



WEAK FORM OF HOLZAPFEL'S CONJECTURE

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Abstract. Let $\mathbb{B} \subset \mathbb{C}^2$ be the unit ball and Γ be a lattice of $SU(2, 1)$. Bearing in mind that all compact Riemann surfaces are discrete quotients of the unit disc $\Delta \subset \mathbb{C}$, Holzapfel conjectures that the discrete ball quotients \mathbb{B}/Γ and their compactifications are widely spread among the smooth projective surfaces. There are known ball quotients \mathbb{B}/Γ of general type, as well as rational, abelian, K3 and elliptic ones. The present note constructs three non-compact ball quotients, which are birational, respectively, to a hyperelliptic, Enriques or a ruled surface with an elliptic base. As a result, we establish that the ball quotient surfaces have representatives in any of the eight Enriques classification classes of smooth projective surfaces.

1. Introduction

In his monograph [4] Rolf-Peter Holzapfel states as a working hypothesis or a philosophy that “... up to birational equivalence and compactifications, all complex algebraic surfaces are ball quotients.” By a complex algebraic surface is meant a smooth projective surface over \mathbb{C} . These have smooth minimal models, which are classified by Enriques in eight types - rational, ruled of genus ≥ 1 , abelian, hyperelliptic, K3, Enriques, elliptic and of general type. The compact torsion free ball quotients \mathbb{B}/Γ are smooth minimal surfaces of general type. Ishida [10], Keum [11, 12] and Dzambic [1] obtain elliptic surfaces, which are minimal resolutions of the isolated cyclic quotient singularities of compact ball quotients. Hirzebruch [2] and then Holzapfel [3], [7], [9] have constructed torsion free ball quotient compactifications with abelian minimal models. In [9] Holzapfel provides a ball quotient compactification, which is birational to the Kummer surface of an abelian surface, i.e., to a smooth minimal K3 surface. Rational ball quotient surfaces are explicitly recognized and studied in [6], [8]. The present work constructs smooth ball quotients with a hyperelliptic or, respectively, a ruled model with an elliptic base. It provides also a ball quotient with one double point, which is birational to an Enriques surface. All of them are finite Galois quotients of a