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SEPARABLE INJECTIVITY AND C^* TENSOR PRODUCTS

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ABSTRACT. Let A and B be C*-algebras and let D be a C*-subalgebra of B. We show that if D is separably injective then the triple (A, B, D) verifies the slice map conjecture. As an application, we prove that the minimal C*-tensor product $A \otimes B$ is separably injective if and only if both A and B are separably injective and either A or B is finite-dimensional.

KEY WORDS AND PHRASES. C-algebra, C*-tensor product, injective C*-algebra, separably injective C*-algebra, slice map.*

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1. INTRODUCTION.

Smith and Williams introduced the notion of separable injectivity in connection with the study of completely bounded maps ([7], [8]). As stated in the introduction of [8], it is weaker than the related concept of injectivity and yet is appropriate for certain desirable extension problems. From this point of view, we study C*-tensor products.

Let A and B be C*-algebras and D be a C*-subalgebra of B. If D is injective, then the triple (A, B, D) verifies the slice map conjecture in the sense of Wassermann ([11], [12]). We first show that the separable injectivity is enough for (A, B, D) to verify the slice map conjecture. Also the minimal C*-tensor product $A \otimes B$ is injective if and only if A and B are injective and either A or B is finite-dimensional (see the proof of [9, Theorem]). Using the above result and [5], we give a separably injective version of this theorem.

2. PRELIMINARIES AND NOTATION.

Let A and B be C*-algebras and let $A \otimes B$ denote their minimal (i.e., spatial) tensor product. Let M_n denote the C*-algebra of all $n \times n$ complex matrices for a positive integer n. If $\phi : A \to B$ is a linear map then $\phi \otimes id_n : A \otimes M_n \to B \otimes M_n$ is defined by $(\phi \otimes id_n)(a_{ij}) = (\phi(a_{ij}))$. ϕ is said to be completely positive if each $\phi \otimes id_n$ is positive.

A C*-algebra D is said to be injective if given C*-algebras $E \subseteq F$, any contractive completely positive map $\phi : E \to D$ has a contractive completely positive extension $\psi : F \to D$. We say that a C*-algebra D is separably injective if given separable C*-algebras $E \subseteq F$, any contractive completely positive map $\phi : E \to D$ has a contractive completely positive extension $\psi : F \to D$. The separable injectivity in this paper is weaker than one in ([**7**], [**8**]) and both coincide for commutative C*-algebras. A compact Hausdorff space is said to be substonean if every two disjoint co-zero sets have disjoint closures. A compact Hausdorff space X is substonean if and only if $C(X)$ is separably injective [**7**, Theorem 4.6].

A C*-algebra D is said to be subhomogeneous if every irreducible representation is finite-dimensional with bounded dimension. In particulr, it is said to be n-homogeneous if every irreducible representation is n-dimensional. If D is subhomogeneous then we identify the spectrum $\widehat{D}$ with the primitive ideal space [**2**, Chapters 3 and 4].

For $i = 1, 2$ let D_i be a C*-algebra and let $h_i \in D_i^*$. The right slice map $R_{h_1} : D_1 \otimes D_2 \to D_2$ and the left slice map $L_{h_2} : D_1 \otimes D_2 \to D_1$ are unique bounded linear maps satisfying $R_{h_1}(x_1 \otimes x_2) = h_1(x_1)x_2$ and $L_{h_2}(x_1 \otimes x_2) = h_2(x_2)x_1$ [**10**]. For C*-subalgebras A_i of D_i, we define the Fubini product $F(A_1, A_2, D_1 \otimes D_2)$ of A_1 and A_2 with respect to $D_1 \otimes D_2$ [**11**] by

$$F(A_1, A_2, D_1 \otimes D_2)$$
$$= \{x \in D_1 \otimes D_2 : R_{h_1}(x) \in A_2 \text{ and } L_{h_2}(x) \in A_1 \text{ for all } h_1 \in D_1^* \text{ and } h_2 \in D_2^*\}.$$

For fixed C*-algebras A_1 and A_2, $F(A_1, A_2, D_1 \otimes D_2)$ depends on $D_1 \otimes D_2$. But they are all isomorhic and are the largest among them if D_1 and D_2 are injective. We denote by $A_1 \otimes_F A_2$ any one of these isomorphic Fubini products of A_1 and A_2 [**4**]. Let A and B be C*-algebras and let D be a C*-subalgebra. The triple (A, B, D) is said to verify the slice map conjecture if $F(A, D, A \otimes B) = A \otimes D$ [**12**].

3. THE SLICE MAP PROBLEM.

A C*-algebra A is said to have property (S) if (A, B, D) verifies the slice map conjecture for every C*-algebra B and every C*-subalgebra D of B [**12**]. We now consider a property (S') as follows. A C*-algebra D is said to have property (S') if (A, B, D) verifies the slice map conjecture for every C*-algebra A and every C*-algebra B containing

D. Subhomogeneous or injective C*-algebras have property (S') [**11**].

THEOREM 1. *Let D be a C*-algebra. If D is serarably injective, then D has property (S').*

PROOF: Let A be a C*-algebra and B a C*-algebra containing D. Let $x \in F(A, D, A \otimes B)$. Then there exists a sequence $\{x_n\}$ such that $x_n = \sum_{i=1}^{m(n)} a(i,n) \otimes b(i,n)$, $\lim_n x_n = x$, where each $a(i,n) \in A$ and each $b(i,n) \in B$. Let B_0 be the C*-subalgebra generated by $\{b(i,n) : i = 1, \ldots, m(n), n = 1, 2, \ldots\}$ and let D_0 be the C*-subalgebra of D generated by $\{R_h(x) : h \in A^*\}$. Then we have

$$D_0 \subseteq B_0, \quad x \in F(A, D_0, A \otimes B_0)$$

by a similar argument of [**4**, Lemma 5]. By hypothesis, there exists a contractive completely positive map $\phi : B_0 \to D$ which extends the identity embedding of D_0 into D. Then

$$R_h((I_A \otimes \phi)(x)) = \phi(R_h(x)) = R_h(x) \qquad (h \in A^*).$$

Since $\{R_h : h \in A^*\}$ is total [**10**, Theorem 1], $x = (I_A \otimes \phi)(x) \in A \otimes D$ and so $F(A, D, A \otimes B) \subseteq A \otimes D$.

The opposite inclusion is immediate.

It is known that the direct sum of two C*-algebras having property (S') has property (S'). In order to show that Theorem 1 gives a new example having property (S'), two results will be needed. In the proof of [**7**, Theorem 4.6], Smith and Williams obtained the following lemma.

LEMMA 2. *Let B and D be C*-algebras. Then there exists a one to one correspondence θ between completely positive maps $\phi : B \to D \otimes M_n$ and completely positive maps $\psi : B \otimes M_n \to D$ for any positive integer n.*

We remark that θ is not necessarily norm preserving and that θ satisfies that $\theta(\phi)_{|A \otimes M_n} = \theta(\phi_{|A})$ for a C*-subalgebra A of B, where $\theta(\phi)_{|A \otimes M_n}$ and $\theta(\phi_{|A})$ denote the restrictions of $\theta(\phi)$ and ϕ to $A \otimes M_n$ and A, respectively.

The proof of the following proposition is based on an idea of [**6**, Theorem 2.1].

PROPOSITION 3. *Let D be a C*-algebra. If D is separably injective, then $D \otimes M_n$ is separably injective for any positive integer n.*

PROOF: Let A be a separable C*-algebra and let $\phi : A \to D \otimes M_n$ be a contractive completely positive map. Let B be a separable C*-algebra containing A. We will show that ϕ has a norm preserving, completely positive extension $\psi : B \to D \otimes M_n$.

Since the image $\phi(A)$ is separable, there exists a separable C*-subalgebra D_0 such that $\phi(A) \subseteq D_0 \otimes M_n$. Let A_1 and D_1 denote the C*-algebras obtained by adjoining identities to A and D_0, respectively. Then the unital map $\phi : A_1 \to D_1$ defined by $\phi_1(a+\alpha I) = \phi(a) + \alpha I$ is completely positive by [1, Lemma 3.9]. By hypothesis, there exists a contractive completely positive map $\pi : D_1 \to D$ which extends the identity embedding of D_0 into D. Define the map $\phi_2 : A_1 \to D \otimes M_n$ by $\phi_2(a) = \pi(\phi_1(a))$. Then ϕ_2 is a contractive completely positive map from A_1 to $D \otimes M_n$ which extends ϕ. Hence we may assume that A has the identity u.

Using the same notations as in Lemma 2, we have the map $\theta(\phi) : A \otimes M_n \to D$ associated with ϕ. Since D is separably injective, there exists a completely positive extension $\psi_1 : B \otimes M_n \to D$ of $\theta(\phi)$. Again by Lemma 2 we have the completely positive map $\phi_3 : B \to D \otimes M_n$ such that $\theta(\phi_3) = \psi_1$. By the remark about Lemma 2, ϕ_3 extends ϕ. Define the completely positive map $\psi : B \to D \otimes M_n$ by $\psi(b) = \phi_3(ubu)$. Then ψ is an extension of ϕ. If $b \in B$ with $\| b \| \leq 1$, then

$$\| \psi(b) \| = \| \phi_{3|uBu}(ubu) \| \leq \| \phi_{3|uBu} \| \| ubu \| \leq \| \phi_{3|uBu}(u) \| = \| \phi(u) \| = \| \phi \| .$$

This completes the proof.

EXAMPLE 4. *Let $\beta\mathbf{N}$ be the Stone-Čech compactification of the set $\mathbf{N}$ of positive integers and $\mathbf{N}^*$ the corona set $\beta\mathbf{N} - \mathbf{N}$. For each positive integer n put $D_n = C(\mathbf{N}^*) \otimes M_n$. Let D_∞ denote the C*-algebra of bounded sequences $\{x_n\}$ such that $x_n \in D_n$ for each n. Then D_∞ is separably injective and has no decomposition $D_\infty = D_s \oplus D_i$ such that D_s is subhomogeneous and D_i is injective.*

PROOF: For each n there exists a projection of norm one from D_n onto $C(\mathbf{N}^*) \otimes 1_n$, where 1_n denotes the identity of M_n. The algebra $C(\mathbf{N}^*)$ is separably injective by [7, Theorem 4.6], but is not injective. Then D_n is separably injective by Proposition 3, but is not injective. Hence D_∞ is separably injective, but is not injective.

Suppose that D_∞ has a decomposition $D_\infty = D_s \oplus D_i$. Then there exist central projections p and q of D_∞ such that $p \oplus q = 1$, where 1 denotes the identity of D_∞. We have the sequence $\{p_n\}$ of projections of $C(\mathbf{N}^*)$ such that $p = \{p_n \otimes 1_n\}$. Hence $D_s = \{x \in D_\infty : x = \{x_n\}$ with $x_n \in (C(\mathbf{N}^*)p_n) \otimes M_n$ for all $n\}$. If $p_n \neq 0$, there exists an irreducible representation of D_s with dimension n. Since D_s is subhomogeneous, we have an integer n_0 such that $p_n = 0$ for all $n \geq n_0$. Put $D_{i,n_0} = \{x \in D_\infty : x = \{x_n\}$ with $x_n = 0$ for all $n \leq n_0\}$. Then D_{i,n_0} is not injective. On the other hand, there exists a projection of norm one from D_i onto D_{i,n_0} and hence D_{i,n_0} is injective. This is a contradiction and completes the proof.

4. C*-TENSOR PRODUCTS OF SEPARABLY INJECTIVE C*-ALGEBRAS.

In this section we prove the following theorem.

THEOREM 5. *Let A and B be C^*-algebras. The following two statements are equivalent:*

(i) *$A \otimes B$ is separably injective.*

(ii) *Both A and B are separably injective and either A or B is finite-dimensional.*

We need several lemmas.

LEMMA 6. *Let A_i be a C^*-subalgebra of a C^*-algebra D_i for $i = 1, 2, 3$. Then, under the obvious identification, we have*

$$F(F(A_1 \otimes A_2, D_1 \otimes D_2), A_3, (D_1 \otimes D_2) \otimes D_3)$$
$$= F(A_1, F(A_2, A_3, D_2 \otimes D_3), D_1 \otimes (D_2 \otimes D_3)).$$

PROOF: Let $z \in F(F(A_1, A_2, D_1 \otimes D_2), A_3, (D_1 \otimes D_2) \otimes D_3)$ and $h_i \in D_i^*$ for $i = 1, 2, 3$. Then, we have

$$R_{h_2}(R_{h_1}(z)) = R_{h_1 \otimes h_2}(z) \in A_3,$$
$$L_{h_3}(R_{h_1}(z)) = R_{h_1}(L_{h_3}(z)) \in A_2,$$

because $L_{h_3}(z) \in F(A_1, A_2, D_1 \otimes D_2)$ by the assumption. These imply that

$$R_{h_1}(z) \in F(A_2, A_3, D_2 \otimes D_3).$$

Now, we have

$$L_{h_2 \otimes h_3}(z) = L_{h_2}(L_{h_3}(z)) \in A_1,$$

because $L_{h_3}(z) \in F(A_1, A_2, D_1 \otimes D_2)$ by the assumption. Since the family of all product functionals $h_2 \otimes h_3$ on $D_2 \otimes D_3$ is total, we obtain

$$L_h(z) \in A_1 \qquad \text{for all} \quad h \in (D_2 \otimes D_3)^*$$

by a standard approximation argument (see, for example, [**11**, Lemma 2.1]). Hence we have

$$z \in F(A_1, F(A_2, A_3, D_2 \otimes D_3), D_1 \otimes (D_2 \otimes D_3)).$$

The reverse inclusion can be shown similarly.

LEMMA 7. *Let A be an infinite-dimensional C^*-algebra and D be a non-subhomogeneous C^*-algebra. Then $D \otimes A$ is not separably injective.*

PROOF: Let $B(H)$ be the C*-algebra of all bounded linear operators on a Hilbert space H such that $B(H) \supseteq D \otimes A$. Since A is infinite-dimensional, there exists an orthogonal sequence $\{A_n\}$ of commutative C*-subalgebras of A. The C*-subalgebra generated by $\{A_n\}$ may be identitied with the c_0-sum $\oplus_n A_n$ of $\{A_n\}$.

Suppose that $F(B(H), D \otimes A, B(H) \otimes B(H)) = B(H) \otimes (D \otimes A)$. Then, we have

$$
\begin{aligned}
&B(H)\otimes_F (D\otimes(\oplus_n A_n))\\
&\quad = F(B(H), D\otimes(\oplus_n A_n), B(H)\otimes B(H))\\
&\quad \subseteq F(B(H), D\otimes A, B(H)\otimes B(H)) = B(H)\otimes(D\otimes A).
\end{aligned}
$$

Hence, it follows that

$$
\begin{aligned}
&B(H)\otimes_F (D\otimes(\oplus_n A_n))\\
&\quad = F(B(H), D\otimes(\oplus_n A_n), B(H)\otimes(D\otimes A))\\
&\quad = F(B(H), F(D, \oplus_n A_n, D\otimes A), B(H)\otimes(D\otimes A)) \qquad \text{(by [12, Theorem 4])}\\
&\quad = F(F(B(H), D, B(H)\otimes D), \oplus_n A_n, (B(H)\otimes D)\otimes A) \qquad \text{(by Lemma 6)}\\
&\quad = F(B(H)\otimes D, \oplus_n A_n, (B(H)\otimes D)\otimes A)\\
&\quad = (B(H)\otimes D)\otimes(\oplus_n A_n) \qquad \text{(by [12, Theorem 4])}\\
&\quad = B(H)\otimes(D\otimes(\oplus_n A_n)).
\end{aligned}
$$

Since $D\otimes(\oplus_n A_n)$ is canonically *-isomorphic to $\oplus_n(D\otimes A_n)$, this contradicts [**5**, Theorem 3.2]. Hence $F(B(H), (D\otimes A), B(H)\otimes B(H))$ contains properly $B(H)\otimes(D\otimes A)$, and so $D\otimes A$ is not separably injective by Theorem 1.

LEMMA 8. *Let A and B be C*-algebras. If $A\otimes B$ is separably injective, then both A and B are separably injective.*

PROOF: Let $E\subseteq F$ be separable C*-algebras. Let $\phi : E\to A$ be a contractive completely positive map. Let b be a positive element of B with $\| b \| = 1$. Define $\psi : E\to A\otimes B$ by $\psi(x) = \phi(x)\otimes b$. Then ψ has a contractive completely positive extension $\psi_1 : F\to A\otimes B$. Let h be a state of B such that $h(b) = 1$. Define $\phi_1 : F\to A$ by $\phi_1(x) = L_h(\psi_1(x))$. Then ϕ_1 is the desired extension of ϕ. This implies that A is separably injective. A similar argument shows that B is separably injective.

The following lemma is a slight modification of the proof of [**8**, Propositon 2.6].

LEMMA 9. *Let A and B be C*-algebras and let A^1 and B^1 denote the C*-algebras obtained by adjoining identities to A and B, respectively. If $A\otimes B$ is separably injective then $A^1\otimes B^1$ is separably injective.*

PROOF: Let $E\subseteq F$ be separable C*-algebras and let $\phi : E\to A^1\otimes B$ be a contractive completely positive map. Choose A_0 and B_0 be separable C*-subalgebras such that $\phi(E)\subseteq A_0\otimes B_0 + \mathbf{C}I\otimes B_0$. By [**8**, Proposition 2.5] there exist positive elements $a\in A, b\in B$ and $c\in A\otimes B$ of unit norm such that a, b, and c act as identities of A_0, B_0 and the C*-subalgebra generated by $A_0\otimes B_0$ and $a\otimes b$, respectively. We note that $(a\otimes b)(1\otimes d) = a\otimes bd = (1\otimes d)(a\otimes b)$ for each $d\in B_0$. Let h be the state of A^1 which annihilates A. Define $\psi : E\to A\otimes B$ by $\psi(x) = c\phi(x)c$ and $\theta : E\to B$ by $\theta(x) = R_h(\phi(x))$. By Lemma 8, B is separably injective. Then θ has a contractive completely positive extension $\theta_1 : F \to B$. Define $\theta_2 : F \to \mathbf{C}I \otimes B$ by $\theta_2(x) = I \otimes \theta_1(x)$. By hypothesis ψ has a contractive completely positive extension $\psi_1 : F \to A \otimes B$. Define $\phi_1 : F \to A^1 \otimes B$ by

$$\phi_1(x) = (a \otimes b)\psi_1(x)(a \otimes b) + (I - (a \otimes b)^2)^{\frac{1}{2}}\theta_2(x)(I - (a \otimes b)^2)^{\frac{1}{2}}.$$

Since $\phi_1(x)$ may be written

$$(a \otimes b, (I - (a \otimes b)^2)^{\frac{1}{2}}) \begin{pmatrix} \psi_1(x) & 0 \\ 0 & \theta_2(x) \end{pmatrix} \begin{pmatrix} a \otimes b \\ (I - (a \otimes b)^2)^{\frac{1}{2}} \end{pmatrix},$$

ϕ_1 is a contractive completely positive map. To see ϕ_1 extends ϕ, let $x \in E$ and write $\phi(x) = y + z, y \in A_0 \otimes B_0, z \in \mathbf{C}I \otimes B_0$. Then $\psi_1(x) = y + czc = y + zc^2$ and $\theta_2(x) = z$. Thus

$$\begin{aligned}\phi_1(x) &= (a \otimes b)(y + zc^2)(a \otimes b) + (I - (a \otimes b)^2)^{\frac{1}{2}} z (I - (a \otimes b)^2)^{\frac{1}{2}} \\ &= y + z(a \otimes b)^2 + z(I - (a \otimes b)^2) = \phi(x).\end{aligned}$$

Hence $A^1 \otimes B$ is separably injective.

A symmetric argument shows that $A^1 \otimes B^1$ is separably injective.

LEMMA 10. *Let A be a unital infinite-dimensional subhomogeneous C^*-algebra. Then there exist a *-homomorphism π of A and a norm one projection ϕ such that the image $\phi(\pi(A))$ is *-isomorhpic to the C^*-algebra of all continuous functions on some infinite compact Hausdorff space X.*

PROOF: By [**2**, 3.6.3 Proposition] and the proof of [**8**, Theorem 3.2], we may assume that there exists a closed two-sided ideal J such that A/J is finite-dimensional, J is n-homogeneous and $\widehat{J}$ is an infinite set.

Suppose first that $\widehat{J}$ has a limit point. By [**2**, 3.6.4 Proposition] $\widehat{J}$ is a locally compact Hausdorff space. Thus there exists a closed two-sided ideal J_0 such that $(\widehat{J/J_0})$ is an infinite compact Hausdorff space. Let $(\widehat{J/J_0}) = X$. Then $C(X)$ may be identified with the center of J/J_0. Let $\pi : A \to A/J_0$ be the quotient map. From [**2**, 3.6.4 Proposition] for $a \in A$ the map $\lambda \to tr_n(\lambda(a))$ is continuous on $\widehat{J}$, where tr_n denotes the normalized trace on M_n. Note that $\ker\lambda \supseteq \ker\pi$ for each $\lambda \in X$. Define $\phi : \pi(A) \to C(X)$ by

$$\phi(\pi(a))(\lambda) = tr_n(\lambda(a)) \qquad (a \in A, \lambda \in X).$$

It is easy to see that π and ϕ are desired maps.

Suppose now that $\widehat{J}$ has no limit point. Let T be a non-empty set. Let $\ell_T^\infty(M_n)$ be the C*-algebra of $(x_\lambda) = (x_\lambda)_{\lambda \in T}$ such that $x_\lambda \in M_n$ for all $\lambda \in T$ and $\sup_\lambda \| x_\lambda \| < \infty$ and let $c_T^0(M_n)$ be the ideal of $\ell_T^\infty(M_n)$ such that for each $\varepsilon > 0$ $\| x_\lambda \| \leq \varepsilon$ for all but a finite number of indices λ.

Let $\widehat{J} = Y$. Define $\rho : A \to \ell_Y^\infty(M_n)$ by

$$\rho(a)(\lambda) = \lambda(a) \qquad (a \in A, \lambda \in Y).$$

Since Y is discrete, by [2, 10.10.1] we have $\rho(J) = c_Y^0(M_n)$. Let $\mu : \ell_Y^\infty(M_n) \to \ell_Y^\infty(M_n)/c_Y^0(M_n)$ denote the quotient map. Since $(\mu\rho)^{-1}(0) \supseteq J$, $\mu\rho(A)$ is finite-dimensional. Hence there exists a finite set $\{a_1, \cdots a_k\}$ of A such that $\{\mu\rho(a_1), \cdots, \mu\rho(a_k)\}$ spans $\mu\rho(A)$. Then we have

$$\rho(A) = c_Y^0(M_n) + \mathbf{C}\rho(a_1) + \cdots + \mathbf{C}\rho(a_k).$$

Let X be the one-point compactification of the set $\mathbf{N}$ of positive integers. Then $C(X) \otimes M_n$ may be identified with the C*-algebra of convergent sequences of elements of M_n. Let $\rho(a_i) = (m_\lambda^i) \in \ell_Y^\infty(M_n)$. Passing to convergent subsequences, there exists a sequence $\{\lambda_n\}$ of Y such that $(m_{\lambda_n}^i) \in C(X) \otimes M_n$ for each i. Define $\nu : \ell_Y^\infty(M_n) \to \ell_{\{\lambda_n\}}^\infty(M_n)$ by

$$\nu((a_\lambda)) = (a_{\lambda_n}).$$

Then $\nu\rho(a) \in C(X) \otimes M_n$ for $a \in A$. Let $\pi = \nu\rho$. Then $\pi(A) = C(X) \otimes M_n$. Define $\phi : \pi(A) \to C(X)$ by

$$\phi(\pi(a))(\lambda_n) = tr_n(\rho(a)_{\lambda_n}).$$

It is well known that ϕ has the desired property.

Using Choi-Effros lifting theorem [1], Smith and Williams showed in the proof of [8, Lemma 3.3] that every quotient algebra of a nuclear separably injective C*-algebra is separably injective. By this useful result we have the following lemma.

LEMMA 11. *Let A be a nuclear separably injective C*-algebra. Let π be a *-homomorphism of A and let B be a commutative C*-subalgebra of $\pi(A)$. If there exists a norm one projection $\phi : \pi(A) \to B$ such that $\phi(\pi(A)) = B$, then B is separably injective.*

PROOF: Let $E \subseteq F$ be separable C*-algebras and let $\psi : E \to B$ be a contractive completely positive map. By the above remark, $\pi(A)$ is separably injective. Then ψ has a contractive completely positive extension $\psi_1 : F \to \pi(A)$. Then $\psi_2 = \phi\psi_1 : F \to B$ is a contractive completely positive extension of ψ. Hence B is separably injective.

PROOF OF THEOREM 5: (i) $\Rightarrow$ (ii). By Lemma 8, it suffices to show that either A or B is finite-dimensional. To do this, we may assume that A and B are unital by Lemma 9.

Suppose that A and B are infinite-dimensional. If A or B is non-subhomogeneous, it follows from Lemma 7 that $A \otimes B$ is not separably injective. This is a contradiction. Now if A and B are subhomogeneous, by Lemma 10 there exist *-homomorphisms

π_1, π_2, infinite compact Housdorff spaces X_1, X_2 and norm one projections $\phi_1 : \pi_1(A) \to C(X_1)$, $\phi_2 : \pi_2(B) \to C(X_2)$. By Lemma 11 and [7, Theorem 4.6] X_1 and X_2 are substonean. We may identify $C(X_1) \otimes C(X_2)$ with $C(X_1 \times X_2)$. Then $\phi_1 \odot \phi_2 : \pi_1 \odot \pi_2(A \otimes B) \to C(X_1 \times X_2)$ is a norm one projection such that $\phi_1 \otimes \phi_2(\pi_1 \otimes \pi_2(A \otimes B)) = C(X_1 \times X_2)$. Again by Lemma 11 and [7, Theorem 4.6] $X_1 \times X_2$ is substonean. But this contradicts [3, Proposition 1.7].

(ii) $\Rightarrow$ (i). Since a finite-dimensional C*-algebra is a finite direct sum of matrix algebras, Propositon 3 implies that $A \otimes B$ is separably injective.

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This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

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