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GEOMETRICAL ASPECTS OF PARACONTACT PAIR STRUCTURES

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Abstract. We introduce the notion of (metric) paracontact pair structure and establish certain properties of the characteristic foliations associated to it. We also consider normal paracontact pair structures and provide necessary and sufficient conditions for a paracontact pair structure to be normal. In particular, we formulate an analogue of Morimoto's theorem for product manifolds. Finally, we describe a way to obtain a metric paracontact pair structure on the total space of a principal $\mathbb{S}^1$-bundle via the Boothby-Wang construction.

1. Introduction

The contact pairs were defined by Blair, Ludden and Yano [10] under the name of *bicontact*. Further they were studied by Bande and Hadjar [3, 4], Bande, Ghiggini and Kotschick [2], [5, 6], which considered a special type of f-structure with complementary frames related to a contact pair and called the assambley *contact pair structure* [4]. With these elements, they considered two almost complex structures and in case they are integrable, the contact pair structure is called *normal*. In [5] the authors describe this case and give necessary and sufficient conditions for a contact pair structure to be normal. Remark that the normality condition for different geometric structures is used in many papers, e.g., [15]. Basically, a *contact pair* consists of a pair of one-forms of constant and complementary classes such that each of them induces a contact form on the leaves of the characteristic foliation of the other. Similar notions were considered if instead of two one-forms, one takes a one-form and a closed two-form, respectively, two closed two-forms, satisfying certain conditions. In the first case, the structure is called *contact-symplectic pair* [1] and in the second one, *symplectic pair* [8].

Inspired by these considerations, we shall introduce the notion of (metric) paracontact pair structure and provide necessary and sufficient conditions for it to be normal. We shall also obtain the relations satisfied by the covariant derivatives of the fundamental form and of the endomorphism of the structure with respect to the Levi-Civita connection of the metric considered. The last section describes a