{ for } I_m(-k_3x_2) > 0\\
&\quad k_3\exp(-k_3x_2)\ [E_1(-k_3x_2) - \pi i], \text{ for } I_m(-k_3x_2) < 0\\
I_7 &= k_4\exp(-k_4x_1)\ [E_1(-k_4x_1) + \pi i], \text{ for } I_m(-k_4x_1) > 0\\
&\quad k_4\exp(-k_4x_1)\ [E_1(-k_4x_1) - \pi i], \text{ for } I_m(-k_4x_1) < 0\\
I_8 &= k_4\exp(-k_4x_2)\ [E_1(-k_4x_2) + \pi i], \text{ for } I_m(-k_4x_2) > 0\\
&\quad k_4\exp(-k_4x_4)\ [E_1(-k_4x_2) - \pi i], \text{ for } I_m(-k_4x_2) < 0
\end{aligned} \tag{3.24}
$$

Now adding the terms in G_4 and G_5 given by the equations (3.17) and (3.14), respectively, we obtain

$$
\begin{aligned}
G_4 + iG_5 &= \frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{1}{\sqrt{1-4\tau\cos\theta}}\{I_1 + I_2 - I_3 - I_4 + \pi i\,(I_{11} + I_{12} + I_{21} + I_{22})\}d\theta\\
&+ \frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{1}{\sqrt{1-4\tau\cos\theta}}\{(I_5 + I_6 - I_7 - I_8) + \pi i\,(I_{31} + I_{32} - I_{41} - I_{42})\}d\theta
\end{aligned} \tag{3.25}
$$

where

$$I_{ij} = k_i\ \exp(-k_ix_j),\ j = 1,2,3,4;\ j = 1,2.$$

Thus, if we combine the corresponding integrands of $G_4 + iG_5$, we obtain

First Integral = $\frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_1\exp(-k_1x_1)}{\sqrt{1-4\ \tau\cos\theta}}[E_1(-k_1x_1)+2\pi i]d\theta,$ for $I_m(-k_1x_1) > 0$

$\frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_1\exp(-k_1x_1)}{\sqrt{1-4\ \tau\cos\theta}}E_1(-k_1x_1)d\theta,$ for $I_m(-k_1x_1) < 0$

Second Integral = $\frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_1\exp(-k_1x_2)}{\sqrt{1-4\ \tau\cos\theta}}[E_1(-k_1x_2)+2\pi i]d\theta,$ for $I_m(-k_1x_2) > 0$

$\frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_1\exp(-k_1x_2)}{\sqrt{1-4\ \tau\cos\theta}}E_1(-k_1x_2)d\theta,$ for $I_m(-k_1x_2) < 0$

$$\text{Third Integral} = \frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_2\exp(-k_2x_1)}{\sqrt{1-4\ \tau\cos\theta}}[-E_1(-k_2x_1)+2\pi i]d\theta, \qquad \text{for } I_m(-k_2x_1) > 0$$

$$-\frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_2\exp(-k_2x_1)}{\sqrt{1-4\ \tau\cos\theta}} E_1(-k_2x_1)d\theta, \qquad \text{for } I_m(-k_2x_1) < 0$$

$$\text{Fourth Integral} = \frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_2\exp(-k_2x_2)}{\sqrt{1-4\ \tau\cos\theta}} [-E_1(-k_2x_2)+2\pi]d\theta, \qquad \text{for } I_m(-k_2x_2) < 0$$

$$-\frac{1}{\pi}\int_{\gamma}^{\pi/2} \frac{k_2\exp(-k_2x_2)}{\sqrt{1-4\ \tau\cos\theta}} E_1(-k_2x_2)d\theta, \qquad \text{for } I_m(-k_2x_2) < 0$$

$$\text{Fifth Integral} = \frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_3\exp(-k_3x_1)}{\sqrt{1-4\ \tau\cos\theta}} [E_1(-k_3x_1)+2\pi i]d\theta, \qquad \text{for } I_m(-k_3x_1) > 0$$

$$\frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_3\exp(-k_3x_2)}{\sqrt{1-4\ \tau\cos\theta}} E_1(-k_3x_1)d\theta, \qquad \text{for } I_m(-k_3x_1) < 0$$

$$\text{Sixth Integral} = \frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_3\exp(-k_3x_2)}{\sqrt{1-4\ \tau\cos\theta}} [E_1(-k_3x_2)+2\pi i]d\theta, \qquad \text{for } I_m(-k_3x_2) < 0$$

$$\frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_3\exp(-k_3x_2)}{\sqrt{1-4\ \tau\cos\theta}} E_1(-k_3x_2)d\theta, \qquad \text{for } I_m(-k_3x_2) < 0$$

$$\text{Seventh Integral} = \frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_4\exp(-k_4x_1)}{\sqrt{1-4\ \tau\cos\theta}} [-E_1(-k_4x_1)-2\pi i]d\theta, \qquad \text{for } I_m(-k_4x_1) > 0$$

$$\frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_4\exp(-k_4x_4)}{\sqrt{1-4\ \tau\cos\theta}} [E_1(-k_4x_1)d\theta, \qquad \text{for } I_m(-k_4x_1) < 0$$

$$\text{Eighth Integral} = \frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_4\exp(-k_4x_2)}{\sqrt{1-4\ \tau\cos\theta}} [-E_1(-k_4x_2)-2\pi i]d\theta, \qquad \text{for } I_m(-k_4x_2) > 0$$

$$-\frac{1}{\pi}\int_{\pi/2}^{\pi} \frac{k_4\exp(-k_4x_2)}{\sqrt{1-4\ \tau\cos\theta}} E_1(-k_4x_2)d\theta, \qquad \text{for } I_m(-k_4x_2) < 0 \tag{3.26}$$

Therefore, for Case I, we can evaluate the Green's function given in equation (3.15).

To evaluate the Green's function for Case II given in equation (3.16), we need to express the G_3 term in exponential integrals as given below:

$$G_3 = \frac{2g}{\pi}\int_0^{\gamma} d\theta \int_0^{\infty} \frac{k\ e^{k[(z+c)\ +\ i(x-a)\cos\theta]}\cos[k(y-b)\sin\theta}{gk\ -\ (\omega\ +\ kU\cos\theta)^2}\,dk$$

$$= \frac{1}{\pi i} \int_0^{\gamma} \frac{1}{\sqrt{4\tau\cos\theta - 1}} (J_1 + J_2 - J_3 - J_4) d\theta \tag{3.27}$$

where

$$J_1 = \int_0^{\infty} \frac{k_1}{k-k_1} \exp(-kx_1)dk, \quad J_2 = \int_0^{\infty} \frac{k_1}{k-k_1} \exp(-kx_2)dk$$

$$J_3 = \int_0^{\infty} \frac{k_2}{k-k_2} \exp(-kx_1)dk, \quad J_4 = \int_0^{\infty} \frac{k_2}{k-k_2} \exp(-kx_2)dk \tag{3.28}$$

and k_1 and k_2 are the complex roots of

$$gk - (\omega + kU\cos\theta)^2 = 0.$$

Using the contour in Figure 2, it can be easily shown that

$$J_1 = \begin{array}{ll} k_1\exp(-k_1x_1)E_1(-k_1x_1) & I_m(-k_1x_1) > 0 \\ k_1\exp(-k_1x_1)\ [E_1(-k_1x_1) - 2\pi i] & I_m(-k_1x_1) < 0 \end{array}$$

$$J_2 = \begin{array}{ll} k_1\exp(-k_1x_2)\ E_1(-k_1x_2) & I_m(-k_1x_2) > 0 \\ k_1\exp(-k_1x_2)\ [E_1(-k_1x_2) - 2\pi i] & I_m(-k_1x_2) < 0 \end{array}$$

$$J_3 = \begin{array}{ll} k_2\exp(-k_2x_1)\ E_1(-k_2x_1) & I_m(-k_2x_1) > 0 \\ k_2\exp(-k_2x_1)\ [E_1(-k_2x_1) - 2\pi i] & I_m(-k_2x_1) < 0 \end{array}$$

$$J_4 = \begin{array}{ll} k_2\exp(-k_2x_2)\ [E_1(-k_2x_2) + 2\pi i] & I_m(-k_2x_2) > 0 \\ -k\ x\ )\ E\ (-k\ x\ ) & \end{array}$$

Thus, with this information, we can evaluate the Green's function for Case II from equation (3.16).

4. RESULTS AND CONCLUSIONS.

The present form of Green's function is equivalent to that used by Wu and Taylor, but in a different form. The terms G_1, G_2 and G_3 are all identical to those used by Chen et al [8]. However, in the present study, we have combined the G_4 and G_5 terms to correspond with the form of Wu and Taylor. It appears that our studies are quite similar to those of Wu and Taylor, and Chen et al.

The double integral arising in the evaluation of Green's function has been replaced by a single integral with the use of complex exponential integrals. The present work has provided an alternative form but similar to that of Wu and Taylor, and has been found to be efficient for the analysis of the three dimensional potential problem of ship motion with forward speed.

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