

*Research Article*

## **Generalization of Some Simpson-Like Type Inequalities via Differentiable $s$ -Convex Mappings in the Second Sense**

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The author obtained new generalizations and refinements of some inequalities based on differentiable  $s$ -convex mappings in the second sense. Also, some applications to special means of real numbers are given.

### **1. Introduction**

Recall that the function  $f : [0, b] \rightarrow \mathbb{R}$ ,  $b > 0$  is said to be  $s$ -convex in the second sense for  $s \in [0, 1]$  if

$$f(tx + (1-t)y) \leq t^s f(x) + (1-t)^s f(y), \quad (1.1)$$

for all  $x, y \in [0, b]$  and  $t \in [0, 1]$  [1–5].

In (1.1), if we let  $s = 1$ ,  $f : \mathbb{I} \subset [0, \infty) \rightarrow \mathbb{R}$  is said to be a convex mapping on an interval  $\mathbb{I}$  [1].

Let us denote the set of  $s$ -convex mappings in the second sense on  $\mathbb{I}$  by  $K_s^2(\mathbb{I})$ .

For some further properties of the  $s$ -convex mappings, see [1–3, 6]. In recent, M. Z. Sarikaya et al. [4], and U. S. Kirmaci et al. [7] established a more general result of the Hermite-Hadamard inequalities.

For recent years many authors have established error estimations for the Simpson's inequality: for refinements, counterparts, generalizations, and new Simpson's type inequalities, see [1, 4, 6, 8].

S. S. Dragomir et al. [9], and M. Alomari et al. [8] proved the following developments on Simpson's inequality for which the remainder is expressed in terms of lower derivatives than the twice.

In the sequel, denote the interior of an interval  $\mathbb{I}$  by  $\mathbb{I}^0$ .

**Theorem 1.1.** *Let  $f : \mathbb{I} \subset [0, \infty) \rightarrow \mathbb{R}$  be an absolutely continuous mapping on  $[a, b]$  such that  $f' \in L_p([a, b])$ , where  $a, b \in \mathbb{I}$  with  $a < b$ . Then the following inequality holds:*

$$\left| \frac{1}{3} \left\{ \frac{f(a) + f(b)}{2} + 2f\left(\frac{a+b}{2}\right) \right\} - \frac{1}{b-a} \int_a^b f(x) dx \right| \leq \frac{(b-a)^{1/q}}{6} \left\{ \frac{2^{q+1} + 1}{3(q+1)} \right\}^{1/q} \|f'\|_p. \quad (1.2)$$

In this article, the author gives some generalized Simpson's type inequalities based on  $s$ -convex mappings in the second sense by using the following lemma.

## 2. Generalization of Inequalities Based on $s$ -Convex Mappings

In this article, for the simplicity of the notation, let

$$S_a^b(f)(h, n) = \frac{1}{n} \{f(a) + (n-2)f(hb + (1-h)a) + f(b)\} - \frac{1}{b-a} \int_a^b f(x) dx, \quad (2.1)$$

for  $h \in (0, 1)$  with  $1/n \leq h \leq (n-1)/n$  for any integer  $n \geq 2$ .

In order to generalize the classical Simpson-like type inequalities, we need the following lemma [1, 6].

**Lemma 2.1.** *Let  $f : [0, b] \rightarrow \mathbb{R}$ ,  $b > 0$  be a differentiable mapping on  $\mathbb{I}^0$  such that  $f' \in L[a, b]$ , where  $a, b \in \mathbb{I}$  with  $a < b$  and  $[a, b] \subset [0, b]$ . If  $f' \in L^1([a, b])$ , then, for  $h \in (0, 1)$  with  $1/n \leq h \leq (n-1)/n$  for any  $n \geq 2$  the following equality holds:*

$$S_a^b(f)(h, n) = (b-a) \int_0^1 p(t, h) f'(tb + (1-t)a) dt, \quad (2.2)$$

for each  $t \in [0, 1]$ , where

$$p(t, h) = \begin{cases} t - \frac{1}{n}, & t \in [0, h], \\ t - \frac{n-1}{n}, & t \in (h, 1]. \end{cases} \quad (2.3)$$

*Proof.* By the integration by parts, we have

$$\begin{aligned} & \int_0^h \left( t - \frac{1}{n} \right) f'(tb + (1-t)a) dt + \int_h^1 \left( t - \frac{n-1}{n} \right) f'(tb + (1-t)a) dt \\ &= \frac{1}{b-a} \left\{ \frac{n-2}{n} f(hb + (1-h)a) + \frac{1}{n} (f(a) + f(b)) \right\} - \frac{1}{(b-a)^2} \int_a^b f(x) dx, \end{aligned} \quad (2.4)$$

which completes the proof.  $\square$

**Theorem 2.2.** Let  $f : [0, b] \rightarrow \mathbb{R}$ ,  $b > 0$  be a differentiable mapping on  $\mathbb{I}^0$  such that  $f' \in L[a, b]$ , where  $a, b \in \mathbb{I}$  with  $a < b$  and  $[a, b] \subset [0, b]$ . If  $|f'| \in K_s^2([a, b])$ , for some  $s \in (0, 1]$ , then for  $h \in (0, 1)$  with  $1/n \leq h \leq n-1/n$  for any  $n \geq 2$  the following inequality holds:

$$|S_a^b(f)(h, n)| \leq (b-a) \{ \lambda_{11} |f'(b)| + \mu_{11} |f'(a)| \}, \quad (2.5)$$

where

$$\begin{aligned} \lambda_{11} &= \frac{2+2(n-1)^{s+2}}{n^{s+2}(s+1)(s+2)} + \frac{2+s-n}{n(s+1)(s+2)} \\ &\quad + \frac{h^{s+1}(s(2h-1)+2(h-1))}{(s+1)(s+2)}, \\ \mu_{11} &= \frac{2+2(n-1)^{s+2}}{n^{s+2}(s+1)(s+2)} + \frac{2+s-n}{n(s+1)(s+2)} \\ &\quad + \frac{(1-h)^{s+1}\{s(1-2h)-2h\}}{(s+1)(s+2)}. \end{aligned} \quad (2.6)$$

*Proof.* From Lemma 2.1 and since  $|f'|$  is  $s$ -convex on  $[a, b]$ , by using Hölder integral inequality, we have

$$\begin{aligned} |S_a^b(f)(h, n)| &\leq (b-a) \int_0^{1/n} \left( \frac{1}{n} - t \right) |(t^s |f'(b)| + (1-t)^s |f'(a)|) dt \\ &\quad + (b-a) \int_{1/n}^h \left( t - \frac{1}{n} \right) (t^s |f'(b)| + (1-t)^s |f'(a)|) dt \\ &\quad + (b-a) \int_h^{(n-1)/n} \left( \frac{n-1}{n} - t \right) (t^s |f'(b)| + (1-t)^s |f'(a)|) dt \\ &\quad + (b-a) \int_{(n-1)/n}^1 \left( t - \frac{n-1}{n} \right) (t^s |f'(b)| + (1-t)^s |f'(a)|) dt \\ &= (b-a) \{ \lambda_{11} |f'(b)| + \mu_{11} |f'(a)| \}, \end{aligned} \quad (2.7)$$

which implies the theorem.  $\square$

**Corollary 2.3.** In Theorem 2.2, one has:

(i)

$$\left| S_a^b(f)\left(\frac{1}{2}, n\right) \right| \leq (b-a) \left\{ \frac{2+2(n-1)^{s+2}+n^{s+1}(s-n+2-2^{-(s+1)}n)}{n^{s+2}(s+1)(s+2)} \right\} \{ |f'(a)| + |f'(b)| \}, \quad (2.8)$$

(ii)

$$\left| S_a^b(f)\left(\frac{1}{2}, 6\right) \right| \leq (b-a) \left\{ \frac{6^{-s}(1+5^{s+2}-3^{s+2})+3(s-4)}{18(s+1)(s+2)} \right\} \{ |f'(a)| + |f'(b)| \}, \quad (2.9)$$

which implies that Corollary 2.3 is a generalization of Theorem 1.1.

**Theorem 2.4.** Let  $f : [0, b] \rightarrow \mathbb{R}$ ,  $b > 0$  be a differentiable mapping on  $\mathbb{I}^0$  such that  $f' \in L[a, b]$ , where  $a, b \in \mathbb{I}$  with  $a < b$  and  $[a, b] \subset [0, b]$ . If  $|f'|^q \in K_s^2[a, b]$ , for some fixed  $s \in (0, 1]$  and  $q > 1$  with  $1/p + 1/q = 1$ , then for  $h \in (0, 1)$  with  $1/n \leq h \leq (n-1)/n$  for any  $n \geq 2$  the following inequality holds:

$$\begin{aligned} \left| S_a^b(f)(h, n) \right| &\leq (b-a) \left\{ \frac{1+(hn-1)^{p+1}}{n^{p+1}(p+1)} \right\}^{1/p} \left( \frac{h}{s+1} \right)^{1/q} \\ &\quad \times \{ |f'(hb + (1-h)a)|^q + |f'(a)|^q \}^{1/q} \\ &\quad + (b-a) \left\{ \frac{1+(n-hn-1)^{p+1}}{n^{p+1}(p+1)} \right\}^{1/p} \left( \frac{1-h}{s+1} \right)^{1/q} \\ &\quad \times \{ |f'(b)|^q + |f'(hb + (1-h)a)|^q \}^{1/q}. \end{aligned} \quad (2.10)$$

*Proof.* From Lemma 2.1, using the Hölder inequality we get

$$\begin{aligned} \left| S_a^b(f)(h, n) \right| &\leq (b-a) \left( \int_0^h \left| t - \frac{1}{n} \right|^p dt \right)^{1/p} \left( \int_0^h |f'(tb + (1-t)a)|^q dt \right)^{1/q} \\ &\quad + (b-a) \left( \int_h^1 \left| t - \frac{n-1}{n} \right|^p dt \right)^{1/p} \left( \int_h^1 |f(tb + (1-t)a)|^q dt \right)^{1/q}. \end{aligned} \quad (2.11)$$

Since  $|f'|^q \in K_s^2([a, b])$  for a fixed  $s \in (0, 1]$ , we have

(a)

$$\int_0^h |f'(tb + (1-t)a)|^q dt \leq \left( \frac{h}{s+1} \right) \{ |f'(hb + (1-h)a)|^q + |f'(a)|^q \}, \quad (2.12)$$

(b)

$$\int_h^1 |f'(tb + (1-t)a)|^q dt \leq \left( \frac{1-h}{s+1} \right) \{ |f'(b)|^q + |f'(hb + (1-h)a)|^q \}. \quad (2.13)$$

By (2.11) and (2.12), the assertion (2.10) holds.  $\square$

**Corollary 2.5.** *In Theorem 2.4, letting  $n = 6$  and  $h = 1/2$ , one has*

$$\begin{aligned} |S_a^b(f)\left(\frac{1}{2}, 6\right)| &\leq (b-a) \left( \frac{2^{p+1}+1}{6^{p+1}(p+1)} \right)^{1/p} \left( \frac{1}{2(s+1)} \right)^{1/q} \\ &\times \left[ \left\{ |f'(a)|^q + \left| f'\left(\frac{a+b}{2}\right) \right|^q \right\}^{1/q} + \left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q + |f'(b)|^q \right\}^{1/q} \right]. \end{aligned} \quad (2.14)$$

**Theorem 2.6.** *Let  $f : [0, b] \rightarrow \mathbb{R}$ ,  $b > 0$  be a differentiable mapping on  $\mathbb{I}^0$  such that  $f' \in L[a, b]$ , where  $a, b \in \mathbb{I}$  with  $a < b$  and  $[a, b] \subset [0, b]$ . If  $|f'|^q \in K_s^2([a, b])$ , for some fixed  $s \in (0, 1]$  and  $q > 1$  with  $1/p + 1/q = 1$ , then for  $h \in (0, 1)$  with  $1/n \leq h \leq (n-1)/n$  for any  $n \geq 2$  the following inequality holds:*

$$\begin{aligned} |S_a^b(f)(h, n)| &\leq (b-a) \left\{ \frac{1}{n^{p+1}(p+1)} \right\}^{1/p} \left( \frac{1}{s+1} \right)^{1/q} \\ &\times \left[ \left\{ 1 + (hn-1)^{p+1} \right\}^{1/p} \left\{ (h^{s+1}|f'(b)|^q + (1-(1-h)^{s+1})|f'(a)|^q) \right\}^{1/q} \right. \\ &\left. + \left\{ 1 + (n-nh-1)^{p+1} \right\}^{1/p} \times \left\{ ((1-h^{s+1})|f'(b)|^q + (1-h)^{s+1}|f'(a)|^q) \right\}^{1/q} \right]. \end{aligned} \quad (2.15)$$

*Proof.* Note that

$$\begin{aligned} (a) \int_0^h |f'(tb + (1-t)a)|^q dt &\leq \left( \frac{h^{s+1}}{s+1} \right) |f'(b)|^q + \left( \frac{1-(1-h)^{s+1}}{s+1} \right) |f'(a)|^q, \\ (b) \int_h^1 |f'(tb + (1-t)a)|^q dt &\leq \left( \frac{1-h^{s+1}}{s+1} \right) |f'(b)|^q + \left( \frac{(1-h)^{s+1}}{s+1} \right) |f'(a)|^q. \end{aligned} \quad (2.16)$$

By (2.11) and (2.16), the assertion (2.15) of this theorem holds.  $\square$

**Corollary 2.7.** In Theorem 2.6, letting  $h = 1/2$ , then one has

$$\begin{aligned} \left| S_a^b(f)\left(\frac{1}{2}, n\right) \right| &\leq (b-a) \left\{ \frac{2^{p+1} + (n-2)^{p+1}}{2^{p+1} n^{p+1} (p+1)} \right\}^{1/p} \left( \frac{1}{2^{s+1}(s+1)} \right)^{1/q} \\ &\times \left[ \left\{ |f'(b)|^q + (2^{s+1} - 1) |f'(a)|^q \right\}^{1/q} + \left\{ (2^{s+1} - 1) |f'(b)|^q + |f'(a)|^q \right\}^{1/q} \right], \end{aligned} \quad (2.17)$$

which implies that Theorem 2.6 is a generalization of Theorem 1.1.

**Theorem 2.8.** Let  $f : [0, b] \rightarrow \mathbb{R}$ ,  $b > 0$  be a differentiable mapping on  $\mathbb{I}^0$  such that  $f' \in L[a, b]$ , where  $a, b \in \mathbb{I}$  with  $a < b$  and  $[a, b] \subset [0, b]$ . If  $|f'|^q \in K_s^2([a, b])$ , for some fixed  $s \in (0, 1]$  and  $q \geq 1$  with  $1/p + 1/q = 1$ , then for  $h \in (0, 1)$  with  $1/n \leq h \leq (n-1)/n$  for any  $n \geq 2$  the following inequality holds:

$$\begin{aligned} \left| S_a^b(f)(h, n) \right| &\leq (b-a) \left\{ \frac{1 + (nh-1)^2}{2n^2} \right\}^{1/p} \{ \lambda_{21} |f'(b)|^q + \nu_{21} |f'(a)|^q \}^{1/q} \\ &+ (b-a) \left\{ \frac{1 + (nh-n+1)^2}{2n^2} \right\}^{1/p} \{ \lambda_{22} |f'(b)|^q + \nu_{22} |f'(a)|^q \}^{1/q}, \end{aligned} \quad (2.18)$$

where

$$\begin{aligned} \lambda_{21} &= \frac{2}{n^{s+2}(s+1)(s+2)} + \frac{h^{s+1}(-2-s+hn+hns)}{n(s+1)(s+2)}, \\ \nu_{21} &= \frac{2(n-1)^{s+2}}{n^{s+2}(s+1)(s+2)} + \frac{(s+2-n)(1+(1-h)^{s+1})}{n(s+1)(s+2)} - \frac{h(1-h)^{s+1}}{s+2}, \\ \lambda_{22} &= \frac{2(n-1)^{s+2}}{n^{s+2}(s+1)(s+2)} + \frac{(s+2-n)(1+h^{s+1})}{n(s+1)(s+2)} - \frac{h^{s+1}(1-h)}{s+2}, \\ \nu_{22} &= \frac{2}{n^{s+2}(s+1)(s+2)} + \frac{(1-h)^{s+1}(-2-s+n-hn+ns-hns)}{n(s+1)(s+2)}. \end{aligned} \quad (2.19)$$

*Proof.* Suppose that  $q \geq 1$ . From Lemma 2.1, using the power mean inequality one has

$$\begin{aligned} \left| S_a^b(f)(h, n) \right| &\leq (b-a) \left[ \left( \int_0^h \left| t - \frac{1}{n} \right| dt \right)^{1/p} \left( \int_0^h \left| t - \frac{1}{n} \right| |f'(tb + (1-t)a)|^q dt \right)^{1/q} \right. \\ &\quad \left. + \left( \int_h^1 \left| t - \frac{n-1}{n} \right| dt \right)^{1/p} \left( \int_h^1 \left| t - \frac{n-1}{n} \right| |f'(tb + (1-t)a)|^q dt \right)^{1/q} \right]. \end{aligned} \quad (2.20)$$

Since  $|f'|$  is  $s$ -convex on  $[a, b]$ , we have

$$\begin{aligned} \int_0^h \left| t - \frac{1}{n} \right| |f'(tb + (1-t)a)|^q dt &\leq \lambda_{21} |f'(b)|^q + \nu_{21} |f'(a)|^q \\ \int_h^1 \left| t - \frac{n-1}{n} \right| |f(tb + (1-t)a)|^q dt &\leq \lambda_{22} |f'(b)|^q + \nu_{22} |f'(a)|^q. \end{aligned} \quad (2.21)$$

By the above facts (2.20) and (2.21), the assertion (2.18) in this theorem is proved.  $\square$

**Corollary 2.9.** *In Theorem 2.8, letting  $h = 1/2$ , one has*

$$\begin{aligned} \lambda_{21} = \nu_{22} &= \frac{2}{n^{s+2}(s+1)(s+2)} + \frac{(n-2)(s+1)-2}{2^{s+2}n(s+1)(s+2)}, \\ \lambda_{22} = \nu_{21} &= \frac{n^{s+2}2^{-(s+2)}(s+1) + 2(n-1)^{s+2}}{n^{s+2}(s+1)(s+2)} + \frac{(s-n+2)(2^{s+1}+1)}{2^{s+1}n(s+1)(s+2)}, \end{aligned} \quad (2.22)$$

which implies that

$$\begin{aligned} \left| S_a^b(f)\left(\frac{1}{2}, n\right) \right| &\leq (b-a) \left( \frac{1}{8} - \frac{1}{2n} + \frac{1}{n^2} \right)^{1/p} \\ &\times \left[ \{ \lambda_{21} |f'(b)|^q + \lambda_{22} |f'(a)|^q \}^{1/q} + \{ \lambda_{22} |f'(b)|^q + \lambda_{21} |f'(a)|^q \}^{1/q} \right]. \end{aligned} \quad (2.23)$$

Especially, in Theorem 2.8, letting  $h = 1/2$  and  $m = 1$ , one has

$$\begin{aligned} \left| S_a^b(f)\left(\frac{1}{2}, n\right) \right| &\leq (b-a) \left( \frac{1}{8} - \frac{1}{2n} + \frac{1}{n^2} \right)^{1/p} \\ &\times \left\{ (\lambda_{21} |f'(b)|^q + \lambda_{22} |f'(a)|^q)^{1/q} \right. \\ &\quad \left. + (\lambda_{22} |f'(b)|^q + \lambda_{21} |f'(a)|^q)^{1/q} \right\}. \end{aligned} \quad (2.24)$$

### 3. Applications to Special Means

We now consider the applications of our theorems to the followings special means.

- (a) The arithmetic mean:  $A(a, b) = (a+b)/2, a, b \geq 0$ .
- (b) The  $p$ -logarithmic mean:

$$L_p(a, b) = \begin{cases} \left[ \frac{b^{p+1} - a^{p+1}}{(p+1)(b-a)} \right]^{1/p}, & \text{if } a \neq b, \\ a, & \text{if } a = b, \end{cases} \quad (3.1)$$

for  $p \in \mathbb{R} \setminus \{-1, 0\}$  and  $a, b > 0$ .

Now, using the results of Section 2, some new inequalities are derived for the following means:

(1.1) Let  $f : [a, b] \rightarrow \mathbb{R}$ ,  $(0 < a < b)$ ,  $f(x) = x^s$ ,  $s \in (0, 1]$ .

(a) In Theorem 2.2,

(i) if  $h = 1/2$  and  $n \geq 2$ , then we get

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^s, b^s) + (n-2)A^s(a, b)] - L_s^s(a, b) \right| \\ & \leq (b-a)2s \left\{ \frac{2+2(n-1)^{s+2} + n^{s+1}(2+s-n-2^{-(s+1)}n)}{n^{s+2}(s+1)(s+2)} \right\} A(a^{s-1}, b^{s-1}), \end{aligned} \quad (3.2)$$

and,

(ii) if  $h = 1/2$  and  $n = 6$ , then we have

$$\left| \frac{1}{3} [A(a^s, b^s) + 2A^s(a, b)] - L_s^s(a, b) \right| \leq (b-a)s \left\{ \frac{6^{-s} + 5^{s+2}6^{-s} + 3(s-4-2^{-s}3)}{9(s+1)(s+2)} \right\} A(a^{s-1}, b^{s-1}). \quad (3.3)$$

(b) In Theorem 2.4,

(i) if  $h = 1/2$ ,  $n \geq 2$  and  $q > 1$  then we get:

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^s, b^s) + (n-2)A^s(a, b)] - L_s^s(a, b) \right| \\ & \leq \frac{b-a}{2} \left\{ \frac{1+((n/2)-1)^{p+1}}{n^{p+1}(p+1)} \right\}^{1/p} \left( \frac{s^q}{2(s+1)} \right)^{1/q} \\ & \quad \times \left[ \left\{ A^{(s-1)q}(A(a, b)) + a^{(s-1)q} \right\}^{1/q} + \left\{ A^{(s-1)q}(A(a, b)) + b^{(s-1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.4)$$

and

(ii) if  $h = 1/2$ ,  $n = 6$  and  $q > 1$ , then we have

$$\begin{aligned} & \left| \frac{1}{3} [A(a^s, b^s) + 2A^s(a, b)] - L_s^s(a, b) \right| \\ & \leq \frac{b-a}{12} \left\{ \frac{1+2^{p+1}}{3(p+1)} \right\}^{1/p} \left( \frac{s^q}{(s+1)} \right)^{1/q} \\ & \quad \times \left[ \left\{ A^{(s-1)q}(A(a, b)) + a^{(s-1)q} \right\}^{1/q} + \left\{ A^{(s-1)q}(A(a, b)) + b^{(s-1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.5)$$

(c) In Theorem 2.6,

(i) if  $h = 1/2$ ,  $n \geq 2$  and  $q > 1$  then we get

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^s, b^s) + (n-2)A^s(a, b)] - L_s^s(a, b) \right| \\ & \leq (b-a) \left\{ \frac{1 + ((n/2)-1)^{p+1}}{n^{p+1}(p+1)} \right\}^{1/p} \left\{ \frac{s^q}{2^{s+1}(s+1)} \right\}^{1/q} \\ & \quad \times \left[ \left\{ b^{(s-1)q} + (2^{s+1}-1)a^{(s-1)q} \right\}^{1/q} + \left\{ (2^{s+1}-1)b^{(s-1)q} + a^{(s-1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.6)$$

and

(ii) if  $h = 1/2$ ,  $n = 6$  and  $q > 1$ , then we have

$$\begin{aligned} & \left| \frac{1}{3} [A(a^s, b^s) + 2A^s(a, b)] - L_s^s(a, b) \right| \\ & \leq (b-a) \left\{ \frac{1 + 2^{p+1}}{6^{p+1}(p+1)} \right\}^{1/p} \left\{ \frac{s^q}{2^{s+1}(s+1)} \right\}^{1/q} \\ & \quad \times \left[ \left\{ b^{(s-1)q} + (2^{s+1}-1)a^{(s-1)q} \right\}^{1/q} + \left\{ (2^{s+1}-1)b^{(s-1)q} + a^{(s-1)q} \right\}^{1/q} \right]. \end{aligned} \quad (3.7)$$

In Theorem 2.8,

(i) if  $h = 1/2$ ,  $n \geq 2$  and  $q \geq 1$  then we get

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^s, b^s) + (n-2)A^s(a, b)] - L_s^s(a, b) \right| \\ & \leq (b-a) \left\{ \frac{1 + ((n/2)-1)^2}{2n^2} \right\}^{1/p} \\ & \quad \times \left[ \left\{ \lambda'_{21} b^{(s-1)q} + \lambda'_{22} a^{(s-1)q} \right\}^{1/q} + \left\{ \lambda'_{22} b^{(s-1)q} + \lambda'_{21} b^{(s-1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.8)$$

where

$$\begin{aligned} \lambda'_{21} &= \frac{2}{n^{s+2}(s+1)(s+2)} + \frac{(ns/2) + (n/2) - s - 2}{n2^{s+1}(s+1)(s+2)}, \\ \lambda'_{22} &= \frac{2(n-1)^{s+2}}{n^{s+2}(s+1)(s+2)} + \frac{(s-n+2)(2^{s+1}+1)}{n2^{s+1}(s+1)(s+2)} - \frac{1}{2^{s+2}(s+2)}, \end{aligned} \quad (3.9)$$

and

(ii) if  $h = 1/2$ ,  $n = 6$  and  $q \geq 1$ , then we have

$$\begin{aligned} & \left| \frac{1}{3} \{A(a^s, b^s) + 2A^s(a, b)\} - L_s^s(a, b) \right| \\ & \leq (b-a) \left( \frac{5}{72} \right)^{1/p} \left[ \left\{ \lambda_{21}'' b^{(s-1)q} + \lambda_{22}'' a^{(s-1)q} \right\}^{1/q} + \left\{ \lambda_{22}'' b^{(s-1)q} + \lambda_{21}'' a^{(s-1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.10)$$

where

$$\begin{aligned} \lambda_{21}'' &= \frac{2+3^{s+1}(2s+1)}{6^{s+2}(s+1)(s+2)}, \\ \lambda_{22}'' &= \frac{2 \cdot 5^{s+2} + 3^{s+1}(2^{s+1}-2)s - 3^{s+1}(2^{s+3}+7)}{6^{s+2}(s+1)(s+2)}. \end{aligned} \quad (3.11)$$

(2.2) Let  $f : [a, b] \rightarrow \mathbb{R}$ ,  $(0 < a < b)$ ,  $f(x) = 1/x^s$ ,  $s \in (0, 1]$ .

(a) In Theorem 2.2,

(i) if  $h = 1/2$  and  $n \geq 2$ , then we get

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^{-s}, b^{-s}) + (n-2)A^{-s}(a, b)] - L_{-s}^{-s}(a, b) \right| \\ & \leq (b-a)2s \left\{ \frac{2^{s+2} \left( 1 + (n-1)^{s+2} \right) + 2^{s+1} n^{s+1} (2+s-n) - n^{s+2}}{n^{s+2} 2^{s+1} (s+1)(s+2)} \right\} \\ & \quad \times A(a^{-(s+1)}, b^{-(s+1)}), \end{aligned} \quad (3.12)$$

and

(ii) if  $h = 1/2$  and  $n = 6$ , then we have

$$\begin{aligned} & \left| \frac{1}{3} [A(a^{-s}, b^{-s}) + 2A^{-s}(a, b)] - L_{-s}^{-s}(a, b) \right| \\ & \leq \frac{2}{3} (b-a) s \left\{ \frac{1+5^{s+2}+3(s-4)6^s-3^{s+2}}{6^{s+2}(s+1)(s+2)} \right\} A(a^{-(s+1)}, b^{-(s+1)}). \end{aligned} \quad (3.13)$$

(b) In Theorem 2.4,

(i) if  $h = 1/2$  and  $n \geq 2$ , then we get

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^{-s}, b^{-s}) + (n-2)A^{-s}(a, b)] - L_{-s}^{-s}(a, b) \right| \\ & \leq (b-a) \left\{ \frac{1 + ((n/2)-1)^{p+1}}{n^{p+1}(p+1)} \right\}^{1/p} \left\{ \frac{s^q}{2(s+1)} \right\}^{1/q} \\ & \quad \times \left[ \left\{ A^{-(s+1)q}(a, b) + a^{-(s+1)q} \right\}^{1/q} + \left\{ A^{-(s+1)q}(a, b) + b^{-(s+1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.14)$$

and

(ii) if  $h = 1/2$  and  $n = 6$ , then we have

$$\begin{aligned} & \left| \frac{1}{3} [A(a^{-s}, b^{-s}) + 2A^{-s}(a, b)] - L_{-s}^{-s}(a, b) \right| \\ & \leq (b-a) \left\{ \frac{1 + 2^{p+1}}{6^{p+1}(p+1)} \right\}^{1/p} \left\{ \frac{s^q}{(s+1)} \right\}^{1/q} \\ & \quad \times \left\{ A^{1/q} \left( A^{-(s+1)q}(a, b), a^{-(s+1)q} \right) + A^{1/q} \left( A^{-(s+1)q}(a, b), b^{-(s+1)q} \right) \right\}. \end{aligned} \quad (3.15)$$

(c) In Theorem 2.6,

(i) if  $h = 1/2$  and  $n \geq 2$ , then we get

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^{-s}, b^{-s}) + (n-2)A^{-s}(a, b)] - L_{-s}^{-s}(a, b) \right| \\ & \leq (b-a) \left\{ \frac{1}{n^{p+1}(p+1)} \right\}^{1/p} \left\{ \frac{s^q}{2^{s+1}(s+1)} \right\}^{1/q} \left\{ 1 + \left( \frac{n}{2} - 1 \right)^{p+1} \right\}^{1/p} \\ & \quad \times \left[ \left\{ b^{-(s+1)q} + (2^{s+1} - 1) a^{-(s+1)q} \right\}^{1/q} + \left\{ (2^{s+1} - 1) b^{-(s+1)q} + a^{-(s+1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.16)$$

and

(ii) if  $h = 1/2$  and  $n = 6$ , then we have

$$\begin{aligned} & \left| \frac{1}{3} [A(a^{-s}, b^{-s}) + 2A^{-s}(a, b)] - L_{-s}^{-s}(a, b) \right| \\ & \leq (b-a) \left\{ \frac{1 + 2^{p+1}}{6^{p+1}(p+1)} \right\}^{1/p} \left\{ \frac{s^q}{2^{s+1}(s+1)} \right\}^{1/q} \\ & \quad \times \left[ \left\{ b^{-(s+1)q} + (2^{s+1} - 1) a^{-(s+1)q} \right\}^{1/q} + \left\{ (2^{s+1} - 1) b^{-(s+1)q} + a^{-(s+1)q} \right\}^{1/q} \right]. \end{aligned} \quad (3.17)$$

(d) In Theorem 2.8,

(i) if  $h = 1/2$ ,  $n \geq 2$  and  $q \geq 1$  then we get

$$\begin{aligned} & \left| \frac{1}{n} [2A(a^{-s}, b^{-s}) + 2^{-s}(n-2)A^{-s}(hb, (1-h)a)] - L_{-s}^{-s}(a, b) \right| \\ & \leq (b-a)s \left\{ \frac{(n-2)^2 + 4}{8n^2} \right\}^{1/p} \\ & \quad \times \left[ \left\{ \lambda'_{31} b^{(s-1)q} + \lambda'_{32} a^{(s-1)q} \right\}^{1/q} + \left\{ \lambda'_{32} b^{(s-1)q} + \lambda'_{31} a^{(s-1)q} \right\}^{1/q} \right], \end{aligned} \quad (3.18)$$

where

$$\begin{aligned} \lambda'_{31} &= \frac{2}{n^{s+2}(s+1)(s+2)} + \frac{-2-s+n/2+ns/2}{n2^{s+2}(s+1)(s+2)}, \\ \lambda'_{32} &= \frac{2(n-1)^{s+2}}{n^{s+2}(s+1)(s+2)} + \frac{(s-n+2)(2^{s+1}+1)}{n2^{s+1}(s+1)(s+2)} - \frac{1}{2^{s+2}(s+2)}, \end{aligned} \quad (3.19)$$

and

(ii) if  $h = 1/2$ ,  $n = 6$  and  $q \geq 1$ , then we have

$$\begin{aligned} & \left| \frac{1}{3} [A(a^{-s}, b^{-s}) + 2A^{-s}(a, b)] - L_{-s}^{-s}(a, b) \right| \\ & \leq (b-a)s \left\{ \frac{5}{72} \right\}^{1/p} \times \left[ \left\{ \lambda''_{31} b^{q(s-1)} + \lambda''_{32} a^{q(s-1)} \right\}^{1/q} + \left\{ \lambda''_{32} b^{q(s-1)} + \lambda''_{31} a^{q(s-1)} \right\}^{1/q} \right], \end{aligned} \quad (3.20)$$

where

$$\begin{aligned} \lambda''_{31} &= \frac{12 + 3^{s+2}(2s+1)}{6^{s+3}(s+1)(s+2)}, \\ \lambda''_{32} &= \frac{2 \cdot 5^{s+2} + 3^{s+1}(2^{s+1}-4)s + 3^{s+1}(1-2^{s+3})}{6^{s+2}(s+1)(s+2)}. \end{aligned} \quad (3.21)$$

## Acknowledgments

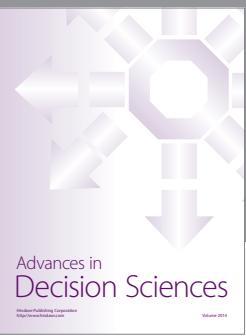
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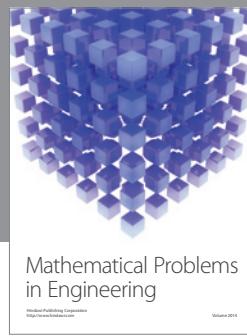
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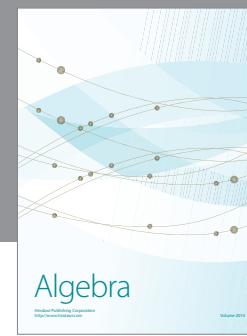
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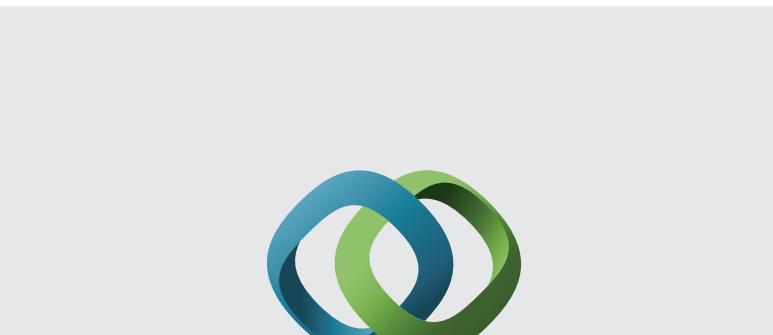
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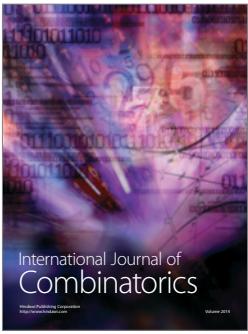


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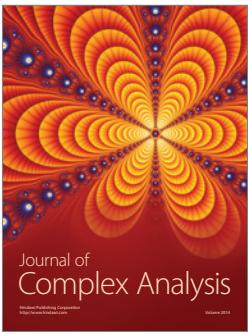
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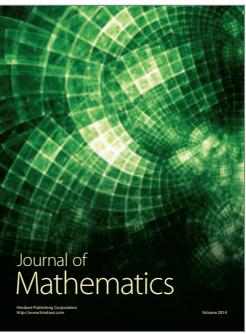
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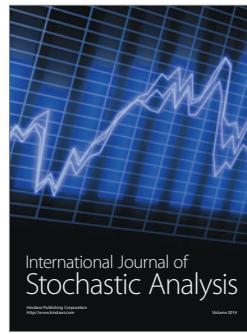
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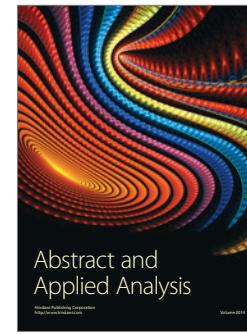
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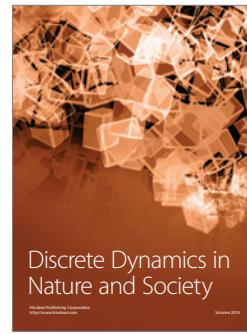
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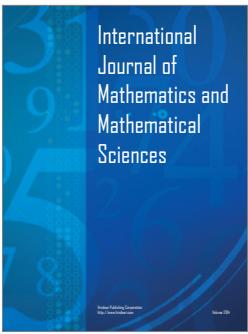
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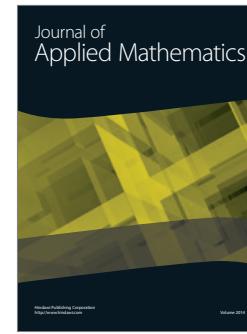
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