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ON A VARIATION OF SANDS' METHOD

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ABSTRACT. A subset of a of a finite additive abelian group G is a Z-set if for all $a \varepsilon G$, $na \varepsilon G$ for all $n \varepsilon Z$. The group G is called "Z-good" if in every factorization $G = A \oplus B$, where A and B are Z-sets at least one factor is periodic. Otherwise G is called "Z-bad."

The purpose of this paper is to investigate factorizations of finite ablian groups which arise from a variation of Sands' method. A necessary condition is given for a factorization $G = A \oplus B$, where A and B are Z-sets, to be obtained by this variation. An example is provided to show that this condition is not sufficient. It is also shown that in general all factorizations $G = A \oplus B$, where A and B are Z-sets, of a "Z-good" group do not arise from this variation of Sands' method.

KEY WORDS AND PHRASES. Finite abelian group, factorization, Z-set, good group, bad group.

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I. INTRODUCTION.

Let G be a finite additive abelian group and let A and B be subsets of G. If every element $g \varepsilon G$ can be uniquely represented in the form $g = a + b$, where $a \varepsilon A$, $b \varepsilon B$, then we write $G = A \oplus B$ and call this a factorization of G. A subset A of G is said to be periodic if there exists an element $g \neq 0$ such that $g + A = A$. Such an element g is called a period of A. The set of all periods of A together with 0 forms a subgroup of G. A subset of G is a Z-set if for all $a \varepsilon A$, $na \varepsilon A$ for all $n \varepsilon Z$. We say G is "good" ("Z-good") if in every factorization $G = A \oplus B$, where A and B are sets (Z-sets) at least one factor is periodic. Otherwise G is called "bad" ("Z-bad").

The problem of classifying a finite abelian group as either "good" or "bad" arose from the solution of G. Hajos [1] to a group-theoretical interpretation of a conjecture of H. Minkowski on homogeneous linear forms. Hajos [1-3], Redei [4-5], de Bruijn [6-7], and Sands [8-11] have completely solved this problem of classification. C. Okuda

[12] classified all finite abelian groups as either "Z-good" of "Z-bad," obtaining quite different results from the "good" - "bad" classification.

Sands [8] gave a method which yields all factorizations of a finite abelian "good" group. His method corrects one given previously by Hajos [2].

The purpose of this paper is to investigate factorizations of finite abelian groups which arise from a variation of Sands' method. A necessary condition is given for a factorization $G = A \oplus B$, where A and B are Z-sets, to be obtained by this variation. An example is provided to show that this condition is not sufficient. It is also shown that in general all factorizations $G = A \oplus B$, where A and B are Z-sets, of a "Z-good" group do not arise from this variation of Sands' method.

2. PRELIMINARIES.

This section provides some basic unpublished results on Okuda's [12] "Z-good" - "Z-bad" classification of finite abelian groups as well as an elementary result concerning factorizations $G = S \oplus A$, where S is a subgroup of G and A is a Z-set. For completeness, we state Sands' Theorem on the factorizations of finite abelian "good" groups.

LEMMA 1 (Okuda [12]). A finite abelian group G is "Z-good" if and only if at least one Sylow p-subgroup of G is "Z-good."

LEMMA 2 (Okuda [12]). Every cyclic group is "Z-good."

LEMMA 3 (Okuda [12]). If $G = A \oplus B$, where A and B are Z-sets, then A and B are pure in G.

LEMMA 4 (Okuda [12]). Let G be a group isomorphic to $Z_p \oplus Z_p + Z_p + Z_p$, where p is an odd prime. Let $\{a_1, a_2, b_1, b_2\}$ be a basis of G and define

$$A = (\langle a_1, a_2\rangle \setminus \langle a_2\rangle) \cup \langle a_2 + b_2\rangle,$$

$$B = (\langle b_1, b_2\rangle \setminus \bigcup_{i=1}^{p-1} \langle b_1 + ib_2\rangle)) \cup (\bigcup_{i=1}^{p-1} \langle b_1 + ib_2 + 2a_2\rangle).$$

Then A and B are non-periodic Z-sets and $G = A + B$.

PROPOSITION 1. Let S be a subgroup of G. G has a factorization $G = S + A$, A a Z-set, if and only if S is pure in G.

PROOF. This is a direct consequence of Lemma 3 and the fact that a pure subgroup of a finite abelian group G is a direct summand of G.

THEOREM 1 (Sands [8]). Let G be a finite abelian "good" group. $G = A + B$ if and only if there exists subsets $H_1, H_2, \ldots, H_n$ such that $H_i + H_{i+1} + \ldots + H_n = K_i$ is a subgroup of G, $1 \le i \le n$, $K_1 = G$, and

$$A = \langle 0\rangle + H_1 \circ H_2 + H_3 \circ H_4 + \ldots,$$

$$B = \langle 0\rangle \circ H_1 + H_2 \circ H_3 + H_4 \circ \ldots,$$

where the notation $C \circ D$ indicates any of the sets formed by adding to each element of C some element of D.

Let us note that the subgroups, K_i, in Sands' Theorem yield the following series for G

$$G = K_1 \supset K_2 \supset K_3 \supset \dots \supset K_n \supset K_{n+1} = \langle 0 \rangle$$

where $K_i = H_i \oplus K_{i+1}$, $1 \leq i \leq n$, $K_n = H_n$. We shall say that the factorization $G = A \oplus B$ arises from the above series if

$$A = H_1 \oplus H_3 \oplus \dots + h_2 + h_4 + \dots ,$$

$$B = H_2 + H_4 \oplus \dots + h_1 + h_3 + \dots ,$$

where H_i is a set of coset representatives for K_i modulo K_{i+1}, and $h_i \in H_i$, $1 \leq i \leq n$.

Factorizations which arise from the above series can be obtained from Sands' method if one computes $C \circ D$ by adding a fixed element of D to the set C. However, as shown in Example 1, there are factorizations which are obtained from Sands' method which do not arise from the corresponding series of subgroups K_i, $1 \leq i \leq n$.

EXAMPLE 1. Let G be the cyclic group of order 81. Consider the series $G = K_1 \supset K_2 \supset K_3 \supset K_4 \supset \langle 0 \rangle$, where $K_4 = \langle 27 \rangle$, $K_3 = \langle 9 \rangle$, $K_2 = \langle 3 \rangle$. If we choose $H_3 = \{0, 9, 18\}$, $H_2 = \{0, 3, 6\}$, $H_1 = \{0, 1, 2\}$ then the following factorization, $G = A \oplus B$, can be obtained from Sands' method.

$$A = \langle 0 \rangle \oplus H_1 \circ H_2 \oplus H_3 \circ H_4 = (H_1 \oplus H_3) \circ H_4$$

$$= (\{0,1,2\} \oplus \{0,9,18\}) \circ \{0,27,54\} = \{0,1,2,9,10,11,18,19,47\}$$

$$B = \langle 0 \rangle \circ H_1 \oplus H_2 \circ H_3 \oplus H_4 = H_2 \oplus H_4 = \{0,3,6\} \oplus \{0,27,54\}.$$

Clearly for every choice of the sets H_i, $1 \leq i \leq 3$, the set A cannot be written in the form $A = H_1 \oplus H_3 + h_2 + h_4$.

3. TRANSLATIONS.

THEOREM 2. Let $G = A + B$ a factorization of G arising from the series $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0 \rangle$ with coset representatives H_i so that $K_i = H_i + K_{i+1}$, $1 \leq i \leq n-1$, $K_n = H_n$. Then $G = A' \oplus B'$ with $A' = A + g_1$, $B' = B + g_2$, $g_1, g_2 \in G$ arises from the same series with coset representatives $H_i' = H_i + u_i$, $u_i \in H_i$, $1 \leq i \leq n$.

PROOF. We will proceed by induction on the length of the series, n. For $n = 2$, we may assume $A = H_1 + h_2$, $B = H_2 + h_1$, $h_i \in H_i$, $i = 1,2$. Suppose $G = A' + B'$ with $A' = A + g_1$, $B' = B + g_2$, $g_1, g_2 \in G$. We can write $g_1 = u_1 + u_2$, $u_i \in H_i$, $i = 1,2$. Since $G = (H_1 + u_1) \oplus H_2$ we have $h_1 + g_2 = z_1 + u_1 + z_2$, $z_i \in H_i$, $i = 1,2$. Let $H_1' = H_1 + u_1$, $H_2' = H_2 + u_2$. Then

$$A' = A + g_1 = H_1 + u_1 + h_2 + u_2 = H_1' + h_2' ,$$

$$B' = B + g_2 = H_2 + z_2 + z_1 + u_1 = H_2 + u_2 + z_1 + u_1 = H_2' + h_1' ,$$

where we have used the fact that $H_2 = H_2 + u_2 = H_2 + z_2$ since $H_2 + K_2$ is a subgroup.

Let us assume the theorem is true for series of length less than n. Let $G = A + B$ be a factorization arising from a series of length n, say $G = B + g_2$, $K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0 \rangle$. We may assume that

$$A = H_1 \oplus H_3 \oplus \cdots + h_2 + h_4 + \cdots ,$$

$$B = H_2 \oplus H_4 \oplus \cdots + h_1 + h_3 + \cdots .$$

Define

$$A_1 = H_3 \oplus H_5 \oplus \cdots + h_2 + h_4 + \cdots ,$$

$$B_1 = H_2 \oplus H_4 \oplus \cdots + h_3 + h_5 + \cdots ,$$

so that $A = A_1 \oplus H_1$, $B = B_1 + h_1$. Suppose $G = A' \oplus B'$, where

$$A' = A + g_1,\ g_1 = s_1 + s_2,\ s_1 \in H_1,\ s_2 \in K_2,$$

$$B' = B + g_2,\ g_2 = t_1 + t_2,\ t_1 \in H_1,\ t_2 \in K_2.$$

Setting $H_1' = H_1 + s_1$ we have

$$A' = H_1' \oplus (A_1 + s_2),$$

$$B' = (h_1 + t_1) + (B_1 + t_2) = h_1' + (B_1 + t_2 + k_2),$$

where $h_1 + t_1 = h_1 + t_1 - s_1 + s_1 = \tilde{h}_1 + k_2 + s_1 = h_1' + k_2$, $k_2 \in K_2$, $\tilde{h}_1 \in H_1$, and $h_1' = \tilde{h}_1 + s_1 \in H_1'$.

Note that $K_2 = A_1 \oplus B_1$ arises from the series $K_2 \supset K_3 \supset \cdots \supset K_n \supset \langle 0 \rangle$. Therefore the factorization $K_2 = (A_1 + s_2) \oplus (B_1 + t_2 + k_2)$ arises from $K_2 \supset K_3 \supset \cdots \supset K_n \supset \langle 0 \rangle$ with coset representatives $H_i' = H_i + u_i$, $u_i \in H_i$, $2 \leq i \leq n$, i.e.,

$$A_1 + s_2 = H_3' \oplus H_5' \oplus \cdots + h_2' + h_4' + \cdots,$$

$$B_1 + t_2 + k_2 = H_2' \oplus H_4' + \cdots + h_3' + h_5' + \cdots .$$

Consequently we have

$$A' = H_1' \oplus (A_1 + s_2) = H_1' \oplus H_3' \oplus \cdots + h_2' + h_4' + \cdots ,$$

$$B' = h_1' + (B_1 + t_2 + k_2) = H_2' \oplus H_4' \oplus \cdots + h_1' + h_3' + \cdots ,$$

which completes the proof.

THEOREM 3. Let $G = A \oplus B$ be a factorization of G arising from the series $G = K_1 \supset K_2 \supset \cdots \supset K_n \supset \langle 0 \rangle$ with coset representatives H_i so that $K_i = H_i \oplus K_{i+1}$, $1 \leq i \leq n-1$, $K_n = H_n$. Then each H_i may be translated to obtain H_i', $1 \leq i \leq n$, in such a way that $0 \in H_i'$, $1 \leq i \leq n$, and the factorization $G = A \oplus B$ arises from the original series with coset representatives H_i', $1 \leq i \leq n$, $K_n = H_n'$.

PROOF. We will use induction on the length of the series, n. For $n = 2$, we may assume $A = H_1 + h_2$, $B = H_2 + h_1$, $h_i \in H_i$, $i = 1,2$. We can write $0 = y_1 + y_2$, $y_i \in H_i$, $i = 1,2$. Define $H_1' = H_1 + y_2$, $H_2' = H_2$, and let $h_1' = h_1 + y_2$. Note that $H_2 = H_2 + y_2$ and $h_2 - y_2 \in H_2$ since $H_2 = K_2$ is a subgroup. Thus,

$$A = H_1 + y_2 + h_2 - y_2 = H_1' + h_2' ,$$

$$B = H_2 + h_1 = H_2 + h_1 + y_2 = H_2' + h_1' .$$

Let us assume the theorem is true for series of length less than n. Let the factorization $G = A \oplus B$ arise from a series of length n, say, $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0 \rangle$. We may assume that

$$A = H_1 \oplus H_3 \oplus \dots + h_2 + h_4 + \dots ,$$

$$B = H_2 \oplus H_4 \oplus \dots + h_1 + h_3 + \dots .$$

Define

$$A_1 = H_3 \oplus H_5 \oplus \dots + h_2 + h_4 + \dots ,$$

$$B_1 = H_2 \oplus H_4 \oplus \dots + h_3 + h_5 + \dots .$$

so that $A = A_1 \oplus H_1$, $B = B_1 + h_1$, and $K_2 + A_1 \oplus B_1$.

We can write $0 = y_1 + y_2 + \dots + y_n$, $y_i \in H_i$, $1 \leq i \leq n$. Set $x_2 = y_2 + y_3 + \dots + y_n$. Then $x_2 \in K_2$. Define $H_1' = H_1 + x_2$ and let $h_1 = h_1' + z_2$, $z_2 \in K_2$. Note that $0 \in H_1'$. We have $K_2 = (A_1 - x_2) \oplus (B_1 + z_2)$. By Theorem 2 there exists coset representatives $H_2', H_3', \dots, H_n'$ translates of $H_2, H_3, \dots, H_n$ respectively such that

$$A_1 - x_2 = H_3' \oplus H_5' \oplus \dots + h_2' + h_4' + \dots ,$$

$$B_1 + z_2 = H_2' \oplus H_4' \oplus \dots + h_3' + h_5' + \dots .$$

By the inductive hypothesis, $0 \in H_i'$, $2 \leq i \leq n$. Hence,

$$A = H_1 \oplus A_1 = H_1' - x_2 \oplus A_1 = H_1' \oplus H_3' \oplus \dots + h_2' + h_4' + \dots$$

$$B = B_1 + h_1 = B_1 + h_1' + z_2 = H_2' \oplus H_4' \oplus \dots + h_1' + h_3' + \dots .$$

This completes the proof.

4. Z-FACTORIZATIONS.

We shall use the term "Z-factorization" when refering to a factorization of the form $G = A \oplus B$, where A and B are Z-sets.

LEMMA 5. Let $G = A \oplus B$ be a Z-factorization of G arising from the series $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0 \rangle$. Then we may choose the coset representatives, H_i', $1 \leq i \leq n$, appearing in the expressions for A and B such that $0 \in H_i'$, $1 \leq i \leq n$, and $h_i' = 0$, $1 \leq i \leq n$.

PROOF. We may assume

$$A = H_1 \oplus H_3 \oplus \dots + h_2 + h_4 + \dots ,$$

$$B = H_2 \oplus H_4 \oplus \dots + h_1 + h_3 + \dots .$$

By theorem 3 there exist coset representatives H_i', $1 \leq i \leq n$, such that $0 \in H_i'$, $1 \leq i \leq n$, and

$$A = H_1' \oplus H_3' \oplus \dots + h_2' + h_4' + \dots ,$$

$$B = H_2' \oplus H_4' \oplus \dots + h_1' + h_3' + \dots .$$

Observe that $0 \in H_1' \oplus H_3' \oplus \dots$ and $0 \in H_2' \oplus H_4' \oplus \dots$. Consequently

$h_2' + h_4' + \dots \in A$. Since A is a Z-set we have $2(h_2' + h_4' + \dots) \in A$. Therefore

$$2(h_2' + h_4' + \dots) = \tilde{h}_2' + \tilde{h}_3' + \dots + h_2'\, h_4' + \dots ,$$

where $\tilde{h}_i' \in H_i'$, $i = 1, 3, 5, \dots$. Thus,

$$h_2' + h_4' + \dots = \tilde{h}_1' + \tilde{h}_3' + \dots .$$

But $G = H_1' \oplus H_2' \oplus H_3' \oplus \dots$ and $0 \in H_i'$, $1 \leq i \leq n$. Hence $h_2' = h_4' = \dots = 0$. Similarly we have $h_1' = h_3' = \dots = 0$, so establishing the lemma.

We shall assume throughout the rest of the paper that whenever a Z-factorization $G = A \oplus B$ arises from the series $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0\rangle$ the coset representatives have been chosen as in Lemma 5 so that $A = H_1 \oplus H_3 \oplus \dots$ and $B = H_2 \oplus H_4 \oplus \dots$, where $0 \in H_i$, $1 \leq i \leq n$.

THEOREM 4. If $G = A \oplus B$ is a Z-factorization of G arising from the series $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0\rangle$ then K_n is pure in K_{n-1}.

PROOF. We prove the result for n odd; the proof for n even is similar.

We may assume

$$A = H_1 \oplus H_3 \oplus \dots \oplus H_n ,$$

$$B = H_2 \oplus H_4 \oplus \dots \oplus H_{n-1} ,$$

where $0 \in H_i$, $1 \leq i \leq n$.

Since $K_{n-1} = H_{n-1} \oplus K_n$ we have that $H_{n-1} = B \cap K_{n-1}$ is a Z-set. The result follows from Proposition 1.

LEMMA 6. Let $G = A \oplus B$ be a Z-factorization of G arising from the series $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0\rangle$. For $3 \leq i \leq n$ let ψ_i be the natural epimorphism with kernel K_i. Then $\psi_i(G) = \psi_i(A) \oplus \psi_i(B)$ is a Z-factorization of $\psi_i(G)$ arising from the series $\psi_i(G) = \psi_i(K_1) \supset \psi_i(K_2) \supset \dots \supset \psi_i(K_{i-1}) \supset \psi_i(K_i)$.

PROOF. The result follows from the homomorphic properties of the epimorphisms ψ_i.

THEOREM 5. If $G = A \oplus B$ is a Z-factorization of G arising from the series $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0\rangle$ then K_i/K_{i+1} is pure in K_{i-1}/K_{i+1}, $2 \leq i \leq n-1$.

PROOF. By Lemma 6 a Z-factorization of G/K_{i+1}, $2 \leq i \leq n-1$, arises from the series $G/K_{i+1} = K_1/K_{i+1} \supset K_2/K_{i+1} \supset \dots \supset K_{i-1}/K_{i+1} \supset K_i/K_{i+1} \supset K_{i+1}/K_{i+1}$. Application of Theorem 4 completes the proof.

5. EXAMPLES.

We now show that the converses of theorems 4 and 5 are false.

EXAMPLE 2. Let G be a group of type $(2^2, 2, 2)$ and let a, b, and c of orders 2^2, 2, and 2 respectively generate G. Consider the series $G = K_1 \supset K_2 \supset K_3 \supset K_4 \supset K_5 = \langle 0\rangle$, where $K_4 = \langle 2a\rangle$, $K_3 = \langle b\rangle \oplus \langle 2a\rangle$, $K_2 = \langle c\rangle \oplus \langle b\rangle \oplus \langle 2a\rangle$. Then K_i/K_{i+1} is pure in $K_{i-1/Ki+1}$, $2 \leq i \leq 4$. Suppose $G = A \oplus B$ is a Z-factorization arising from the above series. We may assume $A = H_1 \oplus H_3$, $B = H_2 \oplus H_4$, $0 \in H_i$, $1 \leq i \leq 4$. The only possible choices for H_3 are $\langle b\rangle$ and $\langle 2a+b\rangle$, and H_1 must have the form $H_1 = \{0,\gamma\}$, $\gamma \neq 0$, $\gamma \in K_2$. Since K_2 contains all the non-zero elements of order 2, γ must be of order 2^2. Thus γ has the form $\gamma = a + k_2$ for some $k_2 \varepsilon K_2$. We have that $\gamma \varepsilon H_1 \oplus H_3$. Therefore $2\gamma \varepsilon H_1 \oplus H_3$. But $2\gamma \varepsilon K_2$. Hence $2\gamma \varepsilon (H_1 \oplus H_3) \cap K_2 = H_3$. Depending on the choice for H_3, we have that $2\gamma = b$ or $2\gamma = 2a+b$. Clearly both cases are impossible and we conclude that for every choice of H_3 we cannot choose H_1 such that $A = H_1 \oplus H_3$ is a Z-set.

Example 3 answers the following questions negatively:

If G is a "bad" group, are all its "good factorizations" (i.e., the factorizations in which at least one factor is periodic) obtained from the variation of Sands' method?

If G is a "Z-good" group, are all its Z-factorizations obtained from the variation of Sands' method?

EXAMPLE 3. Let G be a group of type (p,p,p,p,2), p an odd prime, and let a_1, a_2, b_1, b_2, and c of orders p, p, p, p, and 2 respectively generate G. Let $T = \langle a_1 \rangle \oplus \langle a_2 \rangle \oplus \langle b_1 \rangle \oplus \langle b_2 \rangle$,

$$A' = (\langle a_1, a_2 \rangle \setminus \langle a_2 \rangle) \cup \langle a_2 + b_2 \rangle ,$$

$$B = (\langle b_1, b_2 \rangle \setminus (\bigcup_{i=1}^{p-1} \langle b_1 + ib_2 \rangle)) \cup (\bigcup_{i=1}^{p-1} \langle b_1 + ib_2 + 2a_2 \rangle).$$

By Lemma 4 we have that A' and B are non-periodic Z-sets and $T = A' \oplus B$. Thus T is "Z-bad" and therefore "bad." Consequently, G itself is "bad" [6]. However, in view of Lemma 2, the Sylow 2-subgroup of G, $\langle c \rangle$, is "Z-good" so that G is "Z-good" by Lemma 1.

Let $A = A' \oplus \langle c \rangle$. Clearly A is a periodic Z-set and $\langle c \rangle \subseteq S$, the subgroup of periods of A. Let $s \varepsilon S$ so that for all $a \varepsilon A$, $a+s \varepsilon A$. Then for all $a' \varepsilon A'$, $a'+s \varepsilon A$. Thus $a' + s = \tilde{a} = \tilde{a}' + x$, $\tilde{a} \varepsilon A$, $\tilde{a}' \varepsilon A'$, $x \varepsilon \langle c \rangle$. Hence for all $a' \varepsilon A'$, $a' + s - x \varepsilon A'$ and $s - x$ is a period of A'. Since A' is non-periodic we must have that $s - x = 0$, i.e., $s = x \varepsilon \langle c \rangle$. Therefore $S = \langle c \rangle$.

We have that $G = T \oplus \langle c \rangle = A \oplus B$. Suppose this factorization arises from the series $G = K_1 \supset K_2 \supset \dots \supset K_n \supset \langle 0 \rangle$. Since B is non-periodic, $H_n = K_n$ is not a factor of B. Thus there exist transversals H_i such that $0 \varepsilon H_i$, $1 \leq i \leq n$, and

$$A = H_n + H_{n-2} + \dots ,$$

$$B = H_{n-1} + H_{n-3} + \dots .$$

H_n is contained in the subgroup of periods of A so that $H_n = \langle c \rangle$.

Note that $B \oplus \langle c \rangle$ is not a subgroup. Thus $K_{n-1} \neq B \oplus \langle c \rangle$ and consequently $H_{n-1} \neq B$. But $|H_{n-1}|$ divides $|B| = p^2$. Hence $|H_{n-1}| = p$. $H_{n-1} = B \cap K_{n-1}$ implies that H_{n-1} is a Z-set. Thus H_{n-1} is a subgroup and we conclude that B is periodic, a contradiction.

Let G be a finite abelian group such that all Sylow subgroups of G are "Z-good." It remains an open question as to whether all "Z-factorizations" of G can be obtained from the variation of Sands' method.

REFERENCES

1. HAJOS, G. Uber einfache und merfache Bedeckung des n-dimensionalen Raumes mit einem Wurfelgitter, Math. Zeitschrift 47 (1941), 427-267.

2. HAJOS, G. Sur la factorisation des groupes abeliens, Casopis Pest. Math. Fys. 74 (1950), 157-162.

3. HAJOS, G. Sur le probleme de factorisation des groupes cycliques, Acta Math. Acad. Sci. Hung. 1 (1950), 189-195.

4. REDEI, L. Zwei Luckensatze uber Polynome in endlichen Primkorpern mit Anwendung auf die enlichen Abelschen Gruppen und die Gaussischen Summen, Acta Math 79 (1947), 273-290.

5. REDEI, L. En Beitrag zum Problem der Faktorisation von enlichen Abelschen Gruppen, Acta Math. Acad. Sci. Hung. 1 (1950), 197-207.

6. DE BRUIJN, N.G. On the factorization of finite abelian groups, Indag. Math. Kon. Ned. Akad. Wetensch, Amsterdam 15 (1953), 258-264.

7. DE BRUIJN, N.G. On the factorization of cyclic groups, Indag. Math. Kon. Ned. Akad. Wetensch., Amsterdam 15 (1953), 370-377.

8. SANDS, A.D. On the factorization of finite abelian groups I, Acta Math. Acad. Sci. Hung. 8 (1957), 65-86.

9. SANDS, A.D. The factorization of abelian groups I, Quart. J. Math. (2) 10 (1959), 81-91.

10. SANDS, A.D. On the factorization of finite abelian groups II, Acta Math. Acad. Sci. Hung. 13 (1962), 153-169.

11. SANDS, A.D. The factorization of abelian groups II, Quart. J. Math. (2) 13 (1962), 45-54.

12. OKUDA, C. The Factorization of Abelian Groups, Doctoral Dissertation, The Pennsylvania State University, University Park, PA, 1975.

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The study of dynamic equations on a time scale goes back to its founder Stefan Hilger (1988), and is a new area of still fairly theoretical exploration in mathematics. Motivating the subject is the notion that dynamic equations on time scales can build bridges between continuous and discrete mathematics; moreover, it often revels the reasons for the discrepancies between two theories.

In recent years, the study of dynamic equations has led to several important applications, for example, in the study of insect population models, neural network, heat transfer, and epidemic models. This special issue will contain new researches and survey articles on Boundary Value Problems on Time Scales. In particular, it will focus on the following topics:

- Existence, uniqueness, and multiplicity of solutions
- Comparison principles
- Variational methods
- Mathematical models
- Biological and medical applications
- Numerical and simulation applications

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