

ON A FIXED POINT THEOREM OF PATHAK

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ABSTRACT. It is shown that the continuity of the mapping in Pathak's fixed point theorem for normed spaces is not necessary.

KEY WORDS AND PHRASES. Normed space, Fixed Point.

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1. INTRODUCTION AND MAIN RESULTS.

In [1] Pathak gives the following definitions:

DEFINITION 1. Let X be a normed vector space; then T , a self mapping of X is called a 'generalized contractive mapping' if

$$\begin{aligned} \|Tx - Ty\| \leq q \max \{ & \|x - y\|, \frac{\|x - Tx\| [1 - \|x - Ty\|]}{1 + \|x - Tx\|}, \\ & \frac{\|x - Ty\| [1 - \|x - Tx\|]}{1 + \|x - Ty\|}, \frac{\|y - Tx\| [1 - \|y - Ty\|]}{1 + \|Tx - y\|}, \\ & \frac{\|y - Ty\| [1 - \|Tx - y\|]}{1 + \|y - Ty\|} \}, \end{aligned} \tag{1.1}$$

for all x, y in X , where $0 < q < 1$.

DEFINITION 2. Let T be a self mapping of a Banach space X . The Mann iterative process associated with T is defined in the following manner:

Let x_0 be in X and set $x_{n+1} = (1 - c_n)x_n + c_nTx_n$, for $n \geq 0$, where c_n satisfies (i) $c_0 = 1$, (ii) $0 < c_n < 1$ for $n > 0$, (iii) $\liminf_n c_n = h > 0$.

He then proves the following theorem:

THEOREM. Let X be a closed, convex subset of a normed space N , let T be a generalized contractive mapping of X with T continuous on X , and let $\{x_n\}$, the sequence of Mann iterates associated with T , be the same as defined above where $\{c_n\}$ satisfies (i), (ii) and (iii). If $\{x_n\}$ converges in X , then it converges to a fixed point of T .

Pathak finally asks if the continuity of T is necessary in the theorem for T to have a fixed point.

The answer is in the affirmative. To see this, note that if T is a generalized contractive mapping then T also satisfies the inequality

$$\|Tx - Ty\| \leq q \max \{ \|x - y\|, \|x - Tx\|, \|x - Ty\|, \|y - Tx\|, \|y - Ty\| \} \quad (1.2)$$

for all x, y in X , where $0 < q < 1$.

Using inequality (1.2) now instead of inequality (1.1) to simplify the work, it follows in exactly the same way as in Pathak's proof of the theorem that if $\lim_{n \rightarrow \infty} x_n = z$, then

$$\begin{aligned} \|z - Tz\| &\leq \|z - x_{n+1}\| + (1 - c_n) \|x_n - Tz\| \\ &+ c_n q \max \{ \|x_n - z\|, \|x_n - Tx_n\|, \|x_n - Tz\|, \|Tx_n - z\|, \|z - Tz\| \} \end{aligned} \quad (1.3)$$

It now follows from the definition of x_n in Definition 2 that

$$Tx_n = \frac{x_{n+1} - (1 - c_n)x_n}{c_n}$$

and so

$$\lim_{n \rightarrow \infty} Tx_n = z.$$

On letting n tend to infinity in inequality (1.3) we now have

$$\|z - Tz\| \leq (1 - h) \|z - Tz\| + hq \max \{ 0, \|z - Tz\| \}$$

$$= (1 - h + hq) \|z - Tz\|,$$

where $1 - h + hq < 1$. Thus $Tz = z$.

REFERENCES

1. PATHAK, H.K., Some Fixed Point Theorems on Contractive Mappings, Bull. Cal. Math. Soc. **80**, 183.

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