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# QUANTIZATION OPERATORS AND INVARIANTS OF GROUP REPRESENTATIONS

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**Abstract.** Let $G$ be a semi-simple Lie group and $\pi$ some representation of $G$ belonging to the discrete series. We give interpretations of the constant $\pi(g)$, for $g \in Z(G)$, in terms of geometric concepts associated with the flag manifold $M$ of $G$. In particular, when $G$ is compact this constant is related to the action integral around closed curves in $M$. As a consequence, we obtain a lower bound for de cardinal of the fundamental group of $\mathrm{Ham}(M)$, the Hamiltonian group of $M$. We also interpret geometrically the values of the infinitesimal character of $\pi$ in terms of quantization operators.

## 1. Introduction

Given a Lie group $G$, its coadjoint action on the dual of the Lie algebra of $G$ gives rise to the coadjoint orbits. On the other hand, associated with $G$ we have the set of the irreducible unitary representations of $G$. Thus, the group $G$ has attached a set of "geometric objects", the coadjoint orbits, and a set of "algebraic objects", its irreducible unitary representations.

The study of the possible relations between the set of the coadjoint orbits of $G$ and the unitary dual of $G$, which is the space of the equivalence classes of unitary irreducible representations, is the aim of the Orbit Method.

The origin of the Orbit Method is the following theorem due to Kirillov [2]

**Theorem 1 (Kirillov)**. *Let G be a nilpotent connected simply connected Lie group. Then the unitary dual of G is in bijective correspondence with the space of the coadjoint orbits of G.*

Furthermore, Kirillov gave interpretations of various facts relative to representation theory in terms of the geometry of the coadjoint orbits. For example, if $\mathcal{O}$ is the coadjoint orbit of the element $\mu$ in the dual of the Lie algebra of $G$ and $\pi$ is the