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# NEW RESULTS ON THE GEOMETRY OF TRANSLATION SURFACES

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**Abstract.** In this paper we study the second mean curvature for different hypersurfaces in space forms. We furnish some examples and we remind some connections between $II$-minimality and biharmonicity. The main result consists in proving that there are no $II$-minimal translation surfaces in the Euclidean three-space.

## 1. Introduction

The study of the second fundamental form $II$ was initiated through the early papers of Weingarten [16], Darboux [5] and Cartan [3] where appeared for the first time notions like connection or curvature associated to $II$. Later on, Erard [7] introduced the second fundamental form as metric on the surface. This is possible only when $II$ is non-degenerate and hence it can be regarded as a (pseudo)-Riemannian metric on the surface. At this point one can consider a connected smooth surface $M$ endowed with $II$ as metric in order to study new characteristics associated to $(M, II)$. In the classical case when the metric on the surface is given by the first fundamental form $I$, i.e., for $(M, I)$, there are well known formulae to compute the Gaussian curvature $K$ and the mean curvature $H$ in order to analyze the properties of $M$ that arise from this *"measures"*. In a similar manner, the *second Gaussian curvature* denoted by $K_{II}$ and the *second mean curvature*, denoted by $H_{II}$, were considered. In [3], $K_{II}$ was introduced for the first time by Cartan, as the analogous of the Gaussian curvature. Concerning $H_{II}$, it was defined by Glässner in [8]. An overview over the literature dedicated to the second fundamental form and the associated curvatures for different type of submanifolds in different ambient spaces can be found in [15] and its references. Regarding the second mean curvature, the critical points of the area functional of the second fundamental form are those surfaces for which the mean curvature of the second fundamental form vanishes. A non-developable surface is said to be