

POLYNOMIAL TIME GRAPH FAMILIES FOR ARC KAYLES

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Abstract

ARC KAYLES is a combinatorial game played on a graph. We give nim-sequences for ARC KAYLES on varying classes of graphs including: equimatchable graphs, path graphs, cycle graphs, and wheel graphs. Lastly, we provide an automatic periodicity check for generalized star graphs with three rays (two fixed) and conjecture about the period for generalized star graphs with one ray fixed to two vertices, a second ray fixed to n vertices (in an equivalence class modulo 34) and the third ray varying to infinity.

1. Introduction

Graph theoretic combinatorial games became of interest within the last several decades as a means to model complex networks. One well studied graph game is called NODE KAYLES. NODE KAYLES is a combinatorial game on a graph. Players take turns choosing a single vertex such that it does not repeat, and is not adjacent to, any previously chosen vertices. The last player to move wins. Together, players are forming a maximal independent set. NODE KAYLES is well studied in [4, 6, 8, 14], including analysis on varying graph classes as well as complexity results.

A natural extension is to consider the edge counterpart of this game which is called ARC KAYLES.

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Definition 1. ARC KAYLES is a combinatorial game on a graph. Players take turns choosing a single edge such that it does not repeat, and is not adjacent to, any previously chosen edges. The last player to move wins.

Choosing an edge is the same as deleting that edge and its incident vertices since adjacent edges cannot be chosen during game play. Throughout this paper, we consider edge deletion to make the remaining options for the next player clear.

ARC KAYLES was first introduced by Schaefer in 1978 [14] and to date very little is known about the game (see [12] for some discussion about complexity). Here, we analyze ARC KAYLES on the wheel graph and generalized star graphs, determining their Grundy values and nim-sequences. For the generalized star graphs we present an automatic periodicity check, motivated by octal games described in [1].

Definition 2. A maximal matching is a set of edges, M, from a graph, Γ , in which the following two properties hold:

- 1. no two edges share a vertex; and
- 2. no additional edges from Γ can be added to the set without violating the first condition.

The game of ARC KAYLES ends when there are no more moves. Hence the proof of Lemma 1 is immediate.

Lemma 1. The end state of a game of ARC KAYLES is a maximal matching.

The paper proceeds as follows. In Section 2 we begin by giving necessary background. In Section 3 we examine ARC KAYLES on equimatchable graphs, path graphs, and cycle graphs; determining outcome classes and nim-sequences. In Section 4, we take a non-standard approach to solving ARC KAYLES played on the wheel graph. Next, in Section 5 we examine a generalized star as motivated by [6] and present an automatic periodicity check motivated by [1]. We conclude by discussing future directions for research.

2. Background

A path is a connected, acyclic graph in which all vertices have degree at most two. We denote a path on n vertices by P_n . A cycle is a connected graph where every vertex has degree 2. We denote a cycle on n vertices by C_n .

ARC KAYLES is an *impartial* game since both players have the same options from all positions. Impartial games have two outcome classes: the game is a \mathcal{P} -position if the second (previous) player can force a win; otherwise, it is an \mathcal{N} -position, which means the next (first) player can force a win.

The minimum excluded value (mex) of a set of non-negative integers is the least non-negative integer which does not occur in the set. For example, mex $\{1, 3, 4\} = 0$. An option H of a game G is a subgame of G which can be reached in exactly one move. The value (or nim-value or Grundy value) of an impartial game, denoted by \mathcal{G} , is determined by the mex of the values of its options. If the next player does not have a move, the game position has value 0. There is a natural connection between the impartial game outcome classes and their \mathcal{G} -values: if the \mathcal{G} -value for a game G is 0, then G is a previous player win (or a \mathcal{P} -position); otherwise, it is an \mathcal{N} -position.

Sometimes, making a move in a game splits the game into disjoint boards. Such a game is the disjunctive sum of two (or more) games. Consider two games G and H, their disjunctive sum is denoted by G + H. The value of a disjunctive sum of impartial games is the sum of the values of each component, in binary, without carrying. This is denoted by \oplus (XOR in computer science). For example, suppose we were playing the game G + H, where G and H are games. Then $\mathcal{G}(G + H) = \mathcal{G}(G) \oplus \mathcal{G}(H)$.

Fixing all parameters of a game G and letting one vary, we calculate $\mathcal{G}(G(n))$, for all $n \geq 0$. Arranging these \mathcal{G} -values in a sequence for a particular game G is called the *nim-sequence* of G. Throughout this paper we write $\mathcal{G}(\Gamma)$ or $\mathcal{G}(\Gamma_n)$ to represent the value of the game of ARC KAYLES played on a specific graph Γ , and if there is a parameter varying, this is specified by n.

3. Well-behaved Graph Classes

A graph, Γ , is *equimatchable* if every maximal matching, M, is maximum. Hence, all maximal matchings for an equimatchable graph have the same size. The winner of a game of ARC KAYLES on such a graph is therefore determined by the parity of a maximal matching in Γ .

Theorem 1. The value of a game of ARC KAYLES played on an equimatchable graph Γ , where M is the maximal matching at the end state of the game, is given by

$$\mathcal{G}(\Gamma) = \begin{cases} 1 & \text{if } |\mathbf{M}| \equiv 1 \pmod{2} \\ 0 & \text{otherwise} \end{cases}$$

Proof. Follows directly from Lemma 1 and Definition 2.

If a graph has the property that all maximal matchings have the same parity then its Grundy value is simple to calculate. The following proposition shows that these graphs are all, in fact, equimatchable. This contrasts with the case of independent sets. Graphs where all maximal independent sets have the same size are called *well-covered* [10]. Graphs where all maximal independent sets have the same parity

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are called *parity graphs* and are not equivalent to well-covered graphs [2, 5]. We outline the proof of our proposition, the details are left to the reader.

Proposition 1. Let Γ be a graph such that all maximal matchings have the same parity. Then Γ is equimatchable.

Sketch of proof. Let Γ be a graph and Γ has the property that all maximal matchings have the same parity. Suppose M_1 and M_2 are maximal matchings and without loss of generality $|M_1| < |M_2|$. Consider the symmetric difference of the matchings, $M_1 \triangle M_2 = \{e \in E(\Gamma) : e \text{ is in only one of the matchings}\}$. All connected components of $M_1 \triangle M_2$ are either vertices, paths or cycles and edges in the paths and cycles alternate between the two matchings. An augmenting path is a path component in $M_1 \triangle M_2$. Since $|M_1| < |M_2|$, an augmenting path, P, must exist, which begins and ends with edges from M_2 . Use this to augment matching M_1 to produce $M'_1 = M_1 \triangle P$. Since M_1 and M_2 were maximal, M'_1 is maximal too but has parity opposite to M_1 and M_2 . The result follows.

Complete graphs, K_n , and complete bipartite graphs $K_{m,n}$ are both equimatchable classes of graphs. For characterizations of equimatchable graphs, see [7, 11, 13, 15]. For other games which utilize equimatchable graph theory, see [9].

ARC KAYLES played on path graphs results in several different options: removing end edges results in a path on n-2 vertices, while removing intermediate edges results in a disjunctive sum of paths.

A line graph of a graph Γ is the graph produced from switching all edges of Γ to vertices and these vertices are connected by an edge if the corresponding edges were adjacent in Γ . Choosing an edge in Γ corresponds exactly to choosing a vertex in the line graph of Γ .

ARC KAYLES on paths has the same nim-sequence as that of Berlekamp et al. [3] for NODE KAYLES on paths. Indeed, while playing NODE KAYLES players are deleting a vertex and its neighbours, while in ARC KAYLES players are deleting an edge and its incident vertices. This is the same as playing NODE KAYLES on the line graph. This proves the next theorem.

Theorem 2. ARC KAYLES on Γ is NODE KAYLES on the line graph of Γ .

Corollary 1. ARC KAYLES played on paths, P_n , has pre-period length 53, period length 34.

The nim-sequence for ARC KAYLES on paths is shown in Table 1.

Up to isomorphism, there is one move from C_n ; to P_{n-2} . The value for C_n is 1 if $\mathcal{G}(P_{n-2}) = 0$ and 0 otherwise. The positions with value 1 of the nim-sequence for C_n within the pre-period are $n \in \{3, 7, 11, 17, 23, 27, 31, 37\}$ with pre-period length 38. The period length is 34. Positions with value 1 within the periodic portion of the nim-sequence are

| | $t \rightarrow$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|------------------------|-----------------|---|---|---|---|----------|---|---|---|---|---|----|----|----------|----------|----------|----|----------|
| $\mathbf{s}\downarrow$ | | | | | | | | | | | | | | | | | | |
| 0 | | 0 | 0 | 1 | 1 | 2 | 0 | 3 | 1 | 1 | 0 | 3 | 3 | 2 | 2 | 4 | 0 | 5 |
| 17 | | 2 | 2 | 3 | 3 | 0 | 1 | 1 | 3 | 0 | 2 | 1 | 1 | 0 | 4 | 5 | 2 | 7 |
| 34 | | 4 | 0 | 1 | 1 | 2 | 0 | 3 | 1 | 1 | 0 | 3 | 3 | 2 | 2 | 4 | 4 | 5 |
| 51 | | 5 | 2 | 3 | 3 | 0 | 1 | 1 | 3 | 0 | 2 | 1 | 1 | 0 | 4 | 5 | 3 | 7 |
| 68 | | 4 | 8 | 1 | 1 | 2 | 0 | 3 | 1 | 1 | 0 | 3 | 3 | 2 | 2 | 4 | 4 | 5 |
| 85 | | 5 | 9 | 3 | 3 | 0 | 1 | 1 | 3 | 0 | 2 | 1 | 1 | 0 | 4 | 5 | 3 | 7 |

Table 1: Nim-sequence for ARC KAYLES on paths, P_n , where n = s + t [3].

 $n \in \{41 + 34i, 45 + 34i, 57 + 34i, 61 + 34i, 65 + 34i, i \in \mathbb{N}\}.$

This proves the following result.

Proposition 2. Let $n \geq 3$. The nim-sequence for cycles is given by

$$\mathcal{G}\left(\mathbf{C}_{n}\right) = \begin{cases} 1 & \text{if } n \in S \\ 0 & \text{otherwise} \end{cases}$$

where,

 $S = \{3, 7, 11, 17, 23, 27, 31, 37, 41 + 34i, 45 + 34i, 57 + 34i, 61 + 34i, 65 + 34i, i \in \mathbb{N}\}.$

4. Wheel Graphs

A *wheel* is a connected graph with one distinguished center node, surrounded by a cycle where every node on the cycle is connected by one edge to the center node.

There are two moves for a general wheel graph, W_n , where n is the number of vertices in the cycle surrounding the distinguished node and $n \ge 3$. One move is to choose a spoke edge, which leaves a path on n-1 vertices. The other move is to choose a rim edge which leaves a fan graph with n-2 vertices on the rim. We refer to the fan graph as a *pizza graph*³, Pz_n , since further breakdown of options by removing more rim edges from a fan resembles a partially eaten pizza. Note: the values for $\mathcal{G}(W_0)$, $\mathcal{G}(W_1)$ and $\mathcal{G}(W_2)$ are 0 since we are not allowing multi-edges; the graphs have no edges and the next player does not have a move.

Lemma 2. Let $n \geq 3$, then $\mathcal{G}(\operatorname{Pz}_{n-2}) \geq \mathcal{G}(\operatorname{P}_{n-1})$.

Proof. We begin by breaking down the options of each game. From P_{n-1} , a player may remove any edge. The set of moves are summarized as the set of disjunctive sums of paths $\{P_{n-3-i} + P_i\}$, for $0 \le i \le n-3$. From P_{2n-2} there are two possible

 $^{^{3}}$ Some other authors have used the term *Fan-Star* to describe the same graph.

moves: a player may remove a spoke edge and obtain the same set of disjunctive sums of paths as from P_{n-1} : $\{P_{n-3-i} + P_i\}$, for $0 \le i \le n-3$. Otherwise, a player may remove a rim edge and we obtain another pizza with one or two separate smaller fans joined at the distinguished center node. This set of positions looks like the following set: $\{Pz_{n-i-4,i}\}$, for $0 \le i \le n-4$, where n-i-4 and *i* represent the sizes of the rim edge paths of the fans. Since the set of options of Pz_{n-2} contains the set of options of P_{n-1} , all these options are also contributing to the calculation of the mex of Pz_{n-2} . Therefore, $\mathcal{G}(Pz_{n-2})$ is at least as large as $\mathcal{G}(P_{n-1})$.

Theorem 3. Let $n \geq 3$, then

$$\mathcal{G}(\mathbf{W}_n) = \begin{cases} 1 & \text{if } \mathcal{G}(\mathbf{P}_{n-1}) = 0\\ 0 & \text{otherwise} \end{cases}$$

Proof. Up to isomorphism there are only two options from any wheel graph and hence we are only taking the mex of two values: $mex \{ \mathcal{G}(\operatorname{Pz}_{n-2}), \mathcal{G}(\operatorname{P}_{n-1}) \}$. By Lemma 2, we know that $\mathcal{G}(\operatorname{Pz}_{n-2}) \geq \mathcal{G}(\operatorname{P}_{n-1})$. There are three possibilities:

- 1. $\mathcal{G}(\operatorname{Pz}_{n-2}) \geq \mathcal{G}(\operatorname{P}_{n-1}) > 0$ implying that $\mathcal{G}(\operatorname{W}_n) = 0$.
- 2. $\mathcal{G}(\operatorname{Pz}_{n-2}) > 1 > \mathcal{G}(\operatorname{P}_{n-1}) = 0$ or $\mathcal{G}(\operatorname{Pz}_{n-2}) = \mathcal{G}(\operatorname{P}_{n-1}) = 0$, both implying that $\mathcal{G}(\operatorname{W}_n) = 1$.
- 3. $\mathcal{G}(\operatorname{Pz}_{n-2}) = 1, \mathcal{G}(\operatorname{P}_{n-1}) = 0$ implying that $\mathcal{G}(\operatorname{W}_n) = 2$.

We show that Case 3 cannot occur. If $\mathcal{G}(\mathbf{P}_{n-1}) = 0$ we know all of its options have \mathcal{G} -value greater than 0. Recall that the options of P_{n-1} are also options of Pz_{n-2} . If we can show that all the paths which have value 0 will have an option with value equal to 1, we will have completed the proof. The path sequence is ultimately periodic as shown in Section 3, so we only need to check the zero valued paths within the pre-period and first period of the path nim-sequence; games contributing to subsequent period values will, by definition, have the same values contributing to their mex calculation as did its corresponding position (modulo the period length) within the first period. Hence, there are a finite number of zero valued paths (n = 6, 10, 16, 22, 26, 30, 36, 40, 44, 56, 60, 64, 74, 78) that we need to check. Recall n denotes the number of vertices on the rim of the wheel, the number of vertices on the corresponding path is one less. We know that the options from P_{n-1} are $\{P_i + P_{n-i-3}\}$. For $0 \le i \le 6$, the Grundy values of P_i are (0, 0, 1, 1, 2, 0, 3). The Grundy values required for P_{n-i-3} to yield a sum of 1 are therefore (1, 1, 0, 0, 3, 1, 2)for i in the same range. For $n \in (6, 10, 16, 22, 26, 30, 36, 40, 44, 56, 60, 64, 74, 78)$, the values of i that establish that P_{n-1} has an option with value 1 are therefore i = (0, 0, 5, 6, 0, 0, 5, 0, 0, 6, 0, 0, 0, 0) respectively. This concludes the proof.

5. Generalized Star Graphs

A star is a connected acyclic graph that has one distinguished center node with degree $n \ge 1$ and all other vertices with degree 1. We call a graph a generalized star if we allow for the possibility of all non-center vertices having degree at most 2.

We denote a generalized star graph with three rays as $S_{f,g,n}$, where f, g and n are the number of vertices on each ray. When allowing one ray, n, to vary we denote the class of generalized stars by $S_{f,g}(n)$ or simply $S_{f,g}$ to mean the nim-sequence $\{\mathcal{G}(S_{f,g,n}): n = 1, 2, \ldots\}$. Playing ARC KAYLES on a generalized star graph with three rays, we have the following decomposition.

- 1. A player may remove an edge incident to the center node. This can happen in three ways and produces the following three outcomes:
 - (a) $P_{f-2} + P_{g-1} + P_{n-1}$
 - (b) $P_{f-1} + P_{g-2} + P_{n-1}$
 - (c) $P_{f-1} + P_{q-1} + P_{n-2}$.
- 2. A player may remove the second edge from the center vertex on any of the rays. These will give the following three outcomes:
 - (a) $P_{f-3} + P_{g+n-1}$
 - (b) $P_{g-3} + P_{f+n-1}$
 - (c) $P_{n-3} + P_{f+g-1}$.
- 3. A player may remove any other edge from any of the rays. These will leave a disjunctive sum of a generalized star and a path:
 - (a) $S_{f-i-2,g,n} + P_i, 0 \le i \le f-2$
 - (b) $S_{f,g-i-2,n} + P_i, 0 \le i \le g-2$
 - (c) $S_{f,q,n-i-2} + P_i, 0 \le i \le n-2.$

The decomposition shows that there are many values contributing to the mex calculation of the current game position. We now present a method to implement an automatic check for periodicity for this graph class. This check is motivated by work presented in [1] and [3] with regards to SUBTRACTION and octal games. For notational simplicity, \hat{S} is the expected pre-period associated with the current generalized star $S_{f,g,n}$ and \overline{S} is its expected period. Similarly we reserve \hat{D} for the pre-period and \overline{D} for the period for other nim-sequences with respect to a class of graphs denoted by D.

Theorem 4 (Automatic check for periodicity). Let f and g be positive integers. Suppose $S_{f',g'}$ is periodic, where $f' \leq f$, $g' \leq g$ and f' + g' < f + g. If $\mathcal{G}(S_{f,g,n}) = \mathcal{G}\left(S_{f,g,n+\overline{S}}\right)$ for all $\widehat{S} \leq n \leq \widehat{I} + 2\overline{I} + 2$, then $S_{f,g}$ is periodic with

$$\widehat{I} = max \left\{ \widehat{S} + \widehat{P}_{f,g,n}, \left(\widehat{S}_{f-j-2,g,n} \right)_{j=0}^{f-2}, \left(\widehat{S}_{f,g-j-2,n} \right)_{j=0}^{g-2}, \widehat{P}_{f,g-j-2,n} - \overline{P}, \widehat{P} \right\}$$

and

$$\overline{I} = LCM\left\{\overline{S}, \left(\overline{S}_{f,g-2-j}\right)_{j=0}^{g-2}, \left(\overline{S}_{f-2-j,g}\right)_{j=0}^{f-2}, \overline{P}\right\}$$

Proof. Recall that the path nim-sequence is periodic (presented in Section 3). The value of a generalized star $S_{f,g,n}$ is determined by the minimum excluded value of its options. We want to show that if n is large enough and $\mathcal{G}(S_{f,g,n}) = mex(T)$ and $\mathcal{G}(S_{f,g,n+\overline{S}}) = mex(S)$, then T = S. Hence the nim-sequences of the following options need to be periodic:

- 1. $P_{f-2} + P_{g-1} + P_{n-1}$
- 2. $P_{f-1} + P_{g-2} + P_{n-1}$
- 3. $P_{f-1} + P_{g-1} + P_{n-2}$
- 4. $P_{f-3} + P_{g+n-1}$
- 5. $P_{g-3} + P_{f+n-1}$
- 6. $P_{n-3} + P_{f+g-1}$
- 7. $S_{f-i-2,g,n} + P_i, 0 \le i \le f-2$
- 8. $S_{f,g-i-2,n} + P_i, 0 \le i \le g-2$
- 9. $S_{f,q,n-i-2} + P_i, 0 \le i \le n-2$

Their common period will be \overline{S} .

We now check that each option has a periodic nim-sequence. First consider options 1, 2, 3 as listed above. These options are paths where two of the summands of paths are fixed and the third (involving n) is varying. This is equivalent to taking the nim-sequence of the path (which we know to be periodic) and adding the same constant to every entry. If n is larger than $\hat{P} + \overline{P} + 1$, then $\mathcal{G}(P_{n-1}) = \mathcal{G}(P_{n+\overline{S}-1})$, and so $\mathcal{G}(P_{n-1}) \oplus \mathcal{G}(P_{f-2}) \oplus \mathcal{G}(P_{g-1}) = \mathcal{G}(P_{n+\overline{S}-1}) \oplus \mathcal{G}(P_{f-2}) \oplus \mathcal{G}(P_{g-1})$. These nim-sequences are periodic with period length 34. The same holds for option 2 (indices are switched and the proof is the same). For option 3, the only difference is that n must be larger than $\hat{P} + \overline{P} + 2$. This shows that when n is sufficiently large, the Grundy values of the first three positions are all contained in both T and in S.

The nim-sequence for options 4,5 involving n + f - 1 and n + g - 1, is a path nim-sequence shifted by f-1 and q-1 respectively. The second component of the nim-addition is a fixed length path and so the same explanation as the first set $\{1,2,3\}$ of path options applies. Lastly, the sixth option consists of a path nimsequence (shifted by three) being nim-added to a constant. This also reduces to the earlier cases with the slight modification that n must be as large as $\hat{P} + \overline{P} + 3$. All three options are periodic with period length 34. Once again, n must be larger than $\widehat{P} + \overline{P} + 3$. And so, the Grundy values of the three options $\{4, 5, 6\}$ are in both T and S.

The seventh and eighth options listed rely on the breakdown of the fixed rays fand q respectively. We examine the seventh option in detail, the eighth is symmetric. There are finitely many options of the form $\{S_{f-j-2,g,n} + P_j\}_{j=0}^{f-2}$. The generalized stars considered in this set rely on different generalized star classes with respect to q which we consider to have already been determined. Hence, in order to determine the periodicity of the current class, we need to be large enough with respect to all of the previously calculated classes already attaining periodicity and the periodicity of all classes involved have to be synchronized. And so, we need

$$n \ge max \left\{ \left(\widehat{S}_{f-j-2,g,n} \right)_{j=0}^{f-2} \right\} + LCM \left\{ \left(\overline{S}_{f-j-2,g,n} \right)_{j=0}^{f-2} \right\}.$$

If n is sufficiently large then the Grundy values will be contained in both T and S.

Lastly, the first terms of the options $S_{f,g,n-j-2} + P_j, 0 \le j \le n-2$, remain within the same generalized star class and we look back at previously calculated positions and nim-add a path of the appropriate length. We consider two cases:

<u>Case 1:</u> Suppose $n - j - 2 > \widehat{S} + \overline{I}$. Since $n - j - 2 > \widehat{S} + \overline{I}$, $n - j - 2 > \widehat{S} + \overline{I}$, $n - j - 2 > \widehat{S} + \overline{I}$. $2-\overline{I} > \widehat{S}$, the nim-sequence of $S_{f,g,n}$ is already showing signs of periodicity. And so, $\mathcal{G}\left(S_{f,g,n-j-2+\overline{S}}\right) = \mathcal{G}\left(S_{f,g,n-j-2}\right)$. This implies that adding a constant to both positions will give the same value. Hence $\mathcal{G}\left(S_{f,g,n-j-2+\overline{S}} + \mathbf{P}_{j}\right) =$ $\mathcal{G}(S_{f,g,n-j-2} + \mathbf{P}_j)$. Thus the Grundy values of these positions appear in both T and S.

<u>Case 2:</u> Suppose $n - j - 2 \leq \widehat{S} + \overline{I}$. If $n > \widehat{I} + 2\overline{I} + 2$, then $j > \widehat{P} + \overline{I}$ and $\mathcal{G}\left(\mathbf{P}_{j+\overline{S}}\right) = \mathcal{G}\left(\mathbf{P}_{j}\right)$. Then $\mathcal{G}\left(S_{f,g,n-j-2} + \mathbf{P}_{j+\overline{S}}\right) = \mathcal{G}\left(S_{f,g,n-j-2} + \mathbf{P}_{j}\right)$. Thus the Grundy values of these positions appear in both T and S.

When $n > \hat{I} + 2\overline{I} + 2$, we conclude that S = T, where

$$\widehat{I} = max \left\{ \widehat{S} + \widehat{P}_{f,g,n}, \left(\widehat{S}_{f-j-2,g,n} \right)_{j=0}^{f-2}, \left(\widehat{S}_{f,g-j-2,n} \right)_{j=0}^{g-2}, \widehat{P}_{f,g-j-2,n} - \overline{P}, \widehat{P} \right\}$$

and
$$\overline{I} = LCM \left\{ \overline{S}, \left(\overline{S}_{f,g-j-2,n} \right)_{j=0}^{g-2}, \left(\overline{S}_{f-j-2,g,n} \right)_{j=0}^{f-2}, \overline{P} \right\}.$$

ar

We have investigated the nim-sequences when f = 2, g fixed and the third ray varying off to infinity. Based on our preliminary results it appears as though all classes $g \pmod{34}$ have a stabilizing pre-period (see Table 2) meaning that each class (mod 34) eventually has a stable period also.

The quasi-pre-period for a generalized star with a ray $g \pmod{34}$ is the value of g at which the pre-period for that class of generalized stars becomes stable in n.

Conjecture 1. For all g > quasi-pre-period, and n > pre-period length shown in Table 2, the \mathcal{G} -value of ARC KAYLES played on $S_{2,g,n}$ is equal to the value shown in Table 5, where g and n are modulo 34 in the lookup table.

Let us look at an example to understand how the lookup tables (Table 2 and 5) work. Consider $S_{2,750,673}$. This is a generalized star with one ray fixed with 2 vertices, a second ray fixed with 750 vertices and a third ray fixed with 673 vertices. Now, $750 \equiv 2 \pmod{34}$ and $673 \equiv 27 \pmod{34}$. In Table 2 we see that g = 750 is beyond the value where this class modulo 34 becomes stable in n; it is far enough, because in that column g = 274 is the value at which this class is stable for n. Also, our value for n, namely n = 673, is beyond this last irregularity since for this stabilized class h = 459 was the last irregularity. Proceed to Table 5 and look up the value of $S_{2,750,673}$ which will be found at the intersection of row 2 and column 27. Hence, we conjecture that $\mathcal{G}(S_{2,750,673}) = 47$.

If Conjecture 1 is true, it means that there is indeed an ultimate generalized star $S_{2,g,n}$ which is completely determined by smaller generalized star classes (as was hoped for NODE KAYLES in [6]) for ARC KAYLES. At this time, it is unclear how to prove this conjecture as even the first step of proving boundedness a priori of the nim-sequence is hard. This will be further discussed in the future directions section.

| | $t \rightarrow$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|--------------------------|---|--|--|--|---|---|---|---|--|--|---|---|--|---|---|---|--|
| s↓ | | | | | | | | | | | | | | | | | | |
| 0 | | 52 | 52 | 52 | 196 | 52 | 156 | 349 | 232 | 301 | 191 | 249 | 181 | 190 | 383 | 232 | 188 | 388 |
| 34 | | 349 | 249 | 249 | 249 | 215 | 245 | 216 | 250 | 165 | 256 | 250 | 388 | 258 | 248 | 249 | 250 | 261 |
| 68 | | 279 | 249 | 283 | 204 | 241 | 260 | 383 | 301 | 165 | 261 | 459 | 388 | 301 | 232 | 283 | 279 | 244 |
| 102 | | 336 | 320 | 283 | 204 | 283 | 260 | 349 | 248 | 285 | 299 | 388 | 388 | 294 | 232 | 249 | 250 | 272 |
| 136 | | 313 | 388 | 312 | 228 | 283 | 396 | 349 | 313 | 279 | 283 | 388 | 388 | 260 | 232 | 343 | 333 | 271 |
| 170 | | 313 | 388 | 459 | 221 | 283 | 396 | 349 | 345 | 313 | 283 | 388 | 388 | 396 | 232 | 299 | 252 | 373 |
| 204 | | 316 | 388 | 459 | 340 | 261 | 275 | 349 | 388 | 313 | 283 | 275 | 388 | 459 | 232 | 303 | 461 | 280 |
| 238 | | 333 | 388 | 326 | 340 | 261 | 338 | 349 | 459 | 301 | 283 | 388 | 320 | 459 | 232 | 367 | 461 | 340 |
| 272 | | 299 | 388 | 459 | 340 | 283 | 338 | 349 | 459 | 313 | 351 | 388 | 349 | 459 | 374 | 316 | 337 | 408 |
| 306 | | 299 | 388 | 459 | 340 | 283 | 338 | 349 | 459 | 313 | 351 | 388 | 349 | 459 | 340 | 303 | 461 | 340 |
| 340 | | 299 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 313 | 351 | 388 | 349 | 459 | 374 | 303 | 461 | 340 |
| 374 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 374 | 303 | 461 | 340 |
| 408 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 340 | 329 | 461 | 340 |
| 442 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 340 | 329 | 461 | 340 |
| 476 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 340 | 329 | 461 | 340 |
| 510 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 340 | 329 | 461 | 340 |
| 544 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 340 | 329 | 461 | 340 |
| 578 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 340 | 329 | 461 | 340 |
| 612 | | 461 | 388 | 459 | 340 | 283 | 338 | 392 | 459 | 301 | 351 | 388 | 349 | 459 | 340 | 329 | 461 | 340 |
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| | $\mathrm{t}{ ightarrow}$ | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| s↓ | $t \rightarrow$ | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| s↓ 0 | $t \rightarrow$ | 17 190 | 18 283 | 19 170 | 20 250 | 21 261 | 22 349 | 23 125 | 24 349 | 25 249 | 26 162 | 27 191 | 28 249 | 29 177 | 30 303 | 31 178 | 32 245 | 33 232 |
| s↓ 0 34 | $t \rightarrow$ | 17 190 197 | 18 283 249 | 19 170 245 | 20 250 388 | 21 261 246 | 22 349 249 | 23 125 383 | 24 349 349 | 25 249 249 | 26 162 241 | 27 191 204 | 28 249 284 | 29 177 212 | 30 303 245 | 31 178 250 | 32 245 349 | 33 232 245 |
| s↓ 0 34 68 | $t \rightarrow$ | 17 190 197 349 | 18 283 249 388 | 19 170 245 258 | 20 250 388 260 | 21 261 246 283 | 22 349 249 245 | 23 125 383 383 | 24 349 349 349 | 25 249 249 388 | 26 162 241 260 | 27 191 204 301 | 28 249 284 349 | 29 177 212 246 | 30 303 245 250 | 31 178 250 333 | 32 245 349 314 | 33 232 245 264 |
| $s\downarrow 0 \\ 34 \\ 68 \\ 102$ | $t \rightarrow$ | 17 190 197 349 316 | 18 283 249 388 283 | 19 170 245 258 258 | 20 250 388 260 245 | 21 261 246 283 280 | 22 349 249 245 280 | 23 125 383 383 250 | 24 349 349 349 396 | 25 249 249 388 388 | 26 162 241 260 294 | 27 191 204 301 459 | 28 249 284 349 349 | 29 177 212 246 333 | 30 303 245 250 250 | 31 178 250 333 314 | 32 245 349 314 349 | 33 232 245 264 320 |
| $s\downarrow 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 150$ | $t \rightarrow$ | 17 190 197 349 316 306 | 18 283 249 388 283 303 | 19 170 245 258 258 272 | 20 250 388 260 245 250 | 21 261 246 283 280 280 | 22 349 249 245 280 301 | 23 125 383 383 250 250 | 24 349 349 349 396 396 | 25 249 249 388 388 388 | 26 162 241 260 294 273 | 27 191 204 301 459 459 | 28 249 284 349 303 | 29 177 212 246 333 459 | 30 303 245 250 250 298 | 31 178 250 333 314 300 | 32 245 349 314 349 349 | 33 232 245 264 320 388 |
| $s\downarrow 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 2004$ | t→ | 17 190 197 349 316 306 459 | 18 283 249 388 283 303 283 | 19 170 245 258 258 258 272 459 | 20 250 388 260 245 250 333 | 21 261 246 283 280 280 333 | 22 349 249 245 280 301 396 | 23 125 383 383 250 250 388 | 24 349 349 396 396 396 | 25 249 249 388 388 388 286 | 26 162 241 260 294 273 340 | 27 191 204 301 459 459 461 | 28 249 284 349 349 303 383 | 29 177 212 246 333 459 459 | 30 303 245 250 250 298 250 | 31 178 250 333 314 300 333 | 32 245 349 314 349 349 358 | 33 232 245 264 320 388 459 |
| $s\downarrow 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 204 \\ 204$ | $t \rightarrow$ | 17 190 197 349 316 306 459 374 374 | 18 283 249 388 283 303 283 283 283 | 19 170 245 258 258 272 459 258 | 20 250 388 260 245 250 333 301 | 21 261 246 283 280 280 333 280 | 22 349 249 245 280 301 396 260 | 23 125 383 383 250 250 388 388 | 24 349 349 396 396 396 396 349 | 25 249 249 388 388 388 286 255 | 26 162 241 260 294 273 340 336 | 27 191 204 301 459 461 461 | 28 249 284 349 303 383 383 | 29 177 212 246 333 459 459 459 | 30 303 245 250 250 298 250 298 250 | 31 178 250 333 314 300 333 333 | 32 245 349 314 349 349 358 459 | 33 232 245 264 320 388 459 367 |
| $s\downarrow 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 204 \\ 238 \\ 372 \\ 38 \\ 372 \\ 34 \\ 35 \\ 34 \\ 35 \\ 35 \\ 35 \\ 35 \\ 35$ | $t \rightarrow$ | 17 190 197 349 316 306 459 374 326 | 18 283 249 388 283 303 283 283 341 | 19 170 245 258 258 258 272 459 258 258 258 | 20 250 388 260 245 250 333 301 301 | 21 261 246 283 280 280 333 280 283 280 | 22 349 249 245 280 301 396 260 459 | 23 125 383 383 250 250 388 388 388 | 24 349 349 396 396 396 396 349 459 | 25 249 249 388 388 388 286 255 337 | 26 162 241 260 294 273 340 336 329 | 27 191 204 301 459 469 461 388 | 28 249 284 349 303 383 383 383 383 | 29 177 212 246 333 459 459 459 301 | 30 303 245 250 250 298 250 298 298 298 | 31 178 250 333 314 300 333 333 333 | 32 245 349 314 349 349 358 459 459 | 33 232 245 264 320 388 459 367 459 |
| $s\downarrow 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 204 \\ 238 \\ 272 \\ 202 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $ | t→ | 17 190 197 349 316 306 459 374 326 348 | 18 283 249 388 283 303 283 283 341 341 | 19 170 245 258 258 272 459 258 258 258 258 | 20 250 388 260 245 250 333 301 301 392 | 21 261 246 283 280 280 333 280 283 280 283 261 | 22 349 245 280 301 396 260 459 443 | 23 125 383 383 250 250 388 388 388 388 | 24 349 349 396 396 396 396 349 459 | 25 249 249 388 388 388 286 255 337 303 | 26 162 241 260 294 273 340 336 329 341 | 27 191 204 301 459 459 461 461 388 388 | 28 249 284 349 303 383 383 383 383 383 | 29 177 212 246 333 459 459 459 301 301 | 30 303 245 250 250 298 250 298 298 298 298 | 31 178 250 333 314 300 333 333 333 333 333 | 32 245 349 314 349 349 358 459 459 459 | 33 232 245 264 320 388 459 367 459 459 |
| $s\downarrow 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 204 \\ 238 \\ 272 \\ 306 \\ 646 \\ $ | t→ | 17 190 197 349 316 306 459 374 326 348 313 | 18 283 249 388 283 303 283 283 283 341 341 341 | $ \begin{array}{r} 19\\ 170\\ 245\\ 258\\ 258\\ 272\\ 459\\ 258\\ 258\\ 258\\ 256\\ 256\\ 256\\ 256\\ \end{array} $ | 20 250 388 260 245 250 333 301 301 392 392 | 21 261 246 283 280 280 280 283 280 283 261 261 | 22 349 249 245 280 301 396 260 459 443 443 | 23 125 383 383 250 250 388 388 388 388 388 388 | 24 349 349 396 396 396 349 459 459 459 | 25 249 249 388 388 286 255 337 303 255 | 26 162 241 260 294 273 340 336 329 341 374 | 27 191 204 301 459 461 461 388 388 388 | 28 249 284 349 303 383 383 383 383 303 | 29 177 212 246 333 459 459 301 301 392 | 30 303 245 250 250 298 250 298 298 298 298 298 298 | 31 178 250 333 314 300 333 333 333 333 333 333 | 32 245 349 314 349 358 459 459 459 459 | 33 232 245 264 320 388 459 367 459 459 459 |
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| $\begin{array}{c} \mathrm{s}\downarrow \\ 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 204 \\ 238 \\ 272 \\ 306 \\ 340 \\ 374 \\ 408 \\ 442 \\ 476 \\ 510 \\ 514 \end{array}$ | $t \rightarrow$ | 17 190 197 349 316 306 459 374 326 348 313 313 313 313 313 313 313 313 | 18 283 249 388 283 283 283 341 341 341 343 443 443 443 443 443 443 443 | 19 170 245 258 258 258 258 258 258 256 256 309 309 309 309 309 309 309 | 20 250 388 260 245 250 333 301 301 392 392 392 392 392 392 392 392 | 21 261 246 283 280 280 280 283 261 261 261 261 261 261 261 261 261 | 22 349 245 280 301 396 260 459 443 443 443 443 443 443 443 443 443 | 23 125 383 250 250 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388388 388 388388 388388 388 388388 388388 388388 388388 388388 388388388 388388 388388388 388388 388388388388 388388388388388388388388 | 24 349 349 396 396 396 396 349 459 459 459 459 459 459 459 459 | 25 249 249 388 388 286 255 337 303 255 255 255 255 255 255 255 | 26 162 241 260 294 273 340 336 329 341 374 374 374 374 374 374 374 | 27 191 204 301 459 461 461 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388388 388 388 388 388388 388 388388 388 388388 388388 388 388388 388388 388 388388 388388 388388 388 388388 388388 388 388388388 388388388 3883883883883883883883 | 28 249 284 349 303 383 383 383 383 303 303 303 303 303 | 29 177 212 246 333 459 459 301 301 301 392 392 392 392 392 392 392 392 392 | 30 303 245 250 298 250 298 298 298 298 284 284 284 284 284 284 284 | 31 178 250 333 314 300 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 3333 333 3333 3333 3333 333333333333333 | 32 245 349 314 349 358 459 459 459 459 459 459 459 459 459 459 | 33 232 245 264 320 388 459 367 459 45 |
| $\begin{array}{c} \mathrm{s}\downarrow \\ 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 204 \\ 238 \\ 272 \\ 306 \\ 340 \\ 374 \\ 408 \\ 442 \\ 476 \\ 510 \\ 544 \\ 550 \end{array}$ | t→ | 17 190 197 349 316 306 459 374 326 348 313 313 313 313 313 313 313 313 313 313 313 313 313 313 313 315 | 18 283 249 388 283 283 283 341 341 341 343 443 | 19 245 258 258 272 459 258 258 258 256 256 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 309 3093309 309 30933093309330933093309330933093309330131111111111111 | 20 250 388 260 245 250 333 301 301 392 392 392 392 392 392 392 392 | 21 261 246 283 280 280 280 283 261 261 261 261 261 261 261 261 261 261 | 22 349 245 280 301 396 260 459 443 443 443 443 443 443 443 443 443 44 | 23 125 383 250 250 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388388 388 388388 388 388388 388388 388 388388 388388 388388 388 388388 388388 388388 388388 388388388388388388388388388 | 24 349 349 396 396 396 396 349 459 459 459 459 459 459 459 459 459 | 25 249 249 388 388 286 255 337 303 255 255 255 255 255 255 255 255 | 26 162 241 260 294 273 340 336 329 341 374 374 374 374 374 374 374 374 374 | 27 191 204 301 459 461 461 388 388 388 388 388 388 388 388 388 38 | 28 249 284 349 303 383 383 383 383 303 303 303 303 303 | 29 177 212 246 333 459 459 301 301 392 392 392 392 392 392 392 392 392 392 392 392 392 392 392 | 30 303 245 250 298 250 298 298 298 298 298 298 284 284 284 284 284 284 284 284 | 31 178 250 333 314 300 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 3333 3333 3333 3333 3333333 | 32 245 349 314 349 358 459 459 459 459 459 459 459 459 459 459 | 33 232 245 264 320 388 459 45 |
| $\begin{array}{c} s\downarrow \\ 0 \\ 34 \\ 68 \\ 102 \\ 136 \\ 170 \\ 204 \\ 238 \\ 272 \\ 306 \\ 340 \\ 340 \\ 344 \\ 442 \\ 476 \\ 510 \\ 544 \\ 578 \end{array}$ | t→ | 17 190 197 349 316 306 459 374 326 348 313 313 313 313 313 313 313 313 313 313 313 313 313 313 313 315 | 18 283 249 388 283 303 283 341 341 341 343 443 443 443 443 443 443 443 443 443 443 443 443 443 443 443 443 443 | 19 170 245 258 258 258 258 258 256 256 309 309 309 309 309 309 309 309 309 | 20 250 388 260 245 250 333 301 301 392 392 392 392 392 392 392 392 392 392 | 21 261 246 283 280 280 283 261 261 261 261 261 261 261 261 261 261 | 22 349 249 245 280 301 396 260 459 443 443 443 443 443 443 443 443 443 44 | 23 125 383 250 250 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388388 388 388 388388 388 388388 388388 388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388388 388388 38838838388 388388 388388388 388388 388388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388388 388 388388 388 388388 388 388388 388 388388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 388 38 38 38 38 38 38 38 | 24 349 349 396 396 396 349 459 459 459 459 459 459 459 459 459 4 | 249 249 388 388 285 255 255 255 255 255 255 255 255 2 | 26 162 241 260 294 273 340 329 341 374 374 374 374 374 374 374 374 374 374 | 27 191 204 301 459 461 461 388 388 388 388 388 388 388 388 388 38 | 28 249 284 349 303 383 383 383 303 303 303 303 303 303 | 29 1777 212 246 333 459 459 301 301 392 392 392 392 392 392 392 392 392 392 392 392 392 392 | 30 303 245 250 298 298 298 298 298 298 298 284 284 284 284 284 284 284 284 | 31 178 250 333 314 300 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 333 | 32 245 349 314 349 358 459 459 459 459 459 459 459 459 459 459 | 33 232 245 264 320 388 459 367 459 < |

Table 2: Pre-period lengths for generalized star graphs $S_{f,g,n}$; f = 2, g = s + t and n goes to infinity.

| | $n' \rightarrow$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | |
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| $g'\downarrow$ | | 0 | | 0 | | | | | | 10 | | | 10 | | 10 | | 10 | 0 | | | 20 | | | |
| 0 | | 6 | 8 | 8 | 1 | 26 | 4 | 28 | 3 | 13 | 29 | 41 | 42 | 27 | 13 | 33 | 12 | 8 | 1 | 45 | 29 | 3 | 24 | |
| 1 | | 0 | 1 | 1 | 9 | 0 | 20 | 0 | 20 42 | 0 | 4 | 20 | 2 | 10 | 4 | 10 | Э Е | - 0 - 4.9 | 20 | 3 | 50 | 04 07 | 0 1 | |
| 3 | | 0 | 9 | 2 | 3 | 14 | 26 | 2 | 43 | 8 | 14 | 5 | 20 | 4 | 8 | 5 | 18 | 43 | 3 | 8 | 5 | 13 | 3 | |
| 4 | | 26 | Ő | 0 | 14 | 1 | 2 | 4 | 14 | 14 | 5 | 43 | 4 | 16 | 5 | 31 | 4 | 4 | 24 | 2 | 19 | 36 | Ő | |
| 5 | | 4 | 23 | 37 | 26 | 2 | 4 | 12 | 6 | 13 | 7 | 4 | 42 | 5 | 9 | 33 | 7 | 19 | 2 | 9 | 4 | 16 | 28 | |
| 6 | | 28 | 8 | 1 | 2 | 4 | 12 | 1 | 29 | 7 | 4 | 8 | 2 | 17 | 4 | 4 | 21 | 5 | 51 | 3 | 15 | 37 | 5 | |
| 7 | | 3 | 26 | 43 | 4 | 14 | 6 | 29 | 4 | 12 | 46 | 14 | 44 | 7 | 58 | 49 | 13 | 43 | 43 | 54 | 44 | 37 | 29 | |
| 8 | | 13 | 0 | 0 | 8 | 14 | 13 | 7 | 12 | 8 | 5 | 26 | 23 | 3 | 5 | 5 | 4 | 4 | 12 | 2 | 6 | 9 | 4 | |
| 9 | | 29 | 4 | 28 | 14 E | - 5 - 4.9 | 7 | 4 | 46 | 5 | 5 | 53 | 11 | 24 | 1 24 | 48 | 4 | 0 | 31 | 1 22 | 17 | 26 | 7 | |
| 11 | | 41 42 | 23 | 20 | 20 | 40 | 4 | 2 | 14 | 20 | 7 | 11 | 1 | 18 | 10 | 10 | 20 | 1 | 35 | 1 | 4 | 42 | 6 | |
| 12 | | 27 | 6 | 18 | 4 | 16 | 5 | 17 | 7 | 3 | 24 | 5 | 18 | 4 | 19 | 14 | 25 | 39 | 40 | 4 | 41 | 27 | 40 | |
| 13 | | 13 | 22 | 4 | 8 | 5 | 9 | 4 | 58 | 5 | 1 | 34 | 19 | 19 | 2 | 2 | 26 | 4 | 12 | 14 | 6 | 9 | 3 | |
| 14 | | 33 | 15 | 38 | 5 | 31 | 33 | 4 | 49 | 5 | 48 | 15 | 0 | 14 | 2 | 3 | 3 | 21 | 1 | 1 | 30 | 4 | 7 | |
| 15 | | 12 | 5 | 5 | 18 | 4 | 7 | 21 | 13 | 4 | 4 | 12 | 29 | 2 | 26 | 3 | 1 | 1 | 43 | 38 | 7 | 8 | 9 | |
| 16 | | 8 | 5 | 43 | 41 | 4 | 19 | 5 | 43 | 4 | 0 | 2 | 1 | 39 | 4 | 21 | 1 | 1 | 25 | 4 | 7 | 42 | 1 | |
| 18 | | 45 | 3 | 39 | 3 | 24 | 2 0 | 3 | 40 54 | 12 | 1 | 13 | 30 | 40 | 14 | 1 | 43 38 | 20 | 12 | 5 | 41 | 04 0 | 40 | |
| 19 | | 29 | 3 | 50 | 5 | 19 | 4 | 15 | 44 | 6 | 17 | 4 | 0 | 41 | 6 | 30 | 7 | 7 | 41 | 1 | 26 | 32 | 3 | |
| 20 | | 3 | 34 | 27 | 13 | 36 | 16 | 37 | 37 | 9 | 26 | 7 | 42 | 27 | 9 | 4 | 8 | 42 | 54 | 9 | 32 | 3 | 31 | |
| 21 | | 24 | 5 | 1 | 3 | 0 | 28 | 5 | 29 | 4 | 7 | 27 | 6 | 40 | 3 | 7 | 9 | 1 | 40 | 3 | 3 | 31 | 1 | |
| 22 | | 50 | 21 | 23 | 28 | 33 | 13 | 6 | 18 | 8 | 36 | 6 | 27 | 7 | 8 | 50 | 5 | 40 | 12 | 8 | 3 | 1 | 19 | |
| 23 | | 26 | 3 | 41 | 39 | 2 | 13 | 4 | 4 | 31 | 14 | 13 | 0 | 23 | 39 | 5 | 38 | 7 | 41 | 2 | 13 | 0 | 12 | |
| 24 25 | | 48 | 20 | 30 13 | | 38 | 4 | 10 | о 0 | 10 | 32 | 4 20 | 20 | - 0 - 13 | 7 | 7 | 4 | 39 | 19 | 30 | 22 | 4 | 28 6 | |
| 26 | | 54 | 6 | 51 | 4 | 24 | 20 | 1 | 52 | 7 | 20 | 25 | 5 | 26 | 12 | 42 | 50 | 2 | 16 | 4 | 19 | 34 | 26 | |
| 27 | | 14 | 18 | 47 | 7 | 6 | 5 | 9 | 52 | 8 | 6 | 47 | 43 | 4 | 19 | 32 | 13 | 0 | 12 | 20 | 6 | 5 | 26 | |
| 28 | | 10 | 4 | 4 | 5 | 5 | 10 | 0 | 17 | 5 | 5 | 35 | 3 | 8 | 1 | 2 | 21 | 12 | 28 | 6 | 6 | 7 | 4 | |
| 29 | | 12 | 0 | 4 | 6 | 9 | 7 | 8 | 46 | 5 | 38 | 12 | 3 | 14 | 13 | 0 | 0 | 28 | 48 | 13 | 31 | 4 | 4 | |
| 30 | | 4 | 8 | 5 | 14 | 4 | 8 | 8 | 1 | 9 | 7 | 7 | 36 | 14 | 0 | 4 | 36 | 2 | 6 | 26 | 4 | 21 | 5 | |
| 30 | | 38 5 | 91 | 40 | 10 | 6 | 42 | | 48 | 22 | 30 | 19 | 1 93 | 30 | 4 | 30 | 3 | 47 | 40 | 4 20 | 41 | - Ð - 1.4 | 9 | |
| 33 | | 30 | 19 | 4 | 18 | 5 | 40 | 15 | 20 | 13 | 6 | 0 | 4 | 38 | 45 | 2 | 34 | 46 | 37 | 5 | 5 | 43 | 3 | |
| | | | | | | | | | | | | | | | | | | | | | | | _ | |
| | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | n | ′ → | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | • | | | | | |
| | | | | g' | <i>n</i> | ′ → | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | | | | | | |
| | | | | $g' \downarrow 0 1$ | <i>n</i> | ′ → | 22 50 21 | 23 26 | 24 48 16 | 25 15 2 | 26 54 6 | 27 14 18 | 28 10 4 | 29 12 0 | 30 4 8 | 31 38 5 | 32 5 21 | 33 30 | | | | | | |
| | | | | $g' \downarrow 0 \\ 1 \\ 2$ | <i>n</i> | ′ → | 22 50 21 23 | 23 26 3 41 | 24 48 16 36 | 25 15 2 13 | 26 54 6 51 | 27 14 18 47 | 28 10 4 4 | 29 12 0 4 | 30 4 5 | 31 38 5 40 | 32 5 21 7 | 33 30 19 4 | | | | | | |
| | | | | $g' \downarrow 0 \\ 1 \\ 2 \\ 3$ | | ′ → | 22 50 21 23 28 | 23 26 3 41 39 | 24 48 16 36 5 | 25 15 2 13 14 | 26 54 6 51 4 | 27 14 18 47 7 | 28 10 4 5 | 29 12 0 4 6 | 30 4 8 5 14 | 31 38 5 40 15 | 32 5 21 7 39 | 33 30 19 4 18 | | | | | | |
| | | | | $g' \downarrow 0 \\ 1 \\ 2 \\ 3 \\ 4$ | | ′ → | 22 50 21 23 28 33 | 23 26 3 41 39 2 | 24 48 16 36 5 38 | 25 15 2 13 14 4 | 26 54 6 51 4 24 | 27 14 18 47 7 6 | 28 10 4 5 5 | 29 12 0 4 6 9 | $ \begin{array}{r} 30 \\ 4 \\ 8 \\ 5 \\ 14 \\ 4 \end{array} $ | 31 38 5 40 15 2 | 32 5 21 7 39 6 | 33 30 19 4 18 5 | | | | | | |
| | | | | $g' \downarrow 0 1 2 3 4 5 c$ | | ′→ | 22 50 21 23 28 33 13 6 | 23 26 3 41 39 2 13 | 24 48 16 36 5 38 4 | 25 15 2 13 14 4 8 | 26 54 6 51 4 24 20 | 27 14 18 47 7 6 5 0 | 28 10 4 5 5 10 | 29 12 0 4 6 9 7 | $ \begin{array}{r} 30 \\ 4 \\ 8 \\ 5 \\ 14 \\ 4 \\ 8 \\ 0 \end{array} $ | 31 38 5 40 15 2 42 5 | 32 5 21 7 39 6 5 | 33 30 19 4 18 5 40 | | | | | | |
| | | | | g'] 0 1 2 3 4 5 6 7 | | ′ → | 22 50 21 23 28 33 13 6 | 23 26 3 41 39 2 13 4 4 | 24 48 16 36 5 38 4 31 5 | 25 15 2 13 14 4 8 19 9 | 26 54 6 51 4 24 20 1 52 | 27 14 18 47 7 6 5 9 52 | 28 10 4 5 5 10 0 17 | 29 12 0 4 6 9 7 8 46 | $ \begin{array}{r} 30 \\ 4 \\ 8 \\ 5 \\ 14 \\ 4 \\ 8 \\ 8 \\ 1 \end{array} $ | 31 38 5 40 15 2 42 5 48 | 32 5 21 7 39 6 5 40 48 | 33 30 19 4 18 5 40 15 20 | | | | | | |
| | | | | $g' \downarrow 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8$ | | ′ → | 22 50 21 23 28 33 13 6 18 8 | 23 26 3 41 39 2 13 4 4 31 | 24 48 16 36 5 38 4 31 5 1 | 25 15 2 13 14 4 8 19 9 10 | 26 54 6 51 4 24 20 1 52 7 | $ \begin{array}{r} 27 \\ 14 \\ 18 \\ 47 \\ 7 \\ 6 \\ 5 \\ 9 \\ 52 \\ 8 \\ \end{array} $ | 28 10 4 5 5 10 0 17 5 | 29 12 0 4 6 9 7 8 46 5 | $ \begin{array}{r} 30 \\ 4 \\ 8 \\ 5 \\ 14 \\ 4 \\ 8 \\ 8 \\ 1 \\ 9 \end{array} $ | 31 38 5 40 15 2 42 5 48 11 | 32 5 21 7 39 6 5 40 48 22 | 33 30 19 4 18 5 40 15 20 13 | | | | | | |
| | | | | $g' \downarrow 0 1 2 3 4 5 6 7 8 9$ | | ′ → | 22 50 21 23 28 33 13 6 18 8 36 | 23 26 3 41 39 2 13 4 4 4 31 14 | 24 48 16 36 5 38 4 31 5 1 18 | 25 15 2 13 14 4 8 19 9 10 32 | 26 54 6 51 4 24 20 1 52 7 20 | $ \begin{array}{r} 27 \\ 14 \\ 18 \\ 47 \\ 7 \\ 6 \\ 5 \\ 9 \\ 52 \\ 8 \\ 6 \\ 6 \end{array} $ | 28 10 4 4 5 10 0 17 5 5 | 29 12 0 4 6 9 7 8 46 5 38 | 30 4 8 5 14 4 8 1 9 7 | 31 38 5 40 15 2 42 5 48 11 35 | 32 5 21 7 39 6 5 40 48 22 13 | 33 30 19 4 18 5 40 15 20 13 6 | | | | | | |
| | | | | $g' \downarrow 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10$ | <i>n</i> | | 22 50 21 23 28 33 13 6 18 8 36 6 | 23 26 3 41 39 2 13 4 4 4 31 14 13 | 24 48 16 36 5 38 4 31 5 1 18 4 | 25 15 2 13 14 4 8 19 9 10 32 20 | 26 54 6 51 4 24 20 1 52 7 20 25 | 27 14 18 47 7 6 5 9 52 8 6 47 | 28 10 4 4 5 5 10 0 17 5 5 35 | 29 12 0 4 6 9 7 8 46 5 38 12 | 30 4 8 5 14 4 8 8 1 9 7 7 | 31 38 5 40 15 2 42 5 48 11 35 19 | 32 5 21 7 39 6 5 40 48 22 13 14 | 33 30 19 4 18 5 40 15 20 13 6 0 | | | | | | |
| | | | | $g' \downarrow 0 1 2 3 4 5 6 7 8 9 100 111 1100 11100 110000 110000 110000 110000 110000 110000 110000 110000 110000 110000 110000 110000 110000 110000 11000000$ | n | ´ → | 22 50 21 23 28 33 13 6 18 8 36 6 27 7 | 23 26 3 41 39 2 13 4 4 31 14 13 0 22 | $ \begin{array}{r} 24 \\ 48 \\ 16 \\ 36 \\ 5 \\ 38 \\ 4 \\ 31 \\ 5 \\ 1 \\ 18 \\ 4 \\ 26 \\ 5 \end{array} $ | 25 15 2 13 14 4 8 19 9 10 32 20 5 12 | 26 54 6 51 4 24 20 1 52 7 20 25 5 52 | 27 14 18 47 7 6 5 9 52 8 6 47 43 43 | 28 10 4 5 5 10 0 17 5 5 35 3 3 | 29 12 0 4 6 9 7 8 46 5 38 12 3 12 3 | 30 4 8 5 14 4 8 1 9 7 36 5 | 31 38 5 40 15 2 42 5 48 11 35 19 1 | 32 5 21 7 39 6 5 40 48 22 13 14 23 23 | 33 30 19 4 18 5 40 15 20 13 6 0 4 20 | | | | | | |
| | | | | $g' \downarrow 0$ 1 2 3 4 5 6 7 8 9 10 11 12 12 12 12 12 12 12 12 12 | <u>n</u> | ′ → | 22 50 21 23 28 33 13 6 18 8 36 6 27 7 8 | 23 26 3 41 39 2 13 4 4 31 14 13 0 23 20 | $ \begin{array}{r} 24 \\ 48 \\ 16 \\ 36 \\ 5 \\ 38 \\ 4 \\ 31 \\ 5 \\ 1 \\ 18 \\ 4 \\ 26 \\ 5 \\ 5$ | 25 15 2 13 14 4 8 19 9 10 32 20 5 13 7 | $ \begin{array}{r} 26 \\ 54 \\ 6 \\ 51 \\ 4 \\ 24 \\ 20 \\ 1 \\ 52 \\ 7 \\ 20 \\ 25 \\ 5 \\ 26 \\ 12 \\ \end{array} $ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 10\end{array}$ | 28 10 4 4 5 5 10 0 17 5 5 35 3 8 1 | 29 12 0 4 6 9 7 8 46 5 38 12 3 14 12 12 12 12 12 12 12 12 12 12 | $ \begin{array}{r} 30 \\ 4 \\ 8 \\ 5 \\ 14 \\ 4 \\ 8 \\ 1 \\ 9 \\ 7 \\ 7 \\ 36 \\ 14 \\ 0 \\ \end{array} $ | 31 38 5 40 15 2 42 5 48 11 35 19 1 36 4 | 32 5 21 7 39 6 5 40 48 22 13 14 23 0 2 | 33 30 19 4 18 5 40 15 20 13 6 0 4 38 45 | | | | | | |
| | | | | $g' \downarrow 0$ 1 2 3 4 5 6 7 8 9 10 11 12 13 14 14 | <u>n</u> | ✓ → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ \end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\end{array}$ | 25 15 2 13 14 4 8 19 9 10 32 20 5 13 7 7 7 | 26 54 6 51 4 24 20 1 52 7 20 25 5 26 12 42 | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32 \end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0 \end{array}$ | $ \begin{array}{r} 30 \\ 4 \\ 8 \\ 5 \\ 14 \\ 4 \\ 8 \\ 8 \\ 1 \\ 9 \\ 7 \\ 7 \\ 366 \\ 14 \\ 0 \\ 4 \end{array} $ | 31 38 5 40 15 2 42 5 48 11 35 19 1 36 4 17 | 32 5 21 7 39 6 5 40 48 22 13 14 23 0 2 30 | 33 30 19 4 18 5 40 15 20 13 6 0 4 38 45 2 | | | | | | |
| | | | | $g' \downarrow 0$ 1 2 3 4 5 5 6 6 7 8 9 10 11 12 13 14 15 14 15 15 16 16 16 16 16 16 16 16 16 16 | | ' → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ \end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ \end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\end{array}$ | $ \begin{array}{r} 30 \\ 4 \\ 8 \\ 5 \\ 14 \\ 4 \\ 8 \\ 1 \\ 9 \\ 7 \\ 7 \\ 366 \\ 14 \\ 0 \\ 4 \\ 36 \\ \end{array} $ | 31 38 5 40 15 2 42 5 48 11 35 19 1 36 4 17 9 | 32 5 21 7 39 6 5 40 48 22 13 14 23 0 2 30 | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34 \end{array}$ | | | | | | |
| | | | | $\begin{array}{c}g' \\ 0\\ 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 7\\ 8\\ 9\\ 9\\ 100\\ 111\\ 122\\ 13\\ 14\\ 155\\ 16\end{array}$ | n | <i>′</i> → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ \end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\\ 39\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44 \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ \end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 366\\ 14\\ 0\\ 4\\ 366\\ 2\end{array}$ | $\begin{array}{c} 31\\ 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ \end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ \end{array}$ | | | | | | |
| | | | | $\begin{array}{c} g' \downarrow \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 111 \\ 122 \\ 133 \\ 144 \\ 155 \\ 166 \\ 177 \\ \end{array}$ | <u>n</u> | <i>′</i> → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ \end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ \end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ \end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ 12\\ 13\\ 0\\ 12\\ 13\\ 0\\ 12\\ 13\\ 0\\ 12\\ 13\\ 0\\ 12\\ 13\\ 0\\ 12\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13\\ 13$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \\ 28 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ \end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ \end{array}$ | $\begin{array}{c} 31 \\ \hline 38 \\ 5 \\ 40 \\ 15 \\ 2 \\ 42 \\ 5 \\ 48 \\ 11 \\ 35 \\ 19 \\ 1 \\ 36 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ \end{array}$ | | | | | | |
| | | | | $\begin{array}{c} g' \downarrow \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 6 \\ 7 \\ 8 \\ 9 \\ 9 \\ 9 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 8 \\ 12 \\ 10 \\ 17 \\ 18 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10$ | <u>n</u> | <i>′</i> → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 8\\ 28\\ 20\\ 12\\ 8\\ 20\\ 12\\ 8\\ 20\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 21\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 39\\ 30\\ 5\\ 38\\ 7\\ 41\\ 2\\ 39\\ 30\\ 5\\ 38\\ 7\\ 41\\ 2\\ 39\\ 30\\ 30\\ 30\\ 30\\ 30\\ 30\\ 30\\ 30\\ 30\\ 30$ | $\begin{array}{c} 24 \\ 48 \\ 16 \\ 36 \\ 5 \\ 38 \\ 4 \\ 31 \\ 5 \\ 1 \\ 18 \\ 4 \\ 26 \\ 5 \\ 16 \\ 4 \\ 39 \\ 19 \\ 9 \\ 20 \\ \end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 200\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 42\\ 50\\ 2\\ 16\\ 4\\ 10\\ \end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ 12\\ 20\\ c\\ \end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \\ 28 \\ 6 \\ c \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 0\\ 28\\ 48\\ 13\\ 3\\ 14\\ 13\\ 0\\ 28\\ 48\\ 13\\ 3\\ 14\\ 13\\ 0\\ 28\\ 13\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 366\\ 14\\ 0\\ 4\\ 366\\ 2\\ 6\\ 26\\ 1\end{array}$ | $\begin{array}{c} 31 \\ \hline 38 \\ 5 \\ 40 \\ 15 \\ 2 \\ 42 \\ 5 \\ 48 \\ 11 \\ 35 \\ 19 \\ 1 \\ 36 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ 4 \\ 11 \\ \end{array}$ | 32 5 21 7 39 6 5 40 48 22 13 14 23 0 2 30 3 47 35 29 5 | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\end{array}$ | | | | | | |
| | | | | $\begin{array}{c} g' \downarrow \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 111 \\ 122 \\ 133 \\ 144 \\ 155 \\ 166 \\ 177 \\ 188 \\ 199 \\ 200 \end{array}$ | <u>n</u> | <i>′</i> → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ \end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\\ 9\\ 222\\ 4\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\\ 12\\ \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 19\\ 34 \end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 322\\ 13\\ 0\\ 122\\ 0\\ 6\\ 5\end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \\ 28 \\ 6 \\ 6 \\ 7 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 31\\ 4\end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 366\\ 14\\ 0\\ 4\\ 366\\ 2\\ 6\\ 26\\ 4\\ 21 \end{array}$ | $\begin{array}{c} 31 \\ \hline 38 \\ 5 \\ 40 \\ 15 \\ 2 \\ 42 \\ 5 \\ 48 \\ 11 \\ 35 \\ 19 \\ 1 \\ 36 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ 4 \\ 17 \\ 5 \\ 5 \\ 19 \\ 1 \\ 36 \\ 4 \\ 17 \\ 5 \\ 5 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\$ | 32 5 21 7 39 6 5 40 48 22 13 14 23 0 2 30 3 47 329 5 5 40 3 47 329 5 5 40 3 48 22 30 3 47 35 30 3 47 35 30 3 47 35 5 5 40 3 47 35 5 5 40 3 47 35 5 5 5 40 3 47 35 5 5 5 40 3 47 35 5 5 5 5 40 3 47 35 5 5 5 5 5 5 5 | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\end{array}$ | | | | | | |
| | | | | $\begin{array}{c}g'\\0\\1\\2\\3\\4\\5\\6\\6\\7\\8\\9\\9\\100\\111\\122\\13\\14\\15\\166\\17\\18\\19\\9\\200\\21\end{array}$ | <i>n</i> | <i>′</i> → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ \end{array}$ | $\begin{array}{c} 23\\ \hline 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ \end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 5\\ 38\\ 4\\ 311\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\\ 39\\ 9\\ 9\\ 22\\ 4\\ 28\\ \end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\\ 12\\ 6\end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26 \end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ 12\\ 20\\ 6\\ 5\\ 26\end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \\ 28 \\ 6 \\ 6 \\ 7 \\ 4 \end{array}$ | $\begin{array}{c} 29\\ \hline 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 311\\ 4\\ 4\end{array}$ | $\begin{array}{c} 30\\ \\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 21\\ 5\end{array}$ | $\begin{array}{c} 31 \\ \hline 38 \\ 5 \\ 40 \\ 15 \\ 2 \\ 42 \\ 5 \\ 48 \\ 11 \\ 35 \\ 19 \\ 1 \\ 36 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ 4 \\ 41 \\ 5 \\ 9 \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ \end{array}$ | $\begin{array}{c} 33\\ \hline 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ \end{array}$ | | | | | | |
| | | | | $\begin{array}{c}g' \\ 0\\ 1\\ 2\\ 3\\ 4\\ 5\\ 6\\ 6\\ 7\\ 8\\ 9\\ 9\\ 100\\ 111\\ 122\\ 133\\ 14\\ 155\\ 166\\ 177\\ 188\\ 199\\ 200\\ 211\\ 222\\ \end{array}$ | <i>n</i> | · → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 311\\ 5\\ 1\\ 18\\ 4\\ 266\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\\ 9\\ 222\\ 4\\ 28\\ 6\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 322\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\\ 12\\ 6\\ 9\end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 419\\ 34\\ 26\\ 42 \end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 322\\ 13\\ 0\\ 12\\ 200\\ 6\\ 5\\ 26\\ 4\end{array}$ | $\begin{array}{c} 28\\ 10\\ 4\\ 5\\ 5\\ 10\\ 0\\ 17\\ 5\\ 5\\ 35\\ 3\\ 8\\ 1\\ 2\\ 21\\ 12\\ 28\\ 6\\ 6\\ 7\\ 4\\ 15 \end{array}$ | $\begin{array}{c} 29\\ \hline 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 11\\ 4\\ 4\\ 5\end{array}$ | $\begin{array}{c} 30\\ \\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 21\\ 5\\ 5\end{array}$ | $\begin{array}{c} 31\\ \hline \\ 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\\ 40\\ 4\\ 41\\ 5\\ 9\\ 51\\ \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 300\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\end{array}$ | $\begin{array}{c} 33\\ \hline 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ \end{array}$ | | | | | | |
| | | | | $\begin{array}{c} g' \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 6 \\ 7 \\ 8 \\ 9 \\ 9 \\ 10 \\ 111 \\ 122 \\ 133 \\ 144 \\ 155 \\ 166 \\ 177 \\ 188 \\ 199 \\ 200 \\ 211 \\ 222 \\ 233 \\ 234 \\ 2$ | | · → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 266\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\\ 9\\ 222\\ 4\\ 28\\ 6\\ 6\\ 6\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 0\\ 0\\ 12\\ 6\\ 9\\ 29 \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26\\ 42\\ 3\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ 12\\ 20\\ 6\\ 5\\ 26\\ 4\\ 8\end{array}$ | $\begin{array}{c} 28\\ 10\\ 4\\ 5\\ 5\\ 10\\ 0\\ 17\\ 5\\ 5\\ 35\\ 3\\ 8\\ 1\\ 2\\ 21\\ 12\\ 28\\ 6\\ 6\\ 7\\ 4\\ 15\\ 2\end{array}$ | $\begin{array}{c} 29\\ \hline 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 31\\ 4\\ 4\\ 5\\ 9\end{array}$ | $\begin{array}{c} 30\\ \hline \\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 366\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 21\\ 5\\ 5\\ 3\end{array}$ | $\begin{array}{c} 31 \\ \hline 38 \\ 5 \\ 40 \\ 15 \\ 2 \\ 42 \\ 5 \\ 48 \\ 11 \\ 35 \\ 19 \\ 1 \\ 36 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ 4 \\ 15 \\ 9 \\ 51 \\ 3 \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 36\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\end{array}$ | | | | | | |
| | | | | $\begin{array}{c} g' \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 6 \\ 7 \\ 8 \\ 9 \\ 9 \\ 10 \\ 111 \\ 12 \\ 133 \\ 144 \\ 155 \\ 166 \\ 177 \\ 188 \\ 199 \\ 200 \\ 211 \\ 222 \\ 233 \\ 244 \\ 245 \\ 24$ | | → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 6\\ \end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 3\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 266\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\\ 9\\ 22\\ 4\\ 28\\ 6\\ 6\\ 7\\ .\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\\ 12\\ 6\\ 9\\ 29\\ 4\\ \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26\\ 42\\ 3\\ 36\\ 5\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 20\\ 6\\ 5\\ 26\\ 4\\ 8\\ 5\\ \end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \\ 28 \\ 6 \\ 6 \\ 7 \\ 4 \\ 15 \\ 2 \\ 9 \\ 9 \\ \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 311\\ 4\\ 4\\ 5\\ 9\\ 344\\ 4\\ 5\\ 9\\ 34\\ 4\\ 5\\ 9\\ 34\\ 4\\ 4\\ 5\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\$ | $\begin{array}{c} 30\\ \hline \\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 366\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 21\\ 5\\ 5\\ 3\\ 3\\ 3\end{array}$ | $\begin{array}{c} 31 \\ \hline 38 \\ 5 \\ 40 \\ 15 \\ 2 \\ 42 \\ 5 \\ 48 \\ 11 \\ 35 \\ 19 \\ 1 \\ 36 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ 4 \\ 17 \\ 9 \\ 6 \\ 40 \\ 4 \\ 15 \\ 9 \\ 51 \\ 3 \\ 32 \\ 2 \\ 32 \\ 2 \\ 32 \\ 2 \\ 32 \\ 2 \\ $ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 36\\ 6\\ 6\\ 18\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 3\\ 50\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\$ | | | | | | |
| | | | | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | | → | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 9\\ 12\\ 8\\ 19\\ 7\\ 28\\ 6\\ 9\\ 12\\ 28\\ 6\\ 9\\ 12\\ 28\\ 6\\ 9\\ 12\\ 28\\ 6\\ 9\\ 12\\ 28\\ 6\\ 12\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 29\\ 2\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 266\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\\ 9\\ 22\\ 4\\ 28\\ 6\\ 6\\ 7\\ 4\\ 26\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\\ 12\\ 6\\ 9\\ 29\\ 4\\ 8\\ 5\end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 24\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26\\ 42\\ 3\\ 36\\ 5\\ 6\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 20\\ 6\\ 5\\ 26\\ 4\\ 8\\ 5\\ 9\\ 4\end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \\ 28 \\ 6 \\ 6 \\ 7 \\ 4 \\ 15 \\ 2 \\ 9 \\ 32 \\ 7 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 31\\ 4\\ 4\\ 5\\ 9\\ 34\\ 12\\ 22\\ \end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 21\\ 5\\ 5\\ 3\\ 3\\ 34\\ 5\end{array}$ | $\begin{array}{c} 31\\ \hline 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\\ 40\\ 4\\ 41\\ 5\\ 9\\ 51\\ 3\\ 32\\ 13\\ 32\\ 13\\ 48 \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 36\\ 6\\ 13\\ 28\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\\ 12\\ 5\\ 5\\ 5\\ 12\\ 5\\ 5\\ 5\\ 5\\ 12\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\$ | | | | | | |
| | | | | $\begin{array}{c} g' \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 7 \\ 8 \\ 9 \\ 9 \\ 100 \\ 111 \\ 122 \\ 133 \\ 144 \\ 155 \\ 166 \\ 177 \\ 188 \\ 199 \\ 200 \\ 211 \\ 223 \\ 244 \\ 255 \\ 266 \\ 277 \\ \end{array}$ | | | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 9\\ 42\\ 4\end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 29\\ 3\\ 8\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 266\\ 5\\ 5\\ 16\\ 4\\ 39\\ 9\\ 222\\ 4\\ 28\\ 6\\ 6\\ 7\\ 4\\ 36\\ 5\\ 5\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 9\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 14\\ 40\\ 0\\ 12\\ 6\\ 9\\ 29\\ 4\\ 8\\ 5\\ 9\end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 511\\ 4\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26\\ 42\\ 3\\ 36\\ 5\\ 6\\ 4\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ 12\\ 20\\ 6\\ 5\\ 26\\ 4\\ 8\\ 5\\ 9\\ 4\\ 4\end{array}$ | $\begin{array}{c} 10\\ 4\\ 4\\ 5\\ 5\\ 10\\ 0\\ 17\\ 5\\ 5\\ 35\\ 3\\ 8\\ 1\\ 2\\ 21\\ 12\\ 28\\ 6\\ 6\\ 7\\ 4\\ 15\\ 2\\ 9\\ 32\\ 7\\ 31 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 311\\ 4\\ 4\\ 5\\ 9\\ 34\\ 12\\ 33\\ 13\\ 13\end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 15\\ 5\\ 3\\ 3\\ 34\\ 5\\ 25\end{array}$ | $\begin{array}{c} 31\\ \hline 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\\ 40\\ 4\\ 41\\ 5\\ 9\\ 51\\ 3\\ 32\\ 13\\ 48\\ 43\\ \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 36\\ 6\\ 13\\ 88\\ 41 \end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\\ 12\\ 56\\ 58\\ 8\end{array}$ | | | | | | |
| | | | | $\begin{array}{ c c c c c }\hline g' \downarrow & & \\ \hline 0 & & \\ 0 & & \\ 1 & & \\ 2 & & \\ 3 & & \\ 4 & & \\ 5 & & \\ 6 & & \\ 7 & & \\ 8 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 11 & & \\ 12 & & \\ 13 & & \\ 14 & & \\ 15 & & \\ 16 & & \\ 17 & & \\ 18 & & \\ 19 & & \\ 20 & & \\ 21 & & \\ 13 & & \\ 14 & & \\ 15 & & \\ 16 & & \\ 17 & & \\ 18 & & \\ 10 & & \\ 11 & & \\ 11 & & \\ 12 & & \\ 13 & & \\ 11 & & \\ 11 & & \\ 12 & & \\ 13 & & \\ 11 & & \\ 11 & & \\ 12 & & \\ 13 & & \\ 11 & & \\ 11 & & \\ 12 & & \\ 13 & & \\ 14 & & \\ 15 & & \\ 16 & & \\ 17 & & \\ 12 & & \\ 22 & & \\$ | | | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 9\\ 42\\ 4\\ 15\\ \end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 29\\ 3\\ 8\\ 2\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 426\\ 5\\ 5\\ 16\\ 4\\ 39\\ 9\\ 222\\ 4\\ 8\\ 6\\ 6\\ 7\\ 4\\ 36\\ 5\\ 9\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 14\\ 40\\ 0\\ 12\\ 6\\ 9\\ 29\\ 4\\ 8\\ 5\\ 9\\ 32\end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 20\\ 1\\ 52\\ 7\\ 20\\ 5\\ 5\\ 26\\ 12\\ 42\\ 50\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26\\ 6\\ 42\\ 3\\ 36\\ 5\\ 6\\ 4\\ 7\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 322\\ 13\\ 0\\ 12\\ 20\\ 6\\ 5\\ 26\\ 4\\ 4\\ 8\\ 5\\ 9\\ 4\\ 4\\ 31\\ \end{array}$ | $\begin{array}{c} 28 \\ 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 5 \\ 3 \\ 8 \\ 1 \\ 2 \\ 21 \\ 12 \\ 28 \\ 6 \\ 6 \\ 7 \\ 4 \\ 15 \\ 2 \\ 9 \\ 32 \\ 7 \\ 31 \\ 1 \end{array}$ | $\begin{array}{c} 29\\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 31\\ 4\\ 4\\ 5\\ 9\\ 34\\ 12\\ 33\\ 13\\ 26\end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 211\\ 5\\ 5\\ 3\\ 34\\ 5\\ 25\\ 9\end{array}$ | $\begin{array}{c} 31\\ \hline 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\\ 40\\ 4\\ 41\\ 5\\ 9\\ 51\\ 3\\ 32\\ 13\\ 48\\ 43\\ 37\\ \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 7\\ 36\\ 6\\ 13\\ 38\\ 41\\ 42\\ \end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\\ 12\\ 56\\ 38\\ 14 \end{array}$ | | | | | | |
| | | | | $\begin{array}{ c c c c c }\hline g' \downarrow & & \\ \hline 0 & & \\ 0 & & \\ 1 & & \\ 2 & & \\ 3 & & \\ 4 & & \\ 5 & & \\ 6 & & \\ 7 & & \\ 8 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 9 & & \\ 1 & & \\ 12 & & \\ 133 & & \\ 144 & & \\ 155 & & \\ 166 & & \\ 177 & & \\ 188 & & \\ 191 & & \\ 200 & & \\ 211 & & \\ 122 & & \\ 223 & & \\ 244 & & \\ 255 & & \\ 266 & & \\ 277 & & \\ 288 & & \\ 299 & & \\ 299 & & \\ 100 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 122 & & \\ 223 & & \\ 224 & & \\ 255 & & \\ 266 & & \\ 277 & & \\ 288 & & \\ 299 & & \\ 100 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 122 & & \\ 211 & & \\ 122 & & \\ 222 & & \\ 223 & & \\ 224 & & \\ 255 & & \\ 266 & & \\ 277 & & \\ 288 & & \\ 299 & & \\ 299 & & \\ 290 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 122 & & \\ 211 & & \\ 122 & & \\ 223 & & \\ 224 & & \\ 255 & & \\ 266 & & \\ 277 & & \\ 288 & & \\ 299 & & \\ 299 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ 111 & & \\ 122 & & \\ $ | | | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 9\\ 42\\ 4\\ 15\\ 5\end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 4\\ 11\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 29\\ 3\\ 8\\ 2\\ 9\end{array}$ | $\begin{array}{c} 24\\ \\ 48\\ 16\\ 36\\ 5\\ 8\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\\ 9\\ 22\\ 4\\ 28\\ 6\\ 6\\ 7\\ 4\\ 36\\ 5\\ 9\\ 34 \end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 0\\ 12\\ 6\\ 9\\ 29\\ 4\\ 8\\ 5\\ 9\\ 229\\ 4\\ 8\\ 5\\ 9\\ 32\\ 12 \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 511\\ 4\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26\\ 422\\ 3\\ 36\\ 5\\ 6\\ 4\\ 7\\ 33\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ 12\\ 200\\ 6\\ 5\\ 266\\ 4\\ 8\\ 5\\ 9\\ 4\\ 4\\ 11\\ 31\\ 13\\ \end{array}$ | $\begin{array}{c} 28\\ 10\\ 4\\ 5\\ 5\\ 10\\ 0\\ 17\\ 5\\ 5\\ 35\\ 3\\ 8\\ 1\\ 2\\ 21\\ 12\\ 28\\ 6\\ 6\\ 7\\ 4\\ 15\\ 2\\ 9\\ 32\\ 7\\ 31\\ 1\\ 26\end{array}$ | $\begin{array}{c} 29\\ \hline \\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 31\\ 4\\ 4\\ 5\\ 9\\ 34\\ 12\\ 33\\ 13\\ 26\\ 1\end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 21\\ 5\\ 5\\ 3\\ 34\\ 5\\ 25\\ 9\\ 15\\ \end{array}$ | $\begin{array}{c} 31\\ \hline 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\\ 40\\ 4\\ 41\\ 5\\ 9\\ 51\\ 3\\ 32\\ 13\\ 48\\ 43\\ 37\\ 2 \end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 36\\ 6\\ 13\\ 38\\ 41\\ 2\\ 2\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\\ 12\\ 56\\ 8\\ 8\\ 14\\ 10\\ \end{array}$ | | | | | | |
| | | | | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | | | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 28\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 9\\ 42\\ 4\\ 15\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 4\\ 11\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 29\\ 3\\ 8\\ 2\\ 9\\ 3\\ 8\\ 2\\ 9\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 5\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 8\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 8\\ 3\\ 8\\ 3\\ 8\\ 3\\ 8\\ 3\\ 8\\ 3\\ 8\\ 8\\ 3\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\ 8\\$ | $\begin{array}{c} 24 \\ \\ 48 \\ 16 \\ 36 \\ 5 \\ 8 \\ 4 \\ 311 \\ 5 \\ 1 \\ 18 \\ 4 \\ 26 \\ 5 \\ 5 \\ 16 \\ 4 \\ 39 \\ 19 \\ 9 \\ 22 \\ 4 \\ 28 \\ 6 \\ 6 \\ 7 \\ 4 \\ 36 \\ 5 \\ 9 \\ 34 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ $ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\\ 12\\ 6\\ 9\\ 29\\ 4\\ 8\\ 5\\ 9\\ 229\\ 4\\ 8\\ 5\\ 9\\ 32\\ 12\\ 34\\ 4\\ 5\\ 12\\ 34\\ 4\\ 5\\ 9\\ 32\\ 12\\ 34\\ 4\\ 5\\ 12\\ 34\\ 4\\ 5\\ 12\\ 34\\ 4\\ 12\\ 12\\ 34\\ 4\\ 12\\ 12\\ 34\\ 12\\ 12\\ 34\\ 12\\ 12\\ 34\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12$ | $\begin{array}{c} 26\\ 54\\ 6\\ 511\\ 4\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 2\\ 16\\ 4\\ 19\\ 34\\ 26\\ 422\\ 3\\ 36\\ 5\\ 6\\ 4\\ 7\\ 333\\ 5\\ 5\end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 13\\ 0\\ 12\\ 20\\ 6\\ 5\\ 266\\ 4\\ 8\\ 5\\ 9\\ 4\\ 4\\ 11\\ 32\\ 5\\ 5\\ 26\\ 4\\ 8\\ 5\\ 9\\ 4\\ 4\\ 113\\ 25\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5$ | $\begin{array}{c} 28 \\ \hline 10 \\ 4 \\ 5 \\ 5 \\ 10 \\ 0 \\ 17 \\ 5 \\ 5 \\ 35 \\ 3 \\ 8 \\ 1 \\ 2 \\ 28 \\ 6 \\ 6 \\ 7 \\ 4 \\ 15 \\ 2 \\ 9 \\ 32 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 31 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 1 \\ 1 \\ 26 \\ 9 \\ 5 \\ 7 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 1 \\ 26 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ $ | $\begin{array}{c} 29\\ \hline \\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 31\\ 4\\ 4\\ 5\\ 9\\ 34\\ 12\\ 33\\ 13\\ 26\\ 1\\ 15\\ \end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 26\\ 4\\ 4\\ 21\\ 5\\ 5\\ 3\\ 34\\ 5\\ 25\\ 9\\ 9\\ 15\\ 2\\ 6\end{array}$ | $\begin{array}{c} 31\\ \hline 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\\ 40\\ 4\\ 41\\ 5\\ 9\\ 51\\ 3\\ 32\\ 13\\ 48\\ 43\\ 37\\ 2\\ 6\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10\\ 10$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 36\\ 6\\ 13\\ 38\\ 41\\ 42\\ 2\\ 13\\ 8\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\\ 12\\ 56\\ 38\\ 14\\ 10\\ 28\\ 5\end{array}$ | | | | | | |
| | | | | $\begin{array}{c} g' \downarrow \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \\ 3 \\ 4 \\ 5 \\ 6 \\ 6 \\ 7 \\ 7 \\ 8 \\ 9 \\ 9 \\ 10 \\ 11 \\ 12 \\ 2 \\ 3 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \\ 6 \\ 27 \\ 7 \\ 28 \\ 29 \\ 30 \\ 31 \\ 1 \\ 22 \\ 26 \\ 29 \\ 30 \\ 31 \\ 21 \\ 22 \\ 26 \\ 27 \\ 28 \\ 29 \\ 30 \\ 31 \\ 21 \\ 22 \\ 26 \\ 27 \\ 28 \\ 29 \\ 30 \\ 31 \\ 21 \\ 22 \\ 26 \\ 29 \\ 30 \\ 31 \\ 21 \\ 22 \\ 26 \\ 27 \\ 28 \\ 29 \\ 30 \\ 31 \\ 21 \\ 21 \\ 21 \\ 21 \\ 21 \\ 21 \\ 21$ | | · → | $\begin{array}{c} 22\\ \hline 50\\ 21\\ 23\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 7\\ 7\\ 8\\ 50\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 9\\ 42\\ 4\\ 15\\ 5\\ 5\\ 5\\ 1\end{array}$ | $\begin{array}{c} 23\\ \hline 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 38\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 29\\ 3\\ 8\\ 2\\ 9\\ 3\\ 3\\ 3\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 2\\ 9\\ 3\\ 3\\ 2\\ 2\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 3\\ 2\\ 2\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\ 3\\$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 31\\ 5\\ 1\\ 18\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\\ 39\\ 9\\ 22\\ 4\\ 28\\ 6\\ 6\\ 6\\ 7\\ 4\\ 36\\ 5\\ 9\\ 34\\ 3\\ 32\\ c\end{array}$ | $\begin{array}{c} 25\\ \hline 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 19\\ 9\\ 9\\ 10\\ 32\\ 200\\ 5\\ 13\\ 7\\ 7\\ 15\\ 14\\ 10\\ 300\\ 0\\ 12\\ 6\\ 9\\ 9\\ 29\\ 4\\ 8\\ 5\\ 9\\ 32\\ 12\\ 34\\ 13\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 6\\ 12\\ 42\\ 2\\ 50\\ 2\\ 16\\ 4\\ 4\\ 19\\ 34\\ 426\\ 42\\ 3\\ 36\\ 5\\ 6\\ 4\\ 7\\ 33\\ 5\\ 88\\ 28\\ \end{array}$ | $\begin{array}{c} 27\\ 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 322\\ 13\\ 0\\ 12\\ 200\\ 6\\ 5\\ 26\\ 4\\ 8\\ 5\\ 9\\ 4\\ 4\\ 31\\ 13\\ 225\\ 43\\ 41\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 43\\ 113\\ 255\\ 113\\ 113\\ 113\\ 113\\ 113\\ 113\\ 113\\ 1$ | $\begin{array}{c} 28\\ \hline 10\\ 4\\ 4\\ 5\\ 5\\ 10\\ 0\\ 17\\ 5\\ 5\\ 35\\ 3\\ 8\\ 1\\ 2\\ 21\\ 12\\ 28\\ 6\\ 6\\ 6\\ 7\\ 4\\ 15\\ 2\\ 9\\ 32\\ 7\\ 31\\ 1\\ 26\\ 9\\ 37\\ 126\\ 9\\ 37\\ 126\\ 9\\ 37\\ 126\\ 9\\ 37\\ 126\\ 9\\ 37\\ 126\\ 9\\ 37\\ 126\\ 9\\ 37\\ 126\\ 126\\ 126\\ 126\\ 126\\ 126\\ 126\\ 126$ | $\begin{array}{c} 29\\ \hline \\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 31\\ 4\\ 4\\ 5\\ 9\\ 34\\ 12\\ 33\\ 13\\ 26\\ 1\\ 15\\ 2\\ 2\\ 2\end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 6\\ 6\\ 26\\ 4\\ 4\\ 21\\ 5\\ 5\\ 3\\ 3\\ 4\\ 5\\ 25\\ 9\\ 15\\ 2\\ 6\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12\\ 12$ | $\begin{array}{c} 31\\ \hline 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 36\\ 4\\ 17\\ 9\\ 6\\ 40\\ 4\\ 41\\ 5\\ 9\\ 51\\ 3\\ 32\\ 13\\ 48\\ 43\\ 37\\ 2\\ 6\\ 10\\ 4\end{array}$ | $\begin{array}{c} 32\\ 5\\ 21\\ 7\\ 39\\ 6\\ 5\\ 40\\ 48\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 30\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 36\\ 6\\ 13\\ 38\\ 41\\ 42\\ 2\\ 13\\ 4\\ 4\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 5\\ 40\\ 15\\ 20\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\\ 5\\ 12\\ 56\\ 38\\ 14\\ 10\\ 28\\ 4\\ 4\\ 27\\ \end{array}$ | | | | | | |
| | | | | $ \begin{array}{ c c c c c c c c c c c c c c c c c c c$ | | | $\begin{array}{c} 22\\ 50\\ 21\\ 23\\ 33\\ 13\\ 6\\ 18\\ 8\\ 36\\ 6\\ 27\\ 7\\ 8\\ 5\\ 40\\ 12\\ 8\\ 3\\ 1\\ 19\\ 7\\ 28\\ 6\\ 9\\ 42\\ 4\\ 15\\ 5\\ 511\\ 7\\ 50\end{array}$ | $\begin{array}{c} 23\\ 26\\ 3\\ 41\\ 39\\ 2\\ 13\\ 4\\ 4\\ 31\\ 14\\ 13\\ 0\\ 23\\ 39\\ 5\\ 5\\ 8\\ 7\\ 41\\ 2\\ 13\\ 0\\ 12\\ 28\\ 5\\ 6\\ 29\\ 3\\ 8\\ 2\\ 9\\ 3\\ 36\\ 5\\ 5\end{array}$ | $\begin{array}{c} 24\\ 48\\ 16\\ 36\\ 5\\ 38\\ 4\\ 1\\ 5\\ 1\\ 18\\ 4\\ 4\\ 26\\ 5\\ 5\\ 16\\ 4\\ 39\\ 19\\ 9\\ 222\\ 4\\ 8\\ 6\\ 6\\ 7\\ 4\\ 36\\ 5\\ 9\\ 34\\ 3\\ 322\\ 6\\ 5\end{array}$ | $\begin{array}{c} 25\\ 15\\ 2\\ 13\\ 14\\ 4\\ 8\\ 9\\ 9\\ 10\\ 32\\ 20\\ 5\\ 13\\ 7\\ 7\\ 15\\ 44\\ 10\\ 30\\ 0\\ 12\\ 6\\ 9\\ 29\\ 4\\ 8\\ 5\\ 9\\ 32\\ 229\\ 4\\ 8\\ 5\\ 9\\ 32\\ 12\\ 34\\ 13\\ 13\\ 13\\ 12\\ \end{array}$ | $\begin{array}{c} 26\\ 54\\ 6\\ 51\\ 4\\ 20\\ 1\\ 52\\ 7\\ 20\\ 25\\ 5\\ 26\\ 12\\ 2\\ 16\\ 4\\ 2\\ 19\\ 34\\ 26\\ 5\\ 6\\ 4\\ 7\\ 33\\ 36\\ 5\\ 6\\ 4\\ 7\\ 33\\ 5\\ 48\\ 386\\ 56\\ \end{array}$ | $\begin{array}{c} 27\\ \hline 14\\ 18\\ 47\\ 7\\ 6\\ 5\\ 9\\ 52\\ 8\\ 6\\ 47\\ 43\\ 4\\ 19\\ 32\\ 20\\ 6\\ 5\\ 26\\ 4\\ 8\\ 5\\ 9\\ 4\\ 4\\ 11\\ 13\\ 225\\ 43\\ 41\\ 13\\ 225\\ 43\\ 41\\ 38\\ \end{array}$ | $\begin{array}{c} 28\\ \hline \\ 10\\ 4\\ 4\\ 5\\ 5\\ 10\\ 0\\ 17\\ 5\\ 5\\ 35\\ 3\\ 8\\ 1\\ 2\\ 21\\ 12\\ 28\\ 6\\ 6\\ 6\\ 7\\ 4\\ 15\\ 2\\ 9\\ 32\\ 7\\ 31\\ 1\\ 26\\ 9\\ 37\\ 42\\ 14 \end{array}$ | $\begin{array}{c} 29\\ \hline \\ 12\\ 0\\ 4\\ 6\\ 9\\ 7\\ 8\\ 46\\ 5\\ 5\\ 38\\ 12\\ 3\\ 14\\ 13\\ 0\\ 0\\ 28\\ 48\\ 13\\ 31\\ 4\\ 4\\ 5\\ 9\\ 34\\ 12\\ 2\\ 33\\ 13\\ 26\\ 1\\ 15\\ 2\\ 2\\ 10 \end{array}$ | $\begin{array}{c} 30\\ 4\\ 8\\ 5\\ 14\\ 4\\ 8\\ 8\\ 1\\ 9\\ 7\\ 7\\ 36\\ 14\\ 0\\ 4\\ 36\\ 2\\ 6\\ 626\\ 4\\ 21\\ 5\\ 5\\ 3\\ 34\\ 5\\ 25\\ 9\\ 15\\ 2\\ 6\\ 13\\ 8\end{array}$ | $\begin{array}{c} 31\\ \hline 38\\ 5\\ 40\\ 15\\ 2\\ 42\\ 5\\ 48\\ 11\\ 35\\ 19\\ 1\\ 35\\ 19\\ 1\\ 35\\ 19\\ 1\\ 36\\ 4\\ 41\\ 5\\ 9\\ 51\\ 3\\ 32\\ 13\\ 48\\ 43\\ 37\\ 2\\ 6\\ 100\\ 4\\ 4\end{array}$ | $\begin{array}{c} 32\\ 5\\ 211\\ 7\\ 39\\ 6\\ 5\\ 400\\ 22\\ 13\\ 14\\ 23\\ 0\\ 2\\ 300\\ 3\\ 47\\ 35\\ 29\\ 5\\ 14\\ 20\\ 7\\ 366\\ 6\\ 13\\ 38\\ 411\\ 42\\ 2\\ 13\\ 4\\ 8\\ 5\end{array}$ | $\begin{array}{c} 33\\ 30\\ 19\\ 4\\ 18\\ 5\\ 40\\ 13\\ 6\\ 0\\ 4\\ 38\\ 45\\ 2\\ 2\\ 34\\ 46\\ 37\\ 5\\ 5\\ 43\\ 3\\ 50\\ 5\\ 5\\ 12\\ 56\\ 38\\ 14\\ 10\\ 28\\ 4\\ 35\\ 5\\ 5\\ 5\\ 12\\ 28\\ 38\\ 14\\ 10\\ 28\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\ 5\\$ | | | | | | |

Table 3: Grundy values for generalized star graphs $S_{f,g,n}$, with f = 2, g > quasipre-period, n > pre-period length shown in Table 2, where $g \equiv g' \pmod{34}$ and $n \equiv n' \pmod{34}$.

6. Future Directions

Some games have clear bounds on the possible \mathcal{G} -values for the nim-sequence (see [1]). Depending on the underlying graph, ARC KAYLES could potentially have \mathcal{G} -values growing without bound as the number of vertices increases. However, as observed with path graphs and generalized star graphs, even though they could grow without bound, in these cases they do not. It would be interesting to understand why this is the case. Future work involves solving the conjecture presented in Section 5. It would be a great feat to develop methods of determining a priori bounds on \mathcal{G} -values for impartial games. Alternatively, a general theorem on periodic behaviour of games would help this analysis.

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