JGSP **20** (2010) 107–109

JOURNAL OF
Geometry and Symmetry
in Physics

# BOOK REVIEW

*Stability and Chaos in Celestial Mechanics*, by Alessandra Celletti, Springer, Berlin • Heidelberg • New York 2009, xi+257 pp., Published in Association with Praxis, Chichester, UK, ISBN 978-3-540-85145-5.

The author of this book is a Professor at mathematics department, University of Rome "Tor Vergata", Rome, Italy.

The fact that the last decades have marked the beginning of a new era in Celestial Mechanics is a good reason for a new book in the field. The challenges came from several different directions. The stability theory of nearly-integrable systems (a class of problems which includes many models of Celestial Mechanics) profited from the breakthrough provided by the Kolmogorov-Arnold-Moser theory, which also ensures tools for determining explicitly the parameter values allowing for stability. A confinement of the actions for exponential times was guaranteed by Nekhoroshev's theorem, which gives much information about the geography of the resonances. Performing ever-faster computer simulations allowed us to have deeper insights into many questions of Dynamical Systems, most notably chaos theory. In this context several techniques have been developed to distinguish between ordered and chaotic behaviors. In this framework Chapter 1 is devoted to the interplay between conservative and dissipative dynamical systems, their linear stability, the definition of attractors and the discussion of some paradigmatic models, both conservative and dissipative. These concepts are made clear by means of paradigmatic examples, like the logistic map, the standard mapping both in its conservative and dissipative version, and Hénon's mapping.

Chapter 2 presents a qualitative analysis of dynamical systems based on numerical investigations which provide the description of the phase space. Nowadays there exists a large number of numerical tools, some of which are described in this chapter. The Poincaré mapping allows to reduce the analysis of a continuous system to that of a discrete mapping. The stable or chaotic character of the motion can be investigated through the computation of the Lyapunov exponents. Whenever an attractor exists, it is useful to evaluate its dimension. To estimate the attractor's dimension one can implement the Grassberger and Procaccia method, which can be used also for time series analysis.

Chapter 3 is devoted to the Kepler problem. The elliptic motion, which provides also the solutions of the hyperbolic and parabolic dynamics is described in details. After introducing the Delaunay action-angle variables, the two-body problem with variable mass is discussed.

The three-body problem is the content of Chapter 4. Here the author introduces the planar, circular and the restricted three-body problem in terms of the Delaunay variables. This model is described by a nearly-integrable Hamiltonian function, whose perturbing parameter represents the primaries mass ratio.

The Kolmogorov–Arnold–Moser (KAM) theorem is proved in detail for the specific case of the spin-orbit model [5] in Chapter 5. Moreover the choice of the frequency as well as a computer-assisted implementation are discussed.

Perturbation theory is introduced in Chapter 6, where classical, resonant and degenerate perturbation theories are discussed together with some examples, like the computation the precession of the perihelion or of the precession of the equinoxes.

In Chapter 7 is shown that the perturbation theory fails whenever a resonance condition is met. Small divisors might prevent the convergence of the series and therefore the application of perturbation theory. To overcome this problem, a breakthrough came with the work of Kolmogorov [1], that was proved later on in different mathematical settings by Arnold [1–3] and Moser [6] . The overall theory is known with the acronym KAM theory and it allows to prove the persistence of invariant tori. KAM theory was applied to several physical models of interest in Celestial Mechanics

In Chapter 8 the author studies the Hamiltonian systems with more than two degrees of freedom that do not admit a confinement of the phase space by invariant tori. For example, for a three-dimensional Hamiltonian system, the phase space has dimension six and the constant energy level is five-dimensional.

Chapter 9 presents some results on the existence of periodic orbits through a constructive version of the implicit function theorem, both in a conservative and in a dissipative setting. The computing of periodic orbits, like the Lindstedt–Poincaré and the Krylov-Bogolubov-Mitroposlky (KBM) techniques are reviewed. In conclusions the discussion of Lyapunov's theorem on the determination of families of periodic orbits is presented.

Finally, Chapter 10 deals with the regularization theory. Precisely, the Levi-Civita transformation is implemented to regularize collisions in the framework of the planar, restricted, circular, three-body problem.

The regularization of the spatial case is obtained through the Kustaanheimo–Steifel (KS) transformation. Both Levi-Civita and KS regularizations are local techniques, since they allow to regularize collisions with one of the primaries. A global regularization is attained through the implementation of the Birkhoff regularization procedure, which concludes the last chapter.

The book ends with a set of Appendices. They are intended to review basic Hamiltonian mechanics, Floquet theory and Lyapunov exponents, Yoshida's simplectic integrators while the last one contains some astronomical data on planets, dwarf planets and satellites.

This book is a comprehensive introduction to problems of celestial mechanics appropriate for students, PhD students and specialists in the area.

## References

[1] Arnold V., *Proof of a Theorem by A. N. Kolmogorov on the Invariance of Quasiperiodic Motions Under Small Perturbations of the Hamiltonian*, Russ. Math. Surveys **18** (1963) 13-40.

[2] Arnold V., *Small Divisor Problems in Classical and Celestial Mechanics*, Russ. Math. Surveys **18** (1963) 85-191.

[3] Arnold V., *Instability of Dynamical Systems with Several Degrees of Freedom*, Sov. Math. Doklady **5** (1964) 342-355.

[4] Arnold V., *Mathematical Methods of Classical Mechanics*, Springer, New York, 1978.

[5] Celletti A. and Chierchia L., *KAM Stability and Celestial Mechanics*, Memoirs AMS vol.**187**, Providence, R.I. 2007.

[6] Moser J., *Stable and Random Motions in Dynamical Systems*: *with Special Emphasis on Celestial Mechanics*, Hermann Weyl Lectures, Institute for Advanced Study, N. J. Ann. Math. Studies vol. **77**, Princeton University Press, Princeton, 1973.

Nikolay Asenov Kostov
Institute of Nuclear Research and
Nuclear Energy
Bulgarian Academy of Sciences
Tsarigradsko shousse 72, 1784 Sofia, BULGARIA
*E-mail*: nakostov@inrne.bas.bg