JGSP 31 (2013) 93–104

JOURNAL OF
Geometry and Symmetry
in Physics
ISSN 1312-5192

FOUR POINTS LINEARIZABLE LATTICE SCHEMES

DECIO LEVI AND CHRISTIAN SCIMITERNA

Communicated by Jean-Francois Ganghoffer

Abstract. We provide conditions for a lattice scheme defined on a four points lattice to be linearizable by a point transformation. We apply the obtained conditions to a symmetry preserving difference scheme for the Burgers potential introduced by Dorodnitsyn and show that it is not linearizable.

1. Introduction

In a recent article [4] we extended to lattice equations the theorems introduced by Bluman and Kumei [2] for proving the linearizability of nonlinear Partial Differential Equations (PDEs) (for a recent extended review see [1]) based on the analysis of the symmetry properties of linear PDEs.

Here we extend the results of [4] to the case of a lattice scheme, i.e., when the lattice is not a priory given but it is defined by an equation so as to be able to perform a symmetry preserving discretization of a PDE.

In Section 2 we prove a theorem characterizing the symmetries of linear difference schemes on four lattice points and in Section 3 we apply it to find conditions under which a nonlinear difference scheme is linearizable. These conditions are then applied to the symmetry preserving discretization of the Burgers potential.

2. Symmetries of Linear Schemes

In this Section we define a difference scheme and provide the symmetry conditions under which such a scheme is linearizable. To do so in a definite way we limit ourselves to the case when the equation and the lattice are defined on four points in the plane, i.e., we consider one scalar equation for a continuous function of two (continuous) variables: $u_{m,n} = u(x_{m,n}, t_{m,n})$ defined on four lattice points.

doi: 10.7546/jgsp-31-2013-93-104