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COHOMOLOGY GAPS FOR SHEAVES ON THREEFOLDS

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Abstract. We discuss cohomological invariants of framed reflexive sheaves on $(n \geq 3)$-folds and show that some numerically admissible values of cohomology do not occur.

1. Introduction

We first mention briefly our physics motivation. The paper [3] showed gaps on the values of topological charges for instantons on local surfaces. Using the Kobayashi–Hitchin(KH) correspondence (see [6] for the general theory on KH correspondence) this result is equivalent to the existence of gaps on the value of the holomorphic Euler characteristic of framed sheaves over local surfaces ([3], Lemma 6.5 and Proposition 6.7). For $(n \geq 3)$-folds there is no analogue of the beautiful KH correspondence, simply because the equation defining anti-self-dual connections only makes sense in four dimensions. Nevertheless, sheaves on threefolds are very interesting for questions in string theory and appear in various physical theories that can be regarded as higher dimensional analogues of instanton counting, such as counting of dyons and Bogomolnyi-Pasad-Sommerfeld (BPS) states. In further generality sheaves on threefolds occur as D-branes in string theory and cohomological values of such sheaves can be interpreted as some measure of energy within the charge of a brane. The authors of this paper are mathematicians, and do not aim here to propose a definition of brane charge. We have encountered many different definitions in the literature. Nevertheless, there is a common feature that by and large such definitions involve cohomological invariants of sheaves on threefolds. Thus, we expect that gaps on cohomological invariants of such sheaves shall have interesting physics interpretations. We choose to present this paper in a physics oriented journal in the hope that our result may inspire researchers to find a physical counterpart of the mathematical phenomenon we observed. On the other hand, cohomological invariants of bundles and sheaves are of interest in geometry in their own right and have applications to fundamental questions in geometry, for instance