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ON OBLIQUE WAVES FORCING BY A POROUS CYLINDRICAL WALL

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ABSTRACT. The problem of oblique cylindrical linearized wave motion is considered for a fluid of infinite depth or finite constant depth in the presence of an impermeable cylindrical wall and coaxial porous wall immersed vertically in the fluid The motion is generated once by the oscillations, which are periodic in time and in θ-direction, of the impermeable wall and next by the porous wall. The velocity potentials have been found in closed forms in the different regions of the fluid and then calculating the hydrodynamic pressure distribution on the porous wall and the profile of the free surface. The scattering problem of oblique waves is then considered A wave trapping phenomenon is investigated. Numerical results are given to the case of radial incident waves and the case when the angle of incident waves is 30° to the radial direction.

KEY WORDS AND PHRASES. Surface Waves, Porous Medium
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1. INTRODUCTION.

The scattering of surface waves obliquely incident on partially immersed or completely submerged vertical barriers and plates in infinite fluid were investigated by Faulkner [1,2], Jarvis and Taylor [3], Evans and Morris [4], Rhodes-Robinson [5] and Mandal and Goswami [6]. Levine [7] considered the scattering of surface waves obliquely incident on a submerged circular cylinder. The problem of scattering of oblique waves by a shallow draft cylinder at the free surface was solved by Garrison [8]. Subsequently, Bai [9] studied the more general problem of scattering of oblique waves by a partially immersed cylinder. In all of such works the immersed bodies are assumed to be impermeable. Chwang [10] considered a porous wavemaker oscillating normally to its plane with a constant amplitude. In his linearized analysis, the wavemaker is located in the middle of an infinitely long channel with constant depth. Chwang and Li [11] applied the linearized porous wavemaker method developed in [10] to investigate the small amplitude surface waves produced by a piston-type porous wavemaker near the end of a semi-infinitely long channel of constant depth. Chwang and Dong [12] studied the problem of reflection and transmission of small amplitude surface waves by a vertical porous plate fixed near the end of a semi-infinitely long open channel of constant depth. Gorgui and Faltas [13] extended Chwang's work to include the study of wave motion for a fluid of infinite horizontal extend and of infinite or finite constant depth in the presence of an impermeable plate and a porous wall immersed in the fluid parallel to each other. The waves are generated by arbitrary prescribed horizontal oscillations performed by the impermeable plate or the porous wall.

In the present paper we investigate the case of oblique cylindrical wave motion in fluids of infinite depth or finite constant depth The linearized theory for waves of small amplitude is used to analyze the forced motion in fluids bounded internally by an impermeable vertical circular cylinder surrounded by a coaxial cylindrical porous wall The waves are generated by arbitrary prescribed oscillations, which are periodic in time and in θ-direction, first performed by the impermeable wall and later by the porous wall It is assumed that the pores of the wall are of such nature as to allow the application of Darcy's law that the fluid velocity normal to the wall is linearly proportional to the difference in pressure between its two sides The method of separation of variables is applied to find analytic solution in closed forms for the linearized boundary value problem in the different regions of the fluid The results of Sections 3 and 4 are used to find the reflection coefficient of the reflected waves due to the scattering of time and θ periodic waves incident with angle β to the radial direction In the last section numerical results are presented for the two cases of radial oscillations and the case of $\beta = 30°$

2. BOUNDARY VALUE PROBLEM

We consider here the excitation of gravity waves on the surface of a fluid by an impermeable vertical cylindrical wall of circular cross-section of radius a that performs oscillations which are periodic in time and in θ-direction. A coaxial cylindrical porous wall of circular cross-section of radius $b(>a)$ is fixed in the fluid (see Fig. 1). Let (r, θ, y) be cylindrical coordinates with the origin 0 in the undisturbed free surface such that $0y$ pointing down into the fluid coinciding with the axis of the impermeable and porous walls.

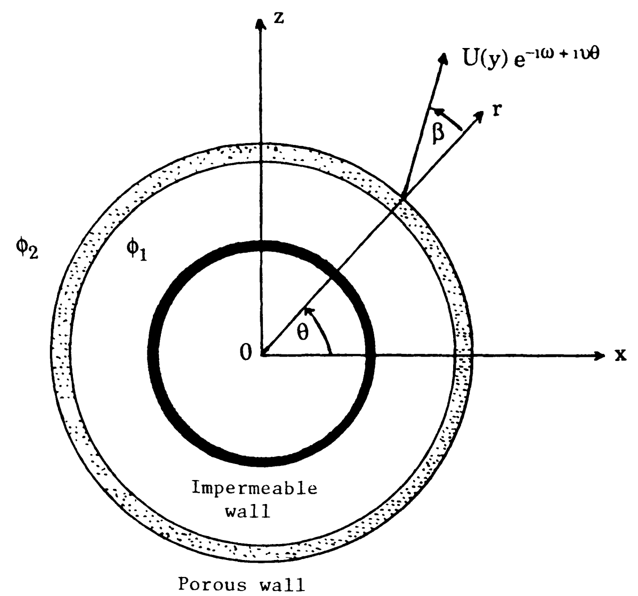

Fig. 1: Schematic diagram of a horizontal cross-section of the physical problem.

Let the velocity of the impermeable wall at time t is $U(y)\exp(-i\omega t + iv\theta)$, where $v = \sin\beta$, β is the angle that the produced train of waves makes with the radial direction and $U(y)$ is a complex valued and suitably limited. The resulting motion is therefore time and θ harmonics with the same ω and v as of the impermeable wall.

We assume that the fluid is incompressible and inviscid and that the motion originates from rest, by virtue of which there exist velocity potentials $\phi_j(r, \theta, y; t)$ such that

$$\phi_j(r, \theta, y; t) = Re[\phi_j(r, y)\exp(- i\omega t + iv\theta)]$$

where the subscripts $j = 1, 2$ refer to the regions $a < r < b$ and $r > b$ respectively Also the motion is assumed small so that the linearization is permissible. We consider here first the case when the fluid is of infinite depth The functions $\phi_j(r, y)$ satisfy

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial y^2} - \frac{v^2}{r^2}\right]\phi_j = 0\ , \qquad y > 0\ , \tag{2 1}$$

The linearized free surface condition is

$$K\phi_j + \frac{\partial}{\partial y}\phi_j = 0\ , \qquad \text{on} \qquad y = 0\ , \tag{2 2}$$

where $K = \frac{\omega^2}{g}$ and g is the gravitational constant.

On the impermeable wall

$$\frac{\partial}{\partial r}\phi_1 = U(y)\ , \qquad \text{on} \qquad r = a\ , \tag{2 3}$$

and on the porous wall

$$\frac{\partial}{\partial r}\phi_1 = \frac{\partial}{\partial r}\phi_2\ , \qquad \text{on} \qquad r = b\ . \tag{2 4}$$

We shall also assume that the porous wall is made of material with very fine pores. Thus according to Taylor's assumption [14] we have

$$\frac{\partial}{\partial r}\phi_j = \frac{d}{\mu}(p_1 - p_2) = iG(\phi_1 - \phi_2)\ , \qquad \text{on} \qquad r = b\ , \tag{2.5}$$

where $g = \rho\omega d/\mu$, μ is the dynamic viscosity, ρ is the constant density of the fluid and d is a coefficient which has the dimension of length. It should be noted here if the porous flow through the wall is significant, condition (2.5) may not be accurate enough. Hence we should confine our investigation to porous walls with fine pores. Finally we have the condition for no motion at infinite depth,

$$\nabla\phi_j \to 0 \qquad \text{as} \qquad y \to \infty \tag{2.6}$$

and the radiation condition for the outgoing waves

$$\phi_2 \to CH_v^{(1)}(Kr)e^{-Ky} \qquad \text{as} \qquad r \to \infty \tag{2.7}$$

where C is multiple constant and $H_v^{(1)}(z)$ is Hankel's Bessel function of third kind of order v. The parameter G is a measure of the porous effect. $G = 0$ means the wall is impermeable, while as G approaches infinity the wall becomes completely permeable to the fluid.

3. SOLUTION

Using the method of separation of variables and superposing basic solutions of Laplace's equation (2.1) appropriate to the present problem, let

$$\begin{aligned}\phi_1(r, y) = &\int_0^\infty [A(k)I_v(kr) + B(k)K_v(kr)]\, f(k, y)\, dk \\ &+ [\alpha J_v(Kr) + \beta H_v^{(1)}(Kr)]\, e^{-Ky}\ ,\end{aligned} \tag{3 2}$$

$$\phi_2(r,y) = \int_0^\infty C(k) K_v(kr)\, f(k,y)\, dk + \gamma H_v^{(1)}(Kr)\, e^{-Ky}\,, \tag{3 2}$$

where $f(k,y) = k\cos ky - K\sin ky$, and, as usual, $(J_v(z), Y_v(z))$ and $(I_v(z), K_v(z))$ are respectively the Bessel and modified Bessel functions of v-th order

These satisfy (2 2), (2 6) and (2 7) Conditions (2 4) and (2 5) give A, B in terms of C and α, β in terms of γ as

$$\begin{gathered} A = -\frac{b}{iG} K_v'^2(kb) C\,, \qquad B = \frac{1}{iG}[iG + bK_v'(kb) I_v'(kb)]C\,, \\ \alpha = -\frac{\pi b}{2G}[H_v^{(1)\prime}(Kb)]^2 \gamma\,, \quad \beta = \frac{1}{G}[G + \frac{\pi b}{2} J_v'(Kb) H_v^{(1)\prime}(Kb)]\gamma\,, \end{gathered} \tag{3 3}$$

where $'$ denotes differentiation with respect to r.

From (3 1), (3 3), (2.3) we get

$$U(y) = \frac{\pi i}{2G} e^{\pi i v/2} \int_0^\infty C(k) \triangle(ik)\, f(k,y)\, dk + \frac{\gamma}{G} \triangle(K)\, e^{-Ky} \tag{3.4}$$

in which

$$\triangle(K) = G H_v^{(1)\prime}(Ka) + M(K)\, H_v^{(1)\prime}(Kb) \tag{3.5}$$

where we have used the Wronskian relations

$$W[I_v(z), K_v(z)] = -\frac{1}{z} \qquad \text{and} \qquad W[H_v^{(1)}(z), J_v(z)] = \frac{2i}{\pi z}$$

in (3.5)

$$M(K) = \frac{1}{2}\pi b [H_v^{(1)\prime}(Ka)\, J_v'(Kb) - H_v^{(1)\prime}(Kb)\, J_v'(Ka)]$$

Multiplying (3.4) by e^{-Ky} and integrating with respect to y from 0 to ∞ we get

$$\gamma = \frac{2\pi K A G}{\triangle(K)}\,, \qquad \text{where} \qquad A = -\frac{1}{\pi K} \int_0^\infty U(y)\, e^{-Ky}\, dy\,.$$

But $U(y)$ has the unique expansion

$$U(y) = -2 \int_0^\infty \frac{k a(k)}{k^2 + K^2} f(k,y)\, dk - 2\pi K A e^{-Ky}\,, \tag{3.6}$$

where

$$a(k) = -\frac{1}{\pi k} \int_0^\infty U(y)\, f(k,y)\, dy\,,$$

which can be easily proved by a straightforward application of the Fourier sine integral of $U(y)$.

Comparing (3.4) and (3.6) we obtain

$$C(k) = \frac{4i e^{-i\pi v/2} k G a(k)}{\pi (k^2 + K^2) \triangle(ik)}\,.$$

Hence

$$\phi_1 = \frac{4}{\pi} e^{-i\pi v/2} \int_0^\infty \frac{ka(k) f(k,y)}{(k^2+K^2)\Delta(ik)} \Big[[iG + bK_v'(kb) I_v'(kb)] K_v(kr)$$
$$- bK_v'^2(kb)\, I_v(kr) \Big] \, dk - \frac{2\pi K A}{\Delta(K)} \Big[\Big[G + \frac{1}{2}\pi b\, J_v'(Kb) H_v^{(1)\prime}(Kb) \Big] H_v^{(1)}(Kr)$$
$$- \frac{1}{2}\pi b [H_v^{(1)\prime}(Kb)]^2 J_v(Kr) \Big] e^{-Ky} , \tag{3 7}$$

$$\phi_2 = \frac{4}{\pi} i\, e^{-i\pi v/2} \int_0^\infty \frac{kGa(k) f(k,y)}{(k^2+K^2)\Delta(ik)} K_v(kr)\, dk - \frac{2\pi K A G}{\Delta(K)} H_v^{(1)}(Kr)\, e^{-Ky} \tag{3.8}$$

The hydrodynamic pressure distribution on the porous wall ($r = b$) is

$$P = \frac{4}{\pi} i \rho\omega e^{-i\pi v/2} \int_0^\infty \frac{ka(k) f(k,y)}{(k^2+K^2)\Delta(ik)} K_v'(kb)\, dk - \rho\omega \frac{2\pi K A}{\Delta(K)} H_v^{(1)\prime}(Kr)\, e^{-Ky} , \tag{3.9}$$

and the free surface elevation is

$$\omega\eta_1(r) = -\frac{4}{\pi} iK e^{-i\pi v/2} \int_0^\infty \frac{k^2 a(k)}{(k^2+K^2)\Delta(ik)} \Big[[iG + bK_v'(kb)] K_v(kr) \tag{3.10}$$
$$- bK_v'^2(kb) I_v(kr) \Big] dk + i \frac{2\pi K^2 A}{\Delta(K)} \Big[\Big[G + \frac{1}{2}\pi b J_v'(Kb) H_v^{(1)\prime}(Kb) \Big] H_v^{(1)}(Kr)$$
$$- \frac{1}{2}\pi b [H_v^{(1)\prime}(Kb)]^2 J_v(Kr) \Big] ,$$

$$\omega\eta_2(r) = \frac{4}{\pi} KG e^{-i\pi v/2} \int_0^\infty \frac{k^2 a(k)}{(k^2+K^2)\Delta(ik)} K_v(kr) dk + iG \frac{2\pi K^2 A}{\Delta(K)} H_v^{(1)}(Kr) \tag{3.11}$$

The second term on the right hand side of equation (3.11) represents the outgoing waves transmitted through the porous cylinder.

When the porous wall ($r = b$) is completely permeable i.e. $G \to \infty$, the velocity potential in the region $r > a$ is

$$-2\int_0^\infty \frac{ka(k) f(k,y)}{k^2+K^2} \frac{K_v(kr)}{K_v'(ka)} dk - 2\pi K A \frac{H_v^{(1)}(Kr)}{H_v^{(1)\prime}(Ka)} e^{-Ky} . \tag{3.12}$$

Also when the porous wall at ($r = b$) becomes impermeable, $G = 0$, the results (3.7), (3.8) reduce to

$$\phi_1 = 2\int_0^\infty \frac{ka(k) f(k,y)}{k^2+K^2} \frac{K_v'(kb) I_v(kr) - I_v'(kb) K_v(kr)}{K_v'(ka) I_v'(kb) - K_v'(kb) I_v'(ka)} dk$$
$$+ 2KA \frac{H_v^{(1)\prime}(Kb) J_v(Kr) - J_v'(Kb) H_v^{(1)}(Kr)}{H_v^{(1)\prime}(Ka) J_v'(Kb) - H_v^{(1)\prime}(Kb) J_v'(Ka)} e^{-Ky} ,$$
$$\phi_2 = 0 .$$

This solution is valid only when the quantity

$$Y_v'(Ka) J_v'(Kb) - Y_v'(Kb) J_v'(Ka) \tag{3.13}$$

is different from zero. However, it indicates that when this quantity vanishes, resonance occurs and linearized theory for small motion cannot be applied.

In the particular case when $U(y) = Ve^{-Ky}$, where V is a real constant, we have

$$\phi_2 = \frac{V}{\triangle(K)}\left[\left[G+\frac{1}{2}\pi b J_v'(Kb)H_v^{(1)\prime}(Kb)\right]H_v^{(1)}(Kr)-\frac{1}{2}\pi b\left[H_v^{(1)\prime}(Kb)\right]^2 \times H_v^{(1)}(Kr)\right]e^{-Ky} \tag{3.14}$$

$$\phi_2 = \frac{GV}{\triangle(K)} H_v^{(1)}(Kr)e^{-Ky}\ , \tag{3.15}$$

The distribution of pressure on $r = b$ is

$$P = \frac{\rho\omega V}{\triangle(K)} H_v^{(1)\prime}(Kb)e^{-Ky}\ . \tag{3.16}$$

4. THE FINITE DEPTH CASE

Now we consider the case of finite depth h. Using the same notation and coordinates, the complex potentials ϕ_j, $j = 1, 2$, for the motion in the fluid regions $a < r < b$, $r > b$ are the solutions of the boundary value problem stated in Section 2 with conditions (2.6), (2.7) replaced by

$$\frac{\partial}{\partial y}\phi_j = 0 \qquad \text{on} \qquad y = h\ , \tag{4.1}$$

$$\phi_2 \to CH_v^{(1)}(k_0 r)\cosh k_0(h-y) \qquad \text{as} \qquad r \to \infty \tag{4.2}$$

when C is a constant multiple, k_0 is the real positive root of

$$k \sinh k h \ - \ K \cosh k h \ = 0\ .$$

The method of separation of variables can also be used here to get solutions for the equations (2.1) that satisfy (2.2), (4.1) and (2.2), (4.1), (4.2). Let then

$$\phi_1(r,y) = \sum_{n=1}^{\infty}[A_n I_v(k_n r) + B_n K_v(k_n r)]\cos k_n(h-y) + [\alpha J_v(k_0 r) + \beta H_v^{(1)\prime}(k_0 r)]\cosh k_0(h-y)\ , \tag{4.3}$$

$$\phi_2(r,y) = \sum_{n=1}^{\infty} C_n K_v(k_n r)\cos k_n(h-y) + H_v^{(1)}(k_0 r)\cosh k_n(h-y)\ , \tag{4.4}$$

where k_n are the real positive roots of

$$k \sin kh + K \cos kh = 0\ .$$

The remaining conditions (2.3)-(2.5) are satisfied if

$$U(y) = \frac{\pi i}{2G}e^{-i\pi v/2}\sum_{n=1}^{\infty} C_n \triangle(ik_n)\cos k_n(h-y) + \frac{\gamma}{G}\triangle(k_0)\cosh k_0(h-y) \tag{4.5}$$

since the eigenfunctions $\cosh k_0(h-y)$ and $\cos k_n(h-y)$ are orthogonal over the inteval $(0,h)$, we obtain the constants as

$$C_n = 8i\,e^{-i\pi v/2}\,\frac{k_n a_n G\cosh k_n h}{\delta_n\ \triangle\ (ik_n)}\ , \qquad \gamma = \ -4\pi\,\frac{k_0 a_0 G\cosh k_0 h}{\delta_0\ \triangle\ (ik_0)}\ ,$$

where

$$\delta_0 = 2k_0 h + \sinh 2k_0 h\ , \qquad \delta_n = 2k_n h + \sinh 2k_n h\ ,$$

$$a_0 = \frac{1}{\pi \cosh k_0 h} \int_0^h U(y) \cosh k_0 h \, dy \ ,$$

$$a_n = - \frac{1}{\pi \cos k_n h} \int_0^h U(y) \cos k_n h dy$$

Consequently

$$\begin{aligned}\phi_1 = & - 8e^{-i\pi v/2} \sum_{n=1}^{\infty} \frac{a_n k_n \cos k_n h}{\delta_n \triangle(ik_n)} \Big[bK_v'^2(k_n b) I_v(k_n r) - \big[iG + bK_v'(k_n b) \times \\ & I_v'(k_n b) \big] K_v(k_n r) \Big] \cos k_n (h-y) \\ & + 4\pi \frac{a_0 k_0 \cosh k_0 h}{\delta_0 \triangle(k_0)} \Big[\frac{1}{2} \pi b \Big(H_v^{(1)}(k_0 b) \Big)^2 J_v(k_0 r) - \Big[G + \frac{1}{2} \pi b H_v^{(1)\prime}(k_0 b) \times \\ & J_v'(k_0 b) \Big] H_v^{(1)}(k_0 r) \Big] \cosh k_0 (h-y) \ ,\end{aligned} \tag{4.6}$$

$$\begin{aligned}\phi_2 = & \, 8\, i\, G\, e^{-i\pi v/2} \sum_{n=1}^{\infty} \frac{a_n k_n \cos k_n h}{\delta_n \triangle(ik_n)} K_v(k_n r) \cos k_n (h-y) \\ & + 4\pi G \frac{a_0 k_0 \cosh k_0 h}{\delta_0 \triangle(k_0)} H_v^{(1)}(k_0 r) \cosh k_0 (h-y) \ ,\end{aligned} \tag{4.7}$$

We consider the following two special cases.

(i) When $U(y) = V$, (V is a constant). In this case

$$a_0 = - \frac{V}{\pi k_0} \tanh k_0 h \ , \qquad a_n = - \frac{V}{\pi k_n} \tanh k_n h$$

Therefore

$$\begin{aligned}\phi_1 = & \frac{8V}{\pi} e^{-i\pi v/2} \sum_{n=1}^{\infty} \frac{\sin k_n h}{\delta_n \triangle(ik_n)} \Big[bK_v'^2(k_n b) I_v(k_n r) - \big[iG + bK_v'(k_n b) \times \\ & I_v'(k_n b) \big] K_v(k_n r) \Big] \cos k_n (h-y) \\ & - 4V \frac{\sinh k_0 h}{\delta_0 \triangle(k_0)} \Big[\frac{1}{2} \pi b \Big(H_v^{(1)}(k_0 b) \Big)^2 J_v(k_0 r) - \Big[G + \frac{1}{2} \pi b H_v^{(1)\prime}(k_0 b) \times \\ & J_v'(k_0 b) \Big] H_v^{(1)}(k_0 r) \Big] \cosh k_0 (h-y) ,\end{aligned} \tag{4.8}$$

$$\begin{aligned}\phi_2 = & \frac{8iVG}{\pi} e^{-i\pi v/2} \sum_{n=1}^{\infty} \frac{\sin k_n h}{\delta_n \triangle(ik_n)} K_v(k_n r) \cos k_n (h-y) \\ & + 4VG \frac{\sinh k_0 h}{\delta_0 \triangle(k_0)} H_v^{(1)}(k_0 b) \cosh k_0 (h-y) \ ,\end{aligned} \tag{4.9}$$

(ii) When $U(y) = V \cosh k_0 (h-y)$, we get

$$a_0 = - \frac{\delta_0 V}{4\pi k_0 \cosh k_0 h} \ , \qquad a_n = 0 \ ,$$

and

$$\phi_1 = -\frac{V}{\triangle(k_0)}\left[\frac{1}{2}\pi b\big(H_v^{(1)}(k_0 b)\big)^2 J_v(k_0 r) - \left[G + \frac{1}{2}\pi b H_v^{(1)\prime}(k_0 b) J_v'(k_0 b)\right] \times \right.$$
$$\left. H_v^{(1)}(k_0 r)\right] \cosh k_0(h-y)\ , \tag{4.10}$$

$$\phi_2 = \frac{VG}{\triangle(k_0)} H_v^{(1)}(k_0 r) \cosh k_0(h-y)\ , \tag{4.11}$$

5. OBLIQUE WAVES GENERATED BY THE POROUS WALL

If we now let the porous wall oscillate obliquely with velocity $U(y)\exp(-i\omega t + iv\theta)$ while the impermeable wall at $r = a$ be kept fixed, then the new boundary value problem is the same as stated in Section 2 except that the boundary conditions (2 3), (2.5) are replaced by

$$\frac{\partial}{\partial r}\phi_1 = 0\ , \qquad \text{on} \qquad r = a \tag{5.1}$$

$$\frac{\partial}{\partial r}\phi_j - U(y) = iG(\phi_1 - \phi_2)\ , \qquad \text{on} \qquad r = b \tag{5.2}$$

Thus when the porous wall is the wave generator we have

$$\phi_1 = \frac{4b}{\pi} e^{-i\pi v/2}\int_0^\infty \frac{ka(k)f(k,y)}{(k^2+K^2)\triangle(ik)}\left[I_v(kr)K_v'(ka) - K_v(kr)I_v'(ka)\right]K_v'(kb)dk$$
$$-\frac{\pi^2 bKA}{\triangle(K)}\left[J_v(kr)H_v^{(1)\prime}(Ka) - H_v^{(1)}(Kr)J_v'(Ka)\right]H_v^{(1)\prime}(Kb)e^{-Ky} \tag{5.3}$$

$$\phi_2 = \frac{4b}{\pi} e^{-i\pi v/2}\int_0^\infty \frac{ka(k)f(k,y)}{(k^2+K^2)\triangle(ik)}\left[I_v(kb)K_v'(ka) - K_v'(kb)I_v'(ka)\right]K_v(kr)dk$$
$$-\frac{\pi^2 bKA}{\triangle(K)}\left[J_v'(Kb)H_v^{(1)\prime}(Ka) - H_v^{(1)\prime}(Kb)J_v'(ka)\right]H_v^{(1)}(Kr)e^{-Ky}\ . \tag{5.4}$$

When $M = 0$, the waves are trapped in the bounded region between the two cylinders $a \le r \le b$ and no waves radiate away from the wall, liquid simply piles up around the wall.

6. WAVE TRAPPING

In this section we investigate an interesting application of the above results to the case of a time cylindrical wave $CH_v^{(2)}(kR)\exp(iv\theta - Ky)$ incident obliquely, proceeding from infinity, the porous cylindrical wall at $r = b$ and the impermeable cylindrical wall at $r = a$ both fixed. The velocity potentials $\phi_j(r,y)$ are functions that satisfy (2.1), (2.2) and (2.6). On the porous wall

$$\left.\begin{aligned}\frac{\partial}{\partial r}\phi_1 &= \frac{\partial}{\partial r}\phi_2 \qquad (6.1)\\ &= iG(\phi_1 - \phi_2)\,, \qquad (6.2)\end{aligned}\right\} r = b$$

and on the impermeable wall

$$\frac{\partial}{\partial r}\phi_1 = 0 \qquad \text{on} \qquad r = a \tag{6.3}$$

Moreover

$$\phi_2 \to CH_v^{(2)}(Kr)e^{-Ky} + AH_v^{(1)}(Kr)e^{-Ky}\,, \qquad \text{as} \qquad r \to \infty \tag{6.4}$$

Here A (to be determined) is a complex constant relating to the amplitude and phase of the reflected wave

Consider the functions

$$\Psi_j = \phi_j(r,y) - 2C\,J_v(Kr)e^{-Ky}$$

These new functions satisfy equations (2 1) and the free surface boundary conditions On the porous wall $(r = b)$,

$$\frac{\partial}{\partial r}\Psi_1 = \frac{\partial}{\partial r}\Psi_2 \tag{6.5}$$

$$= iG(\Psi_1 - \Psi_2) - 2C\,J_v'(Kb)e^{-Ky}\,, \tag{6.6}$$

and on the impermeable wall

$$\frac{\partial}{\partial r}\Psi_1 = -2C\,J_v'(Ka)e^{-Ky}\,. \tag{6.7}$$

And

$$\Psi_2 \to (A-C)H_v^{(1)}(Kr)e^{-Ky}\,, \qquad \text{as} \qquad r \to \infty\,. \tag{6.8}$$

Since the present problem is linear, Ψ_1, Ψ_2 can be obtained by a suitable superposition of the results (3 7), (5.3) and (3.8), (5 4) respectively. Hence

$$\phi_1 = \frac{2CG}{\Delta(K)}\left[J_v(Kr)H_v^{(1)\prime}(Ka) - J_v'(Ka)H_v^{(1)}(Kr)\right]e^{-Ky} \tag{6.9}$$

$$\phi_2 = \left[-C\,\frac{\Delta*(K)}{\Delta(K)}\,H_v^{(1)}(Kr) + C\,H_v^{(2)}(Kr)\right]e^{-Ky}\,, \tag{6.10}$$

where $\Delta^*(K) = G\,H_v^{(2)\prime}(Ka) + M(K)H_v^{(2)\prime}(Kb)$.

The coefficient of reflection R is defined as the square of the ratio of the amplitude of the reflected wave to the amplitude to the incident wave i.e.

$$R = \left|\frac{\Delta^*(K)}{\Delta(K)}\right| = \frac{\alpha^2 - 2M^2G + \beta^2M^2}{\alpha^2 + 2M^2G + \beta^2M^2} \tag{6.11}$$

where $\alpha^2 = \frac{1}{2}\pi b\left[J_v'^2(Ka) + Y_v'^2(Ka)\right]$, $\beta^2 = \frac{1}{2}\pi b\left[J_v'^2(Kb) + Y_v'^2(Kb)\right]$ when the wall at $r = b$ is impermeable i.e., when $G = 0$, the incident wave is totally reflected by it. We get the same situation when the wall $(r = b)$ is completely permeable but now the wave is totally reflected by the impermeable wall at $r = a$. We note also that when $M = 0$ i.e. when a and b has values satisfying the equation

$$J_v'(Kb)Y_v'(Ka) - J_v'(Ka)Y_v'(Kb) = 0\,, \tag{6.12}$$

the incident wave is totally reflected $(R = 1)$ at $r = b$ irrespective of the value of G. By simple differentiation of (6.11) with respect to G for any fixed values of a and b, R reduces to a minimum,

$$R_{\min} = \frac{\alpha\beta - M}{\alpha\beta + M}\,, \tag{6.13}$$

when $G = \frac{M\beta}{\alpha}$; this minimum value vanishes when $\alpha\beta = M$ i.e., when a and b satisfy the equation

$$J_v'(Kb)J_v'(Ka) + Y_v'(Kb)Y_v'(Ka) = 0\,, \tag{6.14}$$

That is $R = 0$ when $G = \beta^2$ and a, b has values satisfying equation (6.14). Under these circumstances the porous wall acts as an efficient wave absorber or eliminator for the incident waves, i.e., for $G = \beta^2$

and for values of a and b which satisfy equation (6 14), there is a wave trapping phenomenon, that is waves will be trapped inside the region $a \leq r \leq b$

Now we give an estimate for $R_{\min}$ (equation (6 13)) for large Ka and Kb for the case of radial incident waves ($v = 0$) This can be done using the asymptotic formulas of $J_1(Kr)$ and $Y_1(Kr)$ for larger Kr which are

$$J_1(Kr) \sim \sqrt{\frac{2}{\pi Kr}} \sin\left(Kr - \frac{\pi}{4}\right), \qquad Y_1(Kr) \sim -\sqrt{\frac{2}{\pi Kr}} \cos\left(Kr - \frac{\pi}{4}\right).$$

In this case

$$\frac{\alpha^2}{K} \simeq \frac{b}{a}, \qquad \frac{\beta^2}{K} \simeq 1, \qquad \frac{M}{K} \simeq \frac{b}{a} \sin K(b-a)$$

Thus

$$R_{\min} \simeq \frac{1 - \sin K(b-a)}{1 + \sin K(b-a)},$$

and therefore, $R_{\min} \sim 0$ when $K(b-a) = \frac{\pi}{2} + \pi s$, $s = 0, 1, 2, \ldots$. This means the porous wall together with the fluid region between it and the impermeable wall acts as an efficient wave absorber for incident waves of wave length $\frac{4(b-a)}{1+2s}$. In fact this result agrees with that obtained in the two dimensional surface wave case treated in [13].

7. NUMERICAL RESULTS

For radial incident waves ($v = 0$), since

$$J_0'(Kr) = -KJ_1(Kr) \qquad \text{and} \qquad Y_0'(Kr) = -KY_1(Kr),$$

the conditions (6.14) for wave trapping and (6.12) for total reflection become

$$J_1(Kb)J_1(Ka) + Y_1(Kb)Y_1(Ka) = 0, \tag{7.1}$$

$$J_1(Kb)J_1(Ka) - Y_1(Kb)Y_1(Ka) = 0, \tag{7.2}$$

respectively For those values of Ka and Kb which satisfy (7.1), $\frac{G}{K} = \frac{1}{2}\pi bK\left[J_1^2(Kb) + Y_1^2(Kb)\right]$. When the angle of incident waves makes 30° with the radial direction ($v = 0.5$), since

$$J_{1/2}(Kr) = \sqrt{\frac{2}{\pi Kr}} \sin Kr, \qquad Y_{1/2}(Kr) = \sqrt{\frac{2}{\pi Kr}} \cos Kr,$$

conditions (6.14) and (6.12) now take the simpler forms

$$(4abK^2 + 1)\cos K(b-a) + 2K(b-a)\sin K(b-a) = 0, \tag{7.3}$$

$$2K(b-a)\cos K(b-a) - (4abK^2 + 1)\sin K(b-a) = 0. \tag{7.4}$$

Thus for the case of $v = 0.5$ and those values of Ka, Kb which satisfy (6.3), $\frac{G}{K} = 1 + \frac{1}{4K^2b^2}$. For fixed Ka or Kb equations (7.1)-(7.4) has infinite number of real roots. Table 1 lists the first few roots $Kb(> Ka)$ of equations (7.1), (7.3) which correspond $v = 0$ and $v = 0.05$ respectively for fixed $Ka = 2\pi$ and the corresponding value of $\frac{G}{K}$ at each Kb Table 2 lists also the first few roots of (7.2), (7.4) for $Ka = 2\pi$ and $\frac{\pi}{2}$. We note that for all cases listed successive large roots differ approximately by π and that $\frac{G}{K} \simeq 1$.

Table 1 Values of $Kb(> Ka)$ and G/K for wave trapping

$v = 0$	G/K	$v = 0.5$	G/K
7 86568689	1 00597502	6 51475569	1 00589039
11 02072268	1.00306449	9 68134717	1.00266728
14 16981407	1 00185914	12 83560523	1 00151743
17 31618973	1 00124677	15 98486294	1 00097841
20 46109864	1 00089374	19 13159616	1 00068303
23 60512575	1 00067188	22 27677662	1 00050377
26 74858147	1 00052344	25.42124425	1.00038685
29 89164583	1.00041925	28 56500080	1.00030639
33.03443042	1 00034334	31.70832819	1.00024865
36 17700808	1 00028632	34 85134260	1.00020583
39 31942840	1 00024241	37 99412174	1.00017138
42.46172629	1 00020788	41.13671958	1.00014773
45 60392703	1.00018023	44.27917471	1.00012751
48 74604942	1.00015775	47.42151553	1 00011117
51.88810767	1 00013923	50 56376335	1.00009778

Table 2 Values of $Kb(> Ka)$ for complete reflection

$v = 0$		$v = 0.5$	
$Ka = 2\pi$	$Ka = \pi/2$	$Ka = 2\pi$	$Ka = \pi/2$
9.44431648	4.84806631	9 45133478	4.91926471
12.59573441	8.01922751	12 60613831	8.10050436
15.74324000	11 17382989	15 75564940	11.25936504
18.88878187	14.32274063	18.90252072	14.41065337
22.03319888	17.46902417	22.04788468	17.55846006
25.17691168	20.61387985	25.19230664	20.70437653
28.32015444	23.75787344	28.33610051	23.84915194
31.46306786	26.90130671	31.47945463	26.99318555
34.60574159	30.04435529	34.62248884	30.13670974
37.74823545	33.18712837	37.76528307	33.27986897
40.89059088	36 32969739	40.90789266	36.42275777
44 03283750	39.47211105	44 05035712	39.56544062
47.17499704	42.61440369	47 19270547	42.70796301
50.31708581	45.75660024	50.33495946	45.85035794
53.45911628	48.89871921	53 471352	48.99264994

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