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QUASICLASSICAL AND QUANTUM SYSTEMS OF ANGULAR MOMENTUM. PART I. GROUP ALGEBRAS AS A FRAMEWORK FOR QUANTUM-MECHANICAL MODELS WITH SYMMETRIES

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Abstract. We use the mathematical structure of group algebras and H^+-algebras for describing certain problems concerning the quantum dynamics of systems of angular momenta, including also the spin systems. The underlying groups are $\mathrm{SU}(2)$ and its quotient $\mathrm{SO}(3,\mathbb{R})$. The proposed scheme is applied in two different contexts. Firstly, the purely group-algebraic framework is applied to the system of angular momenta of arbitrary origin, e.g., orbital and spin angular momenta of electrons and nucleons, systems of quantized angular momenta of rotating extended objects like molecules. Secondly, the other promising area of applications is Schrödinger quantum mechanics of rigid body with its often rather unexpected and very interesting features. Even within this Schrödinger framework the algebras of operators related to group algebras are a very useful tool. We investigate some problems of composed systems and the quasiclassical limit obtained as the asymptotics of "large" quantum numbers, i.e., "quickly oscillating" wave functions on groups. They are related in an interesting way to geometry of the coadjoint orbits of $\mathrm{SU}(2)$.

1. Introduction

Many physical systems have geometric background based on some groups or their byproducts like homogeneous spaces, Lie algebras and co-algebras, co-adjoint orbits, etc. Those group structures are relevant both for classical and quantum theories. They are basic tools for fundamental theoretical studies. They provide us also with the very effective tool for practical calculations. According to some views [16], such a purely group-theoretical background is characteristic for almost all physical models, or at least for realistic and viable ones. Let us mention a funny fact known to everybody from the process of learning or teaching quantum mechanics. After the primary struggle with elementary introduction to quantum theory, first of all to atomic and molecular physics, students are often convinced that the properties of quantum angular momentum, e.g., its composition rules, so