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Research Article

Implicative Ideals of BCK-Algebras Based on the Fuzzy Sets and the Theory of Falling Shadows

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Based on the theory of falling shadows and fuzzy sets, the notion of a falling fuzzy implicative ideal of a BCK-algebra is introduced. Relations among falling fuzzy ideals, falling fuzzy implicative ideals, falling fuzzy positive implicative ideals, and falling fuzzy commutative ideals are given. Relations between fuzzy implicative ideals and falling fuzzy implicative ideals are provided.

1. Introduction and Preliminaries

1.1. Introduction

In the study of a unified treatment of uncertainty modelled by means of combining probability and fuzzy set theory, Goodman [1] pointed out the equivalence of a fuzzy set and a class of random sets. Wang and Sanchez [2] introduced the theory of falling shadows which directly relates probability concepts with the membership function of fuzzy sets. Falling shadow representation theory shows us the way of selection relaid on the joint degrees distributions. It is reasonable and convenient approach for the theoretical development and the practical applications of fuzzy sets and fuzzy logics. The mathematical structure of the theory of falling shadows is formulated in [3]. Tan et al. [4, 5] established a theoretical approach to define a fuzzy inference relation and fuzzy set operations based on the theory of falling shadows. Jun and Park [6] discussed the notion of a falling fuzzy subalgebra/ideal of a BCK/BCI-algebra. Jun and Kang [7, 8] also considered falling fuzzy positive implicative ideals and falling fuzzy commutative ideals. In this paper, we establish a theoretical approach to define a fuzzy implicative ideal in a BCK-algebra based on the theory of falling shadows.

We consider relations between fuzzy implicative ideals and falling fuzzy implicative ideals. We provide relations among falling fuzzy ideals, falling fuzzy implicative ideals, falling fuzzy positive implicative ideals, and falling fuzzy commutative ideals.

1.2. Basic Results on BCK-Algebras and Fuzzy Aspects

A BCK/BCI-algebra is an important class of logical algebras introduced by Iséki and was extensively investigated by several researchers.

An algebra $(X;*,0)$ of type $(2,0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(i) $(\forall x,y,z \in X)\ (((x*y)*(x*z))*(z*y)=0)$,

(ii) $(\forall x,y \in X)\ ((x*(x*y))*y=0)$,

(iii) $(\forall x \in X)\ (x*x=0)$,

(iv) $(\forall x,y \in X)\ (x*y=0, y*x=0 \Rightarrow x=y)$.

If a BCI-algebra X satisfies the following identity:

(v) $(\forall x \in X)\ (0*x=0)$,

then X is called a *BCK-algebra*. Any BCK-algebra X satisfies the following axioms:

(a1) $(\forall x \in X)\ (x*0=x)$,

(a2) $(\forall x,y,z \in X)\ (x \le y \Rightarrow x*z \le y*z,\ z*y \le z*x)$,

(a3) $(\forall x,y,z \in X)\ ((x*y)*z=(x*z)*y)$,

where $x \le y$ if and only if $x*y=0$.

A subset I of a BCK-algebra X is called an *ideal* of X if it satisfies the following:

(b1) $0 \in I$,

(b2) $(\forall x \in X)\ (\forall y \in I)\ (x*y \in I \Rightarrow x \in I)$.

Every ideal I of a BCK-algebra X has the following assertion:

$$(\forall x \in X)\ (\forall y \in I) \quad (x \le y \Longrightarrow x \in I). \tag{1.1}$$

A subset I of a BCK-algebra X is called a *positive implicative ideal* of X if it satisfies (b1) and

(b3) $(\forall x,y,z \in X)\ ((x*y)*z \in I,\ y*z \in I \Rightarrow x*z \in I)$.

A subset I of a BCK-algebra X is called a *commutative ideal* of X if it satisfies (b1) and

(b4) $(\forall x,y,z \in X)\ ((x*y)*z \in I,\ z \in I \Rightarrow x*(y*(y*x)) \in I)$.

A subset I of a BCK-algebra X is called an *implicative ideal* of X if it satisfies (b1) and

(b5) $(\forall x,y,z \in X)\ ((x*(y*x))*z \in I,\ z \in I \Rightarrow x \in I)$.

We refer the reader to the paper [9] and book [10] for further information regarding BCK-algebras.

A fuzzy set μ in a BCK-algebra X is called a *fuzzy ideal* of X (see [11]) if it satisfies the following:

(c1) $(\forall x \in X)\ (\mu(0) \geq \mu(x))$,

(c2) $(\forall x, y \in X)\ (\mu(x) \geq \min\{\mu(x * y), \mu(y)\})$.

A fuzzy set μ in a BCK-algebra X is called a *fuzzy positive implicative ideal* of X (see [12]) if it satisfies (c1) and

(c3) $(\forall x, y, z \in X)\ (\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y * z)\})$.

A fuzzy set μ in a BCK-algebra X is called a *fuzzy commutative ideal* of X (see [13]) if it satisfies (c1) and

(c4) $(\forall x, y, z \in X)\ (\mu(x * (y * (y * x))) \geq \min\{\mu((x * y) * z), \mu(z)\})$.

A fuzzy set μ in a BCK-algebra X is called a *fuzzy implicative ideal* of X (see [14]) if it satisfies (c1) and

(c5) $(\forall x, y, z \in X)\ (\mu(x) \geq \min\{\mu((x * (y * x)) * z), \mu(z)\})$.

Proposition 1.1 (see [11, 14]). *Let μ be a fuzzy set in a BCK-algebra X. Then μ is a fuzzy (implicative) ideal of X if and only if*

$$(\forall t \in [0,1]) \quad \left(\mu_t \neq \emptyset \Longrightarrow \mu_t \text{ is an (implicative) ideal of } X\right), \tag{1.2}$$

where $\mu_t := \{x \in X \mid \mu(x) \geq t\}$.

1.3. The Theory of Falling Shadows

We first display the basic theory on falling shadows. We refer the reader to the papers [1–5] for further information regarding falling shadows.

Given a universe of discourse U, let $\mathcal{P}(U)$ denote the power set of U. For each $u \in U$, let

$$\dot{u} := \{E \mid u \in E,\ E \subseteq U\}, \tag{1.3}$$

and for each $E \in \mathcal{P}(U)$, let

$$\dot{E} := \{\dot{u} \mid u \in E\}. \tag{1.4}$$

An ordered pair $(\mathcal{P}(U), \mathcal{B})$ is said to be a *hyper-measurable structure* on U if $\mathcal{B}$ is a σ-field in $\mathcal{P}(U)$ and $\dot{U} \subseteq \mathcal{B}$. Given a probability space $(\Omega, \mathcal{A}, P)$ and a hyper-measurable structure $(\mathcal{P}(U), \mathcal{B})$ on U, a *random set* on U is defined to be a mapping $\xi : \Omega \to \mathcal{P}(U)$ which is $\mathcal{A}$-$\mathcal{B}$ measurable, that is,

$$(\forall C \in \mathcal{B}) \quad \left(\xi^{-1}(C) = \{\omega \mid \omega \in \Omega,\ \xi(\omega) \in C\} \in \mathcal{A}\right). \tag{1.5}$$

Suppose that ξ is a random set on U. Let

$$\widetilde{H}(u) := P(\omega \mid u \in \xi(\omega)) \quad \text{for each } u \in U. \tag{1.6}$$

Then $\widetilde{H}$ is a kind of fuzzy set in U. We call $\widetilde{H}$ a *falling shadow* of the random set ξ, and ξ is called a *cloud* of $\widetilde{H}$.

For example, $(\Omega, \mathcal{A}, P) = ([0,1], \mathcal{A}, m)$, where $\mathcal{A}$ is a Borel field on $[0,1]$ and m is the usual Lebesgue measure. Let $\widetilde{H}$ be a fuzzy set in U and $\widetilde{H}_t := \{u \in U \mid \widetilde{H}(u) \geq t\}$ be a t-cut of $\widetilde{H}$. Then

$$\xi : [0,1] \longrightarrow \mathcal{P}(U), \quad t \longmapsto \widetilde{H}_t \tag{1.7}$$

is a random set and ξ is a cloud of $\widetilde{H}$. We will call ξ defined above as the *cut-cloud* of $\widetilde{H}$ (see [1]).

2. Falling Fuzzy Implicative Ideals

In what follows let X denote a BCK-algebra unless otherwise.

Definition 2.1 (see [6–8]). Let $(\Omega, \mathcal{A}, P)$ be a probability space, and let

$$\xi : \Omega \longrightarrow \mathcal{P}(X) \tag{2.1}$$

be a random set. If $\xi(\omega)$ is an ideal (resp., positive implicative ideal and commutative ideal) of X for any $\omega \in \Omega$, then the falling shadow $\widetilde{H}$ of the random set ξ, that is,

$$\widetilde{H}(x) = P(\omega \mid x \in \xi(\omega)) \tag{2.2}$$

is called a *falling fuzzy ideal* (resp., *falling fuzzy positive implicative ideal* and *falling fuzzy commutative ideal*) of X.

Let $(\Omega, \mathcal{A}, P)$ be a probability space and let

$$F(X) := \{f \mid f : \Omega \longrightarrow X \text{ is a mapping}\}, \tag{2.3}$$

where X is a BCK-algebra. Define an operation $\circledast$ on $F(X)$ by

$$(\forall \omega \in \Omega) \quad \big((f \circledast g)(\omega) = f(\omega) * g(\omega)\big), \tag{2.4}$$

for all $f, g \in F(X)$. Let $\theta \in F(X)$ be defined by $\theta(\omega) = 0$ for all $\omega \in \Omega$. Then $(F(X); \circledast, \theta)$ is a BCK-algebra (see [6]).

Definition 2.2. Let $(\Omega, \mathcal{A}, P)$ be a probability space and let

$$\xi : \Omega \longrightarrow \mathcal{P}(X) \tag{2.5}$$

be a random set. If $\xi(\omega)$ is an implicative ideal of X for any $\omega \in \Omega$, then the falling shadow $\widetilde{H}$ of the random set ξ, that is,

$$\widetilde{H}(x) = P(\omega \mid x \in \xi(\omega)) \tag{2.6}$$

is called a *falling fuzzy implicative ideal* of X.

For any subset A of X and $f \in F(X)$, let

$$\begin{gathered} A_f := \{\omega \in \Omega \mid f(\omega) \in A\}, \\ \xi : \Omega \longrightarrow \mathcal{P}(F(X)), \quad \omega \longmapsto \{f \in F(X) \mid f(\omega) \in A\}. \end{gathered} \tag{2.7}$$

Then $A_f \in \mathcal{A}$.

Theorem 2.3. *If A is an implicative ideal of X, then*

$$\xi(\omega) = \{f \in F(X) \mid f(\omega) \in A\} \tag{2.8}$$

is an implicative ideal of $F(X)$.

Proof. Assume that A is an implicative ideal of X and let $\omega \in \Omega$. Since $\theta(\omega) = 0 \in A$, we see that $\theta \in \xi(\omega)$. Let $f, g, h \in F(X)$ be such that $(f \circledast (g \circledast f)) \circledast h \in \xi(\omega)$ and $h \in \xi(\omega)$. Then

$$(f(\omega) * (g(\omega) * f(\omega))) * h(\omega) = ((f \circledast (g \circledast f)) \circledast h)(\omega) \in A \tag{2.9}$$

and $h(\omega) \in A$. Since A is an implicative ideal of X, it follows from (b5) that $f(\omega) \in A$ and so $f \in \xi(\omega)$. Hence $\xi(\omega)$ is an implicative ideal of $F(X)$. □

Since

$$\xi^{-1}(\dot{f}) = \{\omega \in \Omega \mid f \in \xi(\omega)\} = \{\omega \in \Omega \mid f(\omega) \in A\} = A_f \in \mathcal{A}, \tag{2.10}$$

we see that ξ is a random set on $F(X)$. Let

$$\widetilde{H}(f) = P(\omega \mid f(\omega) \in A). \tag{2.11}$$

Then $\widetilde{H}$ is a falling fuzzy implicative ideal of $F(X)$.

Table 1: Cayley table.

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	a
b	b	b	0	0	b
c	c	b	a	0	c
d	d	d	d	d	0

Example 2.4. Consider a BCK-algebra $X = \{0, a, b, c, d\}$ with a Cayley table which is given by Table 1 (see [10, page 274]). Let $(\Omega, \mathcal{A}, P) = ([0,1], \mathcal{A}, m)$ and let $\xi : [0,1] \to \mathcal{P}(X)$ be defined by

$$\xi(t) := \begin{cases} \{0, a\} & \text{if } t \in [0, 0.25), \\ \{0, b\} & \text{if } t \in [0.25, 0.55), \\ \{0, b, d\} & \text{if } t \in [0.55, 0.7), \\ \{0, a, b, c\} & \text{if } t \in [0.7, 1]. \end{cases} \tag{2.12}$$

Then $\xi(t)$ is an implicative ideal of X for all $t \in [0,1]$. Hence $\widetilde{H}$, which is given by $\widetilde{H}(x) = P(t \mid x \in \xi(t))$, is a falling fuzzy implicative ideal of X, and it is represented as follows:

$$\widetilde{H}(x) = \begin{cases} 1 & \text{if } x = 0, \\ 0.55 & \text{if } x = a, \\ 0.75 & \text{if } x = b, \\ 0.3 & \text{if } x = c, \\ 0.15 & \text{if } x = d. \end{cases} \tag{2.13}$$

Then

$$\widetilde{H}_t = \begin{cases} \{0\} & \text{if } t \in (0.75, 1], \\ \{0, a\} & \text{if } t \in (0.55, 0.75], \\ \{0, a, b\} & \text{if } t \in (0.3, 0.55], \\ \{0, a, b, c\} & \text{if } t \in (0.15, 0.3], \\ X & \text{if } t \in [0, 0.15]. \end{cases} \tag{2.14}$$

If $t \in (0.3, 0.55]$, then $\widetilde{H}_t = \{0, a, b\}$ is not an implicative ideal of X since $(c * (b * c)) * a = (c * 0) * a = c * a = b \in \widetilde{H}_t$ and $a \in \widetilde{H}_t$, but $c \notin \widetilde{H}_t$. It follows from Proposition 1.1 that $\widetilde{H}$ is not a fuzzy implicative ideal of X.

Theorem 2.5. *Every fuzzy implicative ideal of X is a falling fuzzy implicative ideal of X.*

Proof. Consider the probability space $(\Omega, \mathcal{A}, P) = ([0,1], \mathcal{A}, m)$, where $\mathcal{A}$ is a Borel field on $[0,1]$ and m is the usual Lebesque measure. Let μ be a fuzzy implicative ideal of X. Then μ_t is an implicative ideal of X for all $t \in [0,1]$ by Proposition 1.1. Let

$$\xi : [0,1] \longrightarrow \mathcal{P}(X) \tag{2.15}$$

be a random set and $\xi(t) = \mu_t$ for every $t \in [0,1]$. Then μ is a falling fuzzy implicative ideal of X. □

Example 2.4 shows that the converse of Theorem 2.5 is not valid.

Theorem 2.6. *Every falling fuzzy implicative ideal is a falling fuzzy ideal.*

Proof. Let $\widetilde{H}$ be a falling fuzzy implicative ideal of X. Then $\xi(\omega)$ is an implicative ideal of X, and hence it is an ideal of X. Thus $\widetilde{H}$ is a falling fuzzy ideal of X. □

The converse of Theorem 2.6 is not true in general as shown by the following example.

Example 2.7. Consider a BCK-algebra $X = \{0, a, b, c, d\}$ with a Cayley table which is given by Table 2 (see [10, page 260]). Let $(\Omega, \mathcal{A}, P) = ([0,1], \mathcal{A}, m)$ and let $\xi : [0,1] \rightarrow \mathcal{P}(X)$ be defined by

$$\xi(t) := \begin{cases} \{0, c\} & \text{if } t \in [0, 0.3), \\ \{0, a, b, d\} & \text{if } t \in [0.3, 1]. \end{cases} \tag{2.16}$$

Then $\xi(t)$ is an ideal of X for all $t \in [0,1]$. Hence $\widetilde{H}(x) = P(t \mid x \in \xi(t))$ is a falling fuzzy ideal of X, and

$$\widetilde{H}(x) = \begin{cases} 0.3 & \text{if } x = c, \\ 0.7 & \text{if } x \in \{a, b, d\}, \\ 1 & \text{if } x = 0. \end{cases} \tag{2.17}$$

In this case, we can easily check that $\widetilde{H}$ is a fuzzy ideal of X (see [6]). If $t \in [0, 0.3)$, then $\xi(t) = \{0, c\}$ is not an implicative ideal of X since $(a * (b * a)) * c \in \xi(t)$ and $c \in \xi(t)$, but $a \notin \xi(t)$. Therefore $\widetilde{H}$ is not a falling fuzzy implicative ideal of X.

Theorem 2.8. *Every falling fuzzy implicative ideal is both a falling fuzzy positive implicative ideal and a falling fuzzy commutative ideal.*

Proof. Let $\widetilde{H}$ be a falling fuzzy implicative ideal of X. Then $\xi(\omega)$ is an implicative ideal of X, and hence it is both a positive implicative ideal and a commutative ideal of X. Thus $\widetilde{H}$ is both a falling fuzzy positive implicative ideal and a falling fuzzy commutative ideal. □

Table 2: Cayley table.

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	a	0
b	b	a	0	b	0
c	c	c	c	0	c
d	d	d	d	d	0

The following example shows that the converse of Theorem 2.8 may not be true.

Example 2.9. (1) Consider a BCK-algebra $X = \{0, a, b, c, d\}$ with a Cayley table which is given by Table 3 (see [10, page 263]). Let $(\Omega, \mathcal{A}, P) = ([0,1], \mathcal{A}, m)$ and let $\xi : [0,1] \to \mathcal{P}(X)$ be defined by

$$\xi(t) := \begin{cases} \{0, c\} & \text{if } t \in [0, 0.15), \\ \{0, d\} & \text{if } t \in [0.15, 0.45), \\ \{0, a, b\} & \text{if } t \in [0.45, 0.75), \\ \{0, c, d\} & \text{if } t \in [0.75, 1]. \end{cases} \tag{2.18}$$

Then $\xi(t)$ is a commutative ideal of X for all $t \in [0,1]$. Hence $\widetilde{H}$, which is given by $\widetilde{H}(x) = P(t \mid x \in \xi(t))$, is a falling fuzzy commutative ideal of X (see [8]). But it is not a falling fuzzy implicative ideal of X because if $t \in [0.75, 1]$ then $\xi(t) = \{0, c, d\}$ is not an implicative ideal of X since $(a * (b * a)) * c = (a * a) * c = 0 * c = 0 \in \xi(t)$ and $c \in \xi(t)$, but $a \notin \xi(t)$.

(2) Let $X = \{0, a, b, c\}$ be a BCK-algebra in which the $*$-multiplication is defined by Table 4. Let $(\Omega, \mathcal{A}, P) = ([0,1], \mathcal{A}, m)$ and let

$$\xi : [0,1] \longrightarrow \mathcal{P}(X), \qquad t \longmapsto \begin{cases} \{0, a\} & \text{if } t \in [0, 0.2), \\ \{0, b\} & \text{if } t \in [0.2, 0.5), \\ \{0, a, b\} & \text{if } t \in [0.5, 0.9), \\ X & \text{if } t \in [0.9, 1]. \end{cases} \tag{2.19}$$

Then $\xi(t)$ is a positive implicative ideal of X for all $t \in [0,1]$. Thus $\widetilde{H}$, which is given by $\widetilde{H}(x) = P(t \mid x \in \xi(t))$, is a falling fuzzy positive implicative ideal of X. But it is not a falling fuzzy implicative ideal of X because if $t \in [0.2, 0.5)$ then $\xi(t) = \{0, b\}$ is not an implicative ideal of X since $(a * (c * a)) * b = (a * c) * b = 0 * b = 0 \in \xi(t)$ and $b \in \xi(t)$, but $a \notin \xi(t)$.

The notions of a falling fuzzy positive implicative ideal and a falling fuzzy commutative ideal are independent, that is, a falling fuzzy commutative ideal need not be a falling fuzzy positive implicative ideal, and vice versa. In fact, the falling fuzzy commutative ideal $\widetilde{H}$ in Example 2.9(1) is not a falling fuzzy positive implicative ideal. Also the falling fuzzy positive implicative ideal $\widetilde{H}$ in Example 2.9(2) is not a falling fuzzy commutative ideal.

Table 3: Cayley table.

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	a	a
b	b	a	0	b	b
c	c	c	c	0	c
d	d	d	d	d	0

Table 4: $*$-Multiplication.

$*$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	c	c	0

Let $(\Omega, \mathcal{A}, P)$ be a probability space and $\widetilde{H}$ a falling shadow of a random set $\xi : \Omega \to \mathcal{P}(X)$. For any $x \in X$, let

$$\Omega(x;\xi) := \{\omega \in \Omega \mid x \in \xi(\omega)\}. \tag{2.20}$$

Then $\Omega(x;\xi) \in \mathcal{A}$.

Proposition 2.10. *If a falling shadow $\widetilde{H}$ of a random set $\xi : \Omega \to \mathcal{P}(X)$ is a falling fuzzy implicative ideal of X, then*

(1) $(\forall x, y, z \in X)\ (\Omega((x * (y * x)) * z;\xi) \cap \Omega(z;\xi) \subseteq \Omega(x;\xi))$,

(2) $(\forall x, y, z \in X)\ (\Omega(x;\xi) \subseteq \Omega((x * (y * x)) * z;\xi))$.

Proof. (1) Let $\omega \in \Omega((x * (y * x)) * z;\xi) \cap \Omega(z;\xi)$. Then $(x * (y * x)) * z \in \xi(\omega)$ and $z \in \xi(\omega)$. Since $\xi(\omega)$ is an implicative ideal of X, it follows from (b5) that $x \in \xi(\omega)$ so that $\omega \in \Omega(x;\xi)$. Therefore

$$\Omega\big((x * (y * x)) * z;\xi\big) \cap \Omega(z;\xi) \subseteq \Omega(x;\xi), \tag{2.21}$$

for all $x, y, z \in X$.

(2) If $\omega \in \Omega(x;\xi)$, then $x \in \xi(\omega)$. Since

$$\begin{aligned}\big((x * (y * x)) * z\big) * x &= \big((x * (y * x)) * x\big) * z \\ &= \big((x * x) * (y * x)\big) * z = \big(0 * (y * x)\big) * z = 0 * z = 0,\end{aligned} \tag{2.22}$$

we have $((x * (y * x)) * z) * x = 0 \in \xi(\omega)$. Since $\xi(\omega)$ is an implicative ideal and hence an ideal of X, it follows that $(x * (y * x)) * z \in \xi(\omega)$ and so $\omega \in \Omega((x * (y * x)) * z;\xi)$. Hence $\Omega(x;\xi) \subseteq \Omega((x * (y * x)) * z;\xi)$ for all $x, y, z \in X$. □

Theorem 2.11. *If $\widetilde{H}$ is a falling fuzzy implicative ideal of X, then*

$$(\forall x, y, z \in X) \quad \left(\widetilde{H}(x) \geq T_m\left(\widetilde{H}((x * (y * x)) * z), \widetilde{H}(z)\right)\right), \tag{2.23}$$

where $T_m(s,t) = \max\{s+t-1, 0\}$ *for any* $s, t \in [0,1]$.

Proof. By Definition 2.2, $\xi(\omega)$ is an implicative ideal of X for any $\omega \in \Omega$. Hence

$$\{\omega \in \Omega \mid (x * (y * x)) * z \in \xi(\omega)\} \cap \{\omega \in \Omega \mid z \in \xi(\omega)\} \subseteq \{\omega \in \Omega \mid x \in \xi(\omega)\}, \tag{2.24}$$

and thus

$$\begin{aligned}\widetilde{H}(x) &= P(\omega \mid x \in \xi(\omega)) \\ &\geq P(\{\omega \mid (x * (y * x)) * z \in \xi(\omega)\} \cap \{\omega \mid z \in \xi(\omega)\}) \\ &\geq P(\omega \mid (x * (y * x)) * z \in \xi(\omega)) + P(\omega \mid z \in \xi(\omega)) \\ &\quad - P(\omega \mid (x * (y * x)) * z \in \xi(\omega) \quad \text{or} \quad z \in \xi(\omega)) \\ &\geq \widetilde{H}((x * (y * x)) * z) + \widetilde{H}(z) - 1.\end{aligned} \tag{2.25}$$

Therefore

$$\begin{aligned}\widetilde{H}(x) &\geq \max\left\{\widetilde{H}((x * (y * x)) * z) + \widetilde{H}(z) - 1, 0\right\} \\ &= T_m\left(\widetilde{H}((x * (y * x)) * z), \widetilde{H}(z)\right).\end{aligned} \tag{2.26}$$

This completes the proof. □

Theorem 2.11 means that every falling fuzzy implicative ideal of X is a T_m-fuzzy implicative ideal of X.

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