Internat. J. Math. & Math. Sci.
VOL. 21 NO. 1 (1998) 33-40



# ON CONNECTEDNESS IN INTUITIONISTIC FUZZY SPECIAL TOPOLOGICAL SPACES

**SELMA ÖZÇAĞ**
Department of Mathematics
Hacettepe University,
Beytepe
06532-Ankara / TURKEY

**DOĞAN ÇOKER**
Department of Mathematics Education
Hacettepe University,
Beytepe
06532-Ankara / TURKEY

(Received May 1, 1996 and in revised form July 7, 1996)

**ABSTRACT.** The aim of this paper is to construct the basic concepts related to connectedness in intuitionistic fuzzy special topological spaces. Here we introduce the concepts of $C_5$-connectedness, connectedness, $C_S$-connectedness, $C_M$-connectedness, strong connectedness, super connectedness, $C_i$-connectedness (i=1,2,3,4), and, obtain several preservation properties and some characterizations concerning connectedness in these spaces.

**KEY WORDS AND PHRASES.** Intuitionistic fuzzy special set; intuitionistic fuzzy special topology, intuitionistic fuzzy special topological space, continuity; $C_5$-connectedness; connectedness; $C_S$-connectedness; $C_M$-connectedness; strong connectedness; super connectedness; $C_i$-connectedness (i=1,2,3,4).

**1992 AMS SUBJECT CLASSIFICATION CODE. 04A99.**

## 1. INTRODUCTION

After the introduction of the concept of fuzzy sets by Zadeh [1] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of intuitionistic fuzzy set was first published by Krassimir Atanassov [2] and many works by the same author appeared in the literature (see Atanassov [2,3]) Later this concept is used to define intuitionistic fuzzy special sets by Coker [4] and intuitionistic fuzzy topological spaces are introduced by Çoker [5], Coker-Es [6].In this direction some preliminary concepts are also defined by Coþkun-Çoker[7].Here we shall give the classical version of this kind of fuzzy topological space in the framework of connectedness;

especially, we shall make use of several types of fuzzy connectedness in intuitionistic fuzzy topological spaces in Turanli-Coker [8].

## 2. PRELIMINARIES

First we shall present the fundamental definitions. The following one is obviously inspired by K. Atanassov [2,3]:

**DEFINITION 2.1.** (see Çoker [4] Let X be a nonempty fixed set. An intuitionistic fuzzy special set (IFSS for short) A is an object having the form $A = \langle x, A_1, A_2 \rangle$, where $A_1$ and $A_2$ are subsets of X satisfying $A_1 \cap A_2 = \varnothing$. The set $A_1$ is called the set of members of A, while $A_2$ is called the set of nonmembers of A.

Obviously every set A on a nonempty set X is obviously an IFSS having the form $\langle x, A, A^c \rangle$ One can define several relations and operations between IFSS's as follows:

**DEFINITION 2.2.** (see Çoker [4,5]) Let X be a nonempty set, and the IFSS's A and B be in the form $A=\langle x, A_1, A_2 \rangle$, $B=\langle x, B_1, B_2 \rangle$, respectively. Furthermore, let $\{ A_i : i \in J\}$ be an arbitrary family of IFSS's in X, where $A_i = \langle x, A_i^{(1)}, A_i^{(2)} \rangle$. Then

(a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$ ; (b) $A=B$ iff $A \subseteq B$ and $B \subseteq A$ ;

(c) $\overline{A} = \langle x, A_2, A_1 \rangle$ ; (d) $[]A = \langle x, A_1, A_1^c \rangle$,

(e) $\langle\rangle A = \langle x, A_2^c, A_2 \rangle$ , (f) $\cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)} \rangle$ ,

(g) $\cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)} \rangle$ ; (h) $\underset{\sim}{\varnothing} = \langle x, \varnothing, X \rangle$ and $\underset{\sim}{X} = \langle x, X, \varnothing \rangle$ .

We shall define the image and preimage of IFSS's. Let X and Y be two nonempty sets and $f : X \to Y$ a function.

**DEFINITION 2.3.** (see Çoker [4,5]) (a) If $B=\langle y, B_1, B_2 \rangle$ is an IFSS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is the IFSS in X defined by $f^{-1}(B) = \langle x, f^{-1}(B_1), f^{-1}(B_2) \rangle$

(b) If $A=\langle x, A_1, A_2 \rangle$ is an IS in X, then the image of A under f, denoted by f(A), is the IFSS in Y defined by $f(A) = \langle y, f(A_1), f\_(A_2) \rangle$, where $f\_(A_2)=(f(A_2^c))^c$

**COROLLARY 2.1.** Let A, $A_i$ ($i \in J$) be IFSS's in X, B, $B_j$ ($j \in K$) IFSS's in Y and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$ (b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(c) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A=f^{-1}(f(A))$ .

(d) $f(f^{-1}(B)) \subseteq B$, and if f is surjective, then $f(f^{-1}(B))=B$

(e) $f^{-1}(\cup B_j)=\cup f^{-1}(B_j)$ (f) $f^{-1}(\cap B_j)=\cap f^{-1}(B_j)$

(g) $f(\cup A_i)=\cup f(A_i)$ (h) $f(\cap A_i) \subseteq \cap f(A_i)$, and if f is injective, then $f(\cap A_i)=\cap f(A_i)$ .

(i) $f^{-1}(\underset{\sim}{Y})=\underset{\sim}{X}$ (j) $f^{-1}(\underset{\sim}{\varnothing})=\underset{\sim}{\varnothing}$

(k) $f(\underset{\sim}{X})=\underset{\sim}{Y}$ if f is surjective. (l) $f(\underset{\sim}{\varnothing})=\underset{\sim}{\varnothing}$

(m) If f is surjective, then $\overline{f(A)} \subseteq f(\overline{A})$; and if, furthermore, f is injective, we have $\overline{(f(A))} = f(\overline{A})$ .

(n) $f^{-1}(\overline{B})=\overline{f^{-1}(B)}$

**DEFINITION 2.4** (see Coker [5,9], Coker-Es [6]) An intuitionistic fuzzy special topology (IFST for short) on a nonempty set X is a family $\tau$ of IFSS's in X containing $\underset{\sim}{\varnothing}$, $\underset{\sim}{X}$, and closed under finite infima and arbitrary suprema. In this case the pair $(X,\tau)$ is called an intuitionistic fuzzy special topological space (IFSTS for short) and any IFSS in $\tau$ is known as an intuitionistic fuzzy special open set (IFSOS for short) in X.

Any topological space can be obviously treated as an IFSTS in a usual manner.

**PROPOSITION 2.1.** Let $(X,\tau)$ be an IFSTS on X. Then, we can also construct several IFSTS's on X in the following way.

(a) $\tau_{0,1}=\{[]G : G\in\tau\}$, (b) $\tau_{0,2}=\{<>G : G\in\tau\}$.

**REMARK 2.1** Let $(X, \tau)$ be an IFSTS $\tau_1=\{ G_1 \quad G=<x,G_1,G_2>\in\tau \}$ is a topological space on X $\tau_2^* = \{ G_2 : G=<x,G_1,G_2>\in\tau \}$ is the family of all closed sets of the topological space ($\tau_2 = \{G_2^c$ $G=<x,G_1,G_2>\in\tau \}$ on X.

The complement $\overline{A}$ of an IFSOS A in an IFSTS $(X, \tau)$ is called an intuitionistic fuzzy special closed set (IFSCS for short) in X, and the interior and closure of an IFSS A are defined by

$cl(A)=\cap\{K : K \text{ is an IFSCS in } X \text{ and } A\subseteq K\}$,

$int(A)=\cup\{G : G \text{ is an IFSOS in } X \text{ and } G\subseteq A\}$

**DEFINITION 2.5.** Let $(X, \tau)$ be an IFSTS on X. If $A=int(cl(A))$, then A is called an intuitionistic fuzzy special regular open set in X

**DEFINITION 2.6.** Let $(X, \tau)$ and $(Y, \psi)$ be two IFSTS's and let $f:X\to Y$ be a function. Then f is said to be continuous iff the preimage of each IFSS in $\psi$ is an IFSS in $\tau$

Here we obtain some characterizations of continuity.

**PROPOSITION 2.2** The following are equivalent to each other:

(a) $f : (X, \tau) \to (Y, \psi)$ is continuous.

(b) The preimage of each IFSCS in Y is an IFSCS in X

(c) $f^{-1}(int(B))\subseteq int(f^{-1}(B))$ for each IFSS B in Y.

(d) $cl(f^{-1}(B))\subseteq ( f^{-1}(cl(B))$ for each IFSS B in Y

## 3. TYPES OF CONNECTEDNESS IN INTUITIONISTIC FUZZY SPECIAL TOPOLOGICAL SPACES

Throughout this section $(X, \tau)$ and $(Y, \psi)$ will always denote IFSTS's We shall define several types of connectedness in IFSTS's

**DEFINITION 3.1.** (see Chaudhuri-Das [10], Turanli-Coker [8])

(a)X is called $C_5$-disconnected, if there exists an IFSS A which is both intuitionistic fuzzy special open and intuitionistic fuzzy special closed, such that $\underset{\sim}{\varnothing}\neq A\neq \underset{\sim}{X}$.

(b) X is called $C_5$-connected, if X is not $C_5$-disconnected.

(c)X is called disconnected, if there exist IFSOS's $A\neq\underset{\sim}{\varnothing}$ and $B\neq\underset{\sim}{\varnothing}$ such that $A\cup B=\underset{\sim}{X}$ and $A\cap B=\underset{\sim}{\varnothing}$

(d) X is called connected, if X is not disconnected.

**PROPOSITION 3.1.** $C_5$-connectedness implies connectedness.

**PROOF.** Suppose that there exist nonempty IFSOS's A and B such that $A \cup B = \underset{\sim}{X}$, $A \cap B = \underset{\sim}{\varnothing}$, from which we get $A_1 \cup B_1 = X$, $A_2 \cap B_2 = \varnothing$, $A_1 \cap B_1 = \varnothing$, $A_2 \cup B_2 = X$, in other words, $A = \overline{B}$. Hence A is intuitionistic fuzzy special clopen, i.e. $(X,\tau)$ is $C_5$-disconnected. ■

**COUNTEREXAMPLE 3.1.** Consider the IFTS $\tau$ on $X=\{a,b,c,d\}$, where $\tau=\{\underset{\sim}{\varnothing}, \underset{\sim}{X}, A_1, A_2, A_3, A_4\}$, $A_1 = \langle x, \{a\}, \{b,c\}\rangle$, $A_2 = \langle x, \{b,c\}, \{a\}\rangle$, $A_3 = \langle x, \varnothing, \{a,b,c\}\rangle$, $A_4 = \langle x, \{a,b,c\}, \varnothing\rangle$ $(X,\tau)$ is connected, but not $C_5$-connected (namely, $A_4$ is intuitionistic fuzzy special clopen in X).

**PROPOSITION 3.2.** Let $f: (X,\tau) \to (Y, \psi)$ be a continuous surjection. If X is connected, then so is Y.

**PROOF.** Assume that Y is disconnected Thus there exist IFSOS's $C \neq \underset{\sim}{\varnothing}$, $D \neq \underset{\sim}{\varnothing}$ in Y such that $C \cup D = \underset{\sim}{Y}$, $C \cap D = \underset{\sim}{\varnothing}$. Now we see that $A = f^{-1}(C)$, $B = f^{-1}(D)$ are IFSOS's in X, since f is continuous From $C \neq \underset{\sim}{\varnothing}$, we get $A = f^{-1}(C) \neq \underset{\sim}{\varnothing}$ (If $f^{1}(C) = \underset{\sim}{\varnothing}$, then $C = ff^{1}(C) = f(\underset{\sim}{\varnothing}) == \underset{\sim}{\varnothing}$, which is a contradiction.) Similarly, we obtain $B \neq \underset{\sim}{\varnothing}$. Now $C \cup D = \underset{\sim}{Y} \Rightarrow f^{-1}(C) \cup f^{-1}(D) = f^{1}(\underset{\sim}{Y}) = \underset{\sim}{X} \Rightarrow A \cup B = \underset{\sim}{X}$, $C \cap D = \underset{\sim}{\varnothing} \Rightarrow f^{1}(C) \cap f^{-1}(D) = f^{1}(\underset{\sim}{\varnothing}) = \underset{\sim}{\varnothing} \Rightarrow A \cap B = \underset{\sim}{\varnothing}$. But this is a contradiction to our hypothesis, thus Y is connected. ■

**PROPOSITION 3.3.** If $(X,\tau)$ is disconnected, then so are the IFSTS's $(X,\tau_{0,1})$ and $(X,\tau_{0,2})$

**PROOF.** Let there exist IFSOS's $A \neq \underset{\sim}{\varnothing}$ and $B \neq \underset{\sim}{\varnothing}$ such that $A \cup B = \underset{\sim}{X}$, $A \cap B = \underset{\sim}{\varnothing}$. In this case we obtain

$$\underset{\sim}{X} = [\,]\, \underset{\sim}{X} = [\,](A \cup B) = ([\,]A) \cup ([\,]B) \Rightarrow ([\,]A) \cup ([\,]B) = \underset{\sim}{X};$$

$$\underset{\sim}{\varnothing} = [\,]\, \underset{\sim}{\varnothing} = [\,](A \cap B) = ([\,]A) \cap ([\,]B) \Rightarrow ([\,]A) \cap ([\,]B) = \underset{\sim}{\varnothing},$$

which is a contradiction. ■

**PROPOSITION 3.4.** $(X,\tau)$ is $C_5$-connected iff there exist no nonempty IFSOS's A and B in X such that $A = \overline{B}$.

**PROOF.** ($\Rightarrow$ :) Suppose that A and B are IFSOS's in X such that $A \neq \underset{\sim}{\varnothing} \neq B$ and $A = \overline{B}$ Since $A = \overline{B}$, B is an IFSCS, and $A \neq \underset{\sim}{\varnothing} \Rightarrow B \neq \underset{\sim}{X}$. But this is a contradiction to the fact that X is $C_5$-connected

($\Leftarrow$ :) Let A be both an IFSOS and IFSCS such that $\underset{\sim}{\varnothing} \neq A \neq \underset{\sim}{X}$. Now take $B = \overline{A}$. In this case B is an IFSOS and $A \neq \underset{\sim}{X} \Rightarrow B = \overline{A} \neq \underset{\sim}{\varnothing}$, which is a contradiction. ■

**PROPOSITION 3.5.** $(X,\tau)$ is $C_5$-connected iff there exist no nonempty IFSS's A and B in X such that $B = \overline{A}$, $B = \overline{cl(A)}$, $A = \overline{cl(B)}$.

**PROOF.** ($\Rightarrow$:) Assume that there exist IFSS's A and B such that $A \neq \underset{\sim}{\varnothing} \neq B$, $B = \overline{A}$, $B = \overline{cl(A)}$, $A = \overline{cl(B)}$. Since $\overline{cl(A)}$ and $\overline{cl(B)}$ are IFSOS's in X, A and B are IFSOS's in X, which is a contradiction (

($\Leftarrow$:) Let A be both an IFSOS and IFSCS in X such that $\underset{\sim}{\emptyset} \neq A \neq \underset{\sim}{X}$. Taking $B=\overline{A}$, we obtain a contradiction. ■

Here we generalize the concepts of $C_S$-connectedness and $C_M$-connectedness given by Chaudhuri - Das [10] to the intuitionistic case:

**LEMMA 3.1.** (a) $A \cap B = \underset{\sim}{\emptyset} \Rightarrow A \subseteq \overline{B}$, (b) $A \not\subseteq \overline{B} \Rightarrow A \cap B \neq \underset{\sim}{\emptyset}$

**DEFINITION 3.2.** Let A and B be nonzero IFSS's in $(X,\tau)$. A and B are said to be weakly separated, if $cl(A) \subseteq \overline{B}$ and $cl(B) \subseteq \overline{A}$ ; and q-separated, if $cl(A) \cap B = \underset{\sim}{\emptyset} = A \cap cl(B)$.

**DEFINITION 3.3.** (see Turanli-Çoker [8]) (a) An IFSTS $(X,\tau)$ is said to be $C_S$-disconnected, if there exist weakly separated nonzero IFSS's A and B in $(X,\tau)$ such that $\underset{\sim}{X} = A \cup B$

(b) $(X,\tau)$ is called $C_S$-connected, if $(X,\tau)$ is not $C_S$-disconnected.

(c) X is said to be $C_M$-disconnected, if there exist q-separated nonzero IFSS's A and B in X such that $\underset{\sim}{X} = A \cup B$.

(d) X is called $C_M$-connected, if X is not $C_M$-disconnected.

Let us give the connection between these two types of connectedness in IFSTS's:

**COROLLARY 3.1.** If the IFSTS X is $C_S$ -connected, then X is also $C_M$ -connected.

**DEFINITION 3.4.** (see Turanli-Çoker [8]) An IFSTS $(X,\tau)$ is said to be strongly connected, if there exit no nonempty IFSCS's A and B in X such that $A \cap B = \underset{\sim}{\emptyset}$.

**PROPOSITION 3.6.** X is strongly connected iff there exist no IFSOS's A and B in X such that $A \neq \underset{\sim}{X} \neq B$ and $A \cup B = \underset{\sim}{X}$.

**PROOF.** ($\Rightarrow$:) Let A and B be IFSOS's in X such that $A \neq \underset{\sim}{X} \neq B$ and $A \cup B = \underset{\sim}{X}$. If we take $C=\overline{A}$ and $D=\overline{B}$, then C and D become IFSCS's in X and $C \neq \underset{\sim}{\emptyset} \neq D$, $C \cap D = \underset{\sim}{\emptyset}$, a contradiction.

( $\Leftarrow$ . ) Use a similar technique as above ■

**PROPOSITION 3.7.** Let $f : (X,\tau) \rightarrow (Y, \psi)$ be a continuous surjection. If X is strongly connected, then so is Y

**PROOF.** Suppose that Y is not strongly connected. In this case there exist IFSCS's C and D in Y such that $C \neq \underset{\sim}{\emptyset} \neq D$, $C \cap D = \underset{\sim}{\emptyset}$. Since f is continuous, $f^{1}(C)$ and $f^{1}(D)$ are IFSCS's in X, and $f^{1}(C) \cap f^{1}(D) = \underset{\sim}{\emptyset}$, $f^{-1}(C) \neq \underset{\sim}{\emptyset}$, $f^{1}(D) \neq \underset{\sim}{\emptyset}$. (If $f^{-1}(C) = \underset{\sim}{\emptyset}$, then $f(f^{-1}(C)) = C \Rightarrow f(\underset{\sim}{\emptyset}) = C \Rightarrow \underset{\sim}{\emptyset} = C$, a contradiction.) But this is a contradiction, hence Y is strongly connected, too. ■

Strong connectedness does not imply $C_5$ -connectedness, and the same is true for IFSTS converse, i.e. $C_5$ connectedness does not imply strong connectedness. For this purpose see the following counterexamples:

**COUNTEREXAMPLES 3.2.** Let $X=\{a,b,c,d\}$ (a) If $\tau=\{\underset{\sim}{\emptyset}, \underset{\sim}{X}, A_1, A_2, A_3, A_4\}$, where $A_1 = \langle x,\{b,c\},\{d\}\rangle$, $A_2 = \langle x,\{d\},\{b,c\}\rangle$, $A_3 = \langle x,\emptyset,\{b,c,d\}\rangle$, $A_4 = \langle x,\{b,c,d\},\emptyset\rangle$, then the IFSTS $(X,\tau)$ is strongly connected, but not $C_5$-connected.

(b) If $\tau=\{\underset{\sim}{\emptyset},\underset{\sim}{X},A_1,A_2,A_3,A_4,A_5\}$, where $A_1=\langle x,\{b,c\},\{d\}\rangle$, $A_2=\langle x,\{a\},\{c\}\rangle$, $A_3=\langle x,\{a,d\},\{c\}\rangle$, $A_4=\langle x,\{a,b,c\},\emptyset\rangle$, $A_5=\langle x,\emptyset,\{c,d\}\rangle$, then the IFSTS $(X,\tau)$ is $C_5$-connected, but not strongly connected

**DEFINITION 3.5.** (see Turanli-Çoker [8]) (a) If there exists an intuitionistic fuzzy special regular open set A in X such that $\underset{\sim}{\emptyset}\neq A\neq\underset{\sim}{X}$, then X is called super disconnected

(b) X is called super connected, if X is not super disconnected.

Now we give some characterizations of super connectedness:

**PROPOSITION 3.8.** The following assertions are equivalent:

(a) X is super connected. (b) For each IFSOS $A\neq\underset{\sim}{\emptyset}$ in X we have $cl(A)=\underset{\sim}{X}$

(c) For each IFSCS $A\neq\underset{\sim}{X}$ in X we have $int(A)=\underset{\sim}{\emptyset}$

(d) There exist no IFSOS's A and B in such that $A\neq\underset{\sim}{\emptyset}\neq B$, $A\subseteq\overline{B}$ .

(e) There exist no IFSOS's A and B in X such that $A\neq\underset{\sim}{\emptyset}\neq B$, $B=\overline{cl(A)}$, $A=\overline{cl(B)}$

(f) There exist no IFSCS's A and B in X such that $A\neq\underset{\sim}{\emptyset}\neq B$, $B=\overline{int(A)}$ , $A=\overline{int(B)}$

**PROOF.** (a)$\Rightarrow$(b) : Assume that there exists an IFSOS $A\neq\underset{\sim}{\emptyset}$ such that $cl(A)\neq\underset{\sim}{X}$. Now take $B=int(cl(A))$. Then B is a proper intuitionistic fuzzy special regular open set in X, and this is in contradiction with the super connectedness of X.

(b) $\Rightarrow$(c) : Let $A\neq\underset{\sim}{X}$ be an IFSCS in X. If we take $B=\overline{A}$, then B is an IFSOS in X and $B\neq\underset{\sim}{\emptyset}$ Hence $cl(B)=\underset{\sim}{X}\Rightarrow\overline{cl(B)}=\underset{\sim}{\emptyset}\Rightarrow int(\overline{B})=\underset{\sim}{\emptyset}\Rightarrow int(A)=\underset{\sim}{\emptyset}$ follows.

(c) $\Rightarrow$ (d) : Let A and B be IFSOS's in X such that $A\neq\underset{\sim}{\emptyset}\neq B$ and $A\subseteq\overline{B}$. Since $\overline{B}$ is an IFCS in X and $B\neq\underset{\sim}{\emptyset}\Rightarrow\overline{B}\neq\underset{\sim}{X}$, we obtain $int(\overline{B})=\underset{\sim}{\emptyset}$ But, from $A\subseteq\overline{B}$, we see that $\underset{\sim}{\emptyset}\neq A=int(A)\subseteq int(\overline{B})=\underset{\sim}{\emptyset}$, which is a contradiction.

(d) $\Rightarrow$ (a) : Let $\underset{\sim}{\emptyset}\neq A\neq\underset{\sim}{X}$ be an intuitionistic fuzzy special regular open set in X. If we take $B=\overline{cl(A)}$, we get $B\neq\underset{\sim}{\emptyset}$. (Because, otherwise we have $B=\underset{\sim}{\emptyset}\Rightarrow\overline{cl(A)}=\underset{\sim}{\emptyset}\Rightarrow cl(A)=\underset{\sim}{X}\Rightarrow A=int(cl(A))=int(\underset{\sim}{X})=\underset{\sim}{X}$, but the last result contradicts the fact $A\neq\underset{\sim}{X}$.) We also have $A\subseteq\overline{B}$, and this is a contradiction, too.

(a) $\Rightarrow$ (e) : Let A and B be IFSOS's in X such that $A\neq\underset{\sim}{\emptyset}\neq B$ and $B=\overline{cl(A)}$, $A=\overline{cl(B)}$ Now we have $int(cl(A))=int(\overline{B})=\overline{cl(B)}=A$ and $A\neq\underset{\sim}{\emptyset}$, $A\neq\underset{\sim}{X}$. (If not, i.e. if $A=\underset{\sim}{X}$, then $\underset{\sim}{X}=\overline{cl(B)}\Rightarrow\underset{\sim}{\emptyset}=cl(B)\Rightarrow B=\underset{\sim}{\emptyset}$.) But this is a contradiction .

(e) $\Rightarrow$ (a) : Let A be an IFSOS in X such that $A=int(cl(A))$, $\underset{\sim}{\emptyset}\neq A\neq\underset{\sim}{X}$. Now take $B=\overline{cl(A)}$. In this case we get $B\neq\underset{\sim}{\emptyset}$ and B is an IFSOS in X and $B=\overline{cl(A)}$ and $\overline{cl(B)}=\overline{cl(\overline{cl(A)})}=\overline{\overline{int(cl(A))}}=int(cl(A))=A$, which is a contradiction.

(e) $\Rightarrow$ (f) : Let A and B IFSCS's in X such that $A \neq \tilde{X} \neq B$, $B=\overline{int(A)}$, $A=\overline{int(B)}$ Taking $C=\overline{A}$ and $D=\overline{B}$, C and D become IFSOS's in X and $C \neq \tilde{\emptyset} \neq D$, $\overline{cl(C)} = \overline{cl(\overline{A})} = \overline{\overline{int(A)}} = int(A) = \overline{B} = D$, and similarly $\overline{cl(D)}=C$. But this is an obvious contradiction.

(f) $\Rightarrow$ (e) : One can use a similar technique as in (e) $\Rightarrow$ (f). ■

**PROPOSITION 3.9.** Super connectedness implies $C_5$-connectedness.

**PROOF.** Obvious. ■

But the reverse implication to Proposition 3.9 does not hold in general·

**COUNTEREXAMPLE 3.3.** Let X={a,b,c,d} and the IFST $\tau=\{\tilde{\emptyset}, \tilde{X}, A_1, A_2, A_3, A_4\}$ on X, where $A_1 = \langle x,\{a\},\{c,d\}\rangle$, $A_2=\langle x,\{d\},\{a,c\}\rangle$, $A_3 = \langle x,\{a,d\},\{c\}\rangle$, $A_4 = \langle x,\emptyset,\{a,c,d\}\rangle$. Then the IFSTS $(X,\tau)$ is $C_5$ -connected, but not super connected

**PROPOSITION 3.10.** Let $f:(X,\tau) \to (Y, \psi)$ be a continuous surjection. If X is super connected, then so is Y.

**PROOF.** Suppose that Y is super disconnected. In this case there exist IFSOS's C and D in Y such that $C \neq \tilde{\emptyset} \neq D$, $C \subseteq \overline{D}$ Since f is continuous, $f^{1}(C)$ and $f^{1}(D)$ are IFSOS's in X, and $C \subseteq \overline{D} \Rightarrow f^{1}(C) \subseteq f^{1}(\overline{D}) = \overline{f^{-1}(D)}$, $f^{-1}(C) \neq \tilde{\emptyset} \neq f^{-1}(\overline{D})$, which means that X is super disconnected ■

Now we shall summarize the interrelations between several types of connectedness in IFSTS's.

super connectedness → $C_5$ -connectedness → connectedness

$C_S$ -connectedness → $C_M$ -connectedness

Here we generalize the idea of fuzzy $C_i$-connectedness in fuzzy topological spaces and in intuitionistic fuzzy topological spaces (see Ajmal-Kohli [11], Chaudhuri-Das [10] and Turanli-Çoker [8] to the intuitionistic case:

**DEFINITION 3.6.** Let N be an IFSS in $(X,\tau)$

(a) If there exist IFSOS's M and W in X satisfying the following properties, then N is called $C_i$-disconnected (i=1,2,3,4) :

$C_1$: $N \subseteq M \cup W, M \cap W \subseteq \overline{N}, N \cap M \neq \tilde{\emptyset}, N \cap W \neq \tilde{\emptyset}$, $C_2$: $N \subseteq M \cup W, N \cap M \cap W = \tilde{\emptyset}, N \cap M \neq \tilde{\emptyset}, N \cap W \neq \tilde{\emptyset}$,

$C_3$: $N \subseteq M \cup W, M \cap W \subseteq \overline{N}, M \not\subseteq \overline{N}, W \not\subseteq \overline{N}$, $C_4$: $N \subseteq M \cup W, N \cap M \cap W = \tilde{\emptyset}, M \not\subseteq \overline{N}, W \not\subseteq \overline{N}$.

(b) N is said to be $C_i$ -connected (i=1,2,3,4), if N is not $C_i$ -disconnected (i=1,2,3,4)

Obviously, one can obtain the following implications between several types of $C_i$ -connectedness (i=1,2,3,4) :

| | | |
|---|---|---|
| $C_1$ -connectedness | → | $C_2$ -connectedness |
| ↓ | | ↓ |
| $C_3$-connectedness | → | $C_4$ -connectedness |

None of these implications are reversible, as the following counterexamples state:

**COUNTEREXAMPLES 3.4.** Consider the IFST $\tau$ on X={a,b},where $\tau$={ $\underset{\sim}{\varnothing}$, $\underset{\sim}{X}$ ,$A_1$,$A_2$,$A_3$,$A_4$,$A_5$,$A_6$,$A_7$}, $A_1$ =<x,{a},∅}>, $A_2$ =<x,{b},∅>, $A_3$ =<x,∅,{a}>, $A_4$ =<x,∅,{b}>, $A_5$ =<x,{a},{b}>, $A_6$ =<x,{b},{a}>, $A_7$ =<x,∅,∅> and take the IFSS N=<x,∅,{a}> in X.

(a) N is $C_2$-connected, but not $C_1$-connected. [Namely, $A_2$ and $A_3$ do satisfy the properties in ($C_1$ ) ]

(b) N is $C_3$ -connected, but not $C_1$-connected

**COUNTEREXAMPLE 3.5.** Consider the IFST $\tau$ on X={a,b,c,d}, where $\tau$={ $\underset{\sim}{\varnothing}$, $\underset{\sim}{X}$ ,$A_1$,$A_2$,$A_3$,$A_4$}, $A_1$ =<x,{a},{b,c}>, $A_2$=<x,{b,c},{a}>, $A_3$ =<x,∅,{a,b,c}>, $A_4$ =<x,{a,b,c},∅> The IFSS N=<x,{a},{b}> in X is $C_4$-connected, but not $C_3$-connected [Namely, $A_1$ and $A_2$ do satisfy the properties in ($C_3$ ).]

**COUNTEREXAMPLE 3.6.** Consider the IFST $\tau$ on X={a,b,c}, where $\tau$={ $\underset{\sim}{\varnothing}$, $\underset{\sim}{X}$ ,$A_1$,$A_2$,$A_3$},$A_1$=<x,∅,{a}>,$A_2$=<x,{a},{b,c}>,$A_3$=<x,{a},∅>. The IFSS N=<x,{a},∅> in X is $C_4$-connected, but not $C_2$-connected. [Namely, $A_1$ and $A_2$ do satisfy the properties in ($C_2$ ).]

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