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LOCALLY NONCOMMUTATIVE SPACETIMES

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Abstract. A rather new physically well-motivated notion of *locally* noncommutative spacetimes – a refinement of the notion noncommutative spacetimes – is reviewed and discussed. It is shown that the latter model can be realized in the framework of formal deformation quantization using star products as well as in the framework of Rieffel's strict deformation quantization. For the convergent setting Rieffel's former results for C^*-algebras are generalized to pro-C^*-algebras and applied to actions well-suited to the idea of locally noncommutative spacetimes.

1. Introduction and Motivation

Since the simultaneous validity of quantum theory and general relativity has to break down at the Planck scale, various models for spacetime geometry at such scales have been discussed over the years. One promising approach is to deform the classical commutative geometry of spacetime into a noncommutative geometry and study dynamics like (quantum) field theories on this noncommutative spacetime, see e.g., the pioneering work [3]. Here many versions have been discussed, though all of them have one feature in common: the noncommutativity is global and hence has global consequences. This is reflected in the famous UV/IR mixing in the Euclidian versions of field theory and in rather absurd dispersion relations in the truly Minkowski versions, see e.g., [1, 8] and references therein.

One particular example which has been studied in detail is the noncommutative Minkowski spacetime with the usual Weyl-Moyal star product quantizing a *constant* Poisson structure. Mathematically speaking, this is equivalent to the quantization of a non-relativistic particle with two-dimensional configuration space $\mathbb{R}^2$ and hence phase space $\mathbb{R}^4$. Though quantum mechanics only appears at small distances in *configuration space*, in *phase space* quantum mechanical effects are visible globally. This simple heuristic consideration already gives a hint why the much more subtle effects of UV/IR mixing etc. have to be expected. The reason is that the geometry of spacetime is globally noncommutative.