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ON THE ESSENTIAL ALGEBRAIC ASPECTS OF SUBMANIFOLD QUANTUM MECHANICS

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Abstract. The submanifold quantum mechanics was opened by Jensen and Koppe and has been studied for more than three decades. This article gives its more algebraic definition and show the essential aspects of the submanifold quantum mechanics from an algebraic viewpoint.

1. Introduction

The submanifold quantum mechanics was initiated by Jensen and Koppe [18] and da Costa [8] and has been developed further by Duclos, Exner, Krejčiřík, Šeba and Šťovíček [11, 10, 13, 19], Ikegami, Nagaoka, Takagi and Tanzawa [16, 17, 33], Clark and Bracken [6, 7], Goldstone and Jaffe [14], Burgess and Jensen [5], Encinosa and Etemadi [12], Mladenov [29], Suzuki, Tsuru and the present author [20–26, 28, 30–32]. In all these theories, we obtain differential operators over a submanifold S in an Euclidean space relying on a confinement potential and taking some squeezing limit of the potential.

The so obtained differential operators however do not strongly depend upon the shape of confinement potentials or in the way the squeezing limit is taken. Further they exhibit geometrical nature of the submanifold. In fact, the Dirac operators obtained in the scheme are related to the Frenet-Serret and the generalized Weierstrass relations [21–28] from which we can recover all geometrical data of the submanifold. Thus we believe that they should be obtained beyond any approximation and defined more algebraically.

In this article, we will give a more algebraic definition of the submanifold quantum mechanics, which is free from any approximation theories, and show what is the essential of the submanifold quantum mechanics from an algebraic point of view.