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THE FIFTH-ORDER KORTEWEG-DE VRIES EQUATION

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ABSTRACT. Decomposition is applied to the 5th-order KdV equation.

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The 5th-order KdV equation [1] is given by:

$$u_t + 6uu_x + u_{xxx} + u_{xxxxx} = 0 \ .$$

Writing $L_t = \partial/\partial t$ and $L_t^{-1} = \int_0^t (\bullet) dt$, we have

$$\begin{aligned} L_t u &= -6uu_x - u_{xxx} - u_{xxxxx} \\ L_t^{-1} L_t u &= -6L_t^{-1} uu_x - L_t^{-1} u_{xxx} - L_t^{-1} u_{xxxxx} \\ u &= u(0) - 6L_t^{-1} A_n\{uu_x\} - L_t^{-1}(\partial^3/\partial x^3)u - L_t^{-1}(\partial^5/\partial x^5)u \end{aligned}$$

where $A_n\{uu_x\}$ represents the Adomian Polynomials [2] for uu_x. Letting $u = \sum_{n=0}^{\infty} u_n$, decomposition yields

$$\begin{aligned} u_0 &= u(0) \\ u_1 &= -6L_t^{-1} A_0 - L_t^{-1}(\partial^3/\partial x^3)u_0 - L_t^{-1}(\partial^5/\partial x^5)u_0 \\ u_2 &= -6L_t^{-1} A_1 - L_t^{-1}(\partial^3/\partial x^3)u_1 - L_t^{-1}(\partial^5/\partial x^5)u_1 \\ &\vdots \\ u_{n+1} &= -6L_t^{-1} A_n - L_t^{-1}(\partial^3/\partial x^3)u_n - L_t^{-1}(\partial^5/\partial x^5)u_n \end{aligned}$$

Using primes to indicate the differentiation with respect to x [2]

$$\begin{aligned} A_0 &= u_0 u_0' \\ A_1 &= u_0 u_1' + u_1 u_0' \\ A_2 &= u_2 u_0' + u_1 u_1' + u_0 u_2' \\ &\vdots \\ A_n &= u_n u_0' + u_{n-1} u_1' + \dots + u_1 u_{n-1}' + u_0 u_n' \end{aligned}$$

Now all components of u are determinable and we can write the n-term approximant

$$\phi_n = \sum_{m=0}^{n-1} u_m$$

which approaches u as $m \to \infty$. It has been shown that high accuracy can be achieved for small values of n.

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