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Abstract. We present a generalized formulation of Poisson dynamics suitable to describe *the n-bodies interactions*. Examples are given of physical systems endowed with such a general structure.

1. Introduction

The Nambu dynamics is an example of n-Poisson structure which is a special n-Lie algebra. The latter was introduced for the fist time by Filippov [4] in 1985 who gave first examples, developed first structural concepts, like simplicity, in this context and classified n-Lie algebras of dimensions $n+1$ which is parallel to the Bianchi classification of three-dimensional Lie algebras.

Filippov defines an *n-Lie algebra structure* to be an n-ary multi-linear and anti-symmetric operation

$$[v_1, \ldots, v_{n-1}]$$

which satisfies the n-ary Jacobi identity:

$$[v_1, \ldots, v_{n-1}, [u_1, \ldots, u_n]] = \sum_{i=1}^{n} [u_1, \ldots, u_{i-1}, [v_1, \ldots, v_{n-1}, u_i,], u_{i+1}, \ldots, u_n] . \quad (1)$$

Such an operation, realized on the smooth function algebra of a manifold and additionally assumed to be an n-derivation, is an *n-Poisson structure*.

This general concept, however, was not introduced by Filippov. A first proposal goes back to Albeggiani [13] who introduced, in a different context, the *Poisson bracket of the n-th order* by the formula

$$\{H_1, H_2, \ldots, H_n\} = \sum_{i=1}^{\nu} \frac{\partial (H_1, H_2, \ldots, H_n)}{\partial (x_{1i}, x_{2i}, \ldots, x_{ni})}$$