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GROUP CLASSIFICATION OF VARIABLE COEFFICIENT $K(m,n)$ EQUATIONS

KYRIAKOS CHARALAMBOUS, OLENA VANEEVA AND CHRISTODOULOS SOPHOCLEOUS

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Abstract. Lie symmetries of $K(m,n)$ equations with time-dependent coefficients are classified. Group classification is presented up to widest possible equivalence groups, the usual equivalence group of the whole class for the general case and the conditional equivalence groups for special values of the exponents m and n. Examples on reduction of $K(m,n)$ equations (with initial and boundary conditions) to nonlinear ordinary differential equations (with initial conditions) are presented.

1. Introduction

In order to understand the role of nonlinear dispersion in the formation of patterns in liquid drops, Rosenau and Hyman [17] introduced a generalization of the KdV equation of the form

$$u_t + \varepsilon(u^m)_x + (u^n)_{xxx} = 0$$

where $\varepsilon = \pm 1$. Such equations, that are known as $K(m,n)$ equations, have the property for certain values of m and n their solitary wave solutions are of compact support. In other words, they vanish identically outside a finite core region. Further study followed in the references [13–16].

Here we consider a class of variable coefficient $K(m,n)$ equations of the form

$$u_t + \varepsilon(u^m)_x + f(t)\,(u^n)_{xxx} = 0 \tag{1}$$

where f is an arbitrary nonvanishing function of the variable t, n and m are arbitrary constants with $n \neq 0$, and $\varepsilon = \pm 1$. Note that the more general class (appeared, e.g., in [24]) of the form

$$u_t + g(t)(u^m)_x + f(t)(u^n)_{xxx} = 0, \qquad fn \neq 0 \tag{2}$$

reduces to class (1) via the transformation $\tilde{t} = \varepsilon \int g(t)\mathrm{d}t$, $\tilde{x} = x$, $\tilde{u} = u$. This transformation maps the class (2) into its subclass (1), where $\tilde{f} = \varepsilon f/g$. This is