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CHANNEL LINEAR WEINGARTEN SURFACES

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Abstract. We demonstrate that every non-tubular channel linear Weingarten surface in Euclidean space is a surface of revolution, hence parallel to a catenoid or a rotational surface of non-zero constant Gauss curvature. We provide explicit parametrizations and deduce existence of complete hyperbolic linear Weingarten surfaces.

MSC: 53A10, 53C42
Keywords: Canal surface, channel surface, constant Gauss curvature, constant mean curvature, Jacobi elliptic function, linear Weingarten surface, parallel surface, surface of revolution, tube

1. Introduction

This note is based on the second author's Bachelor thesis, the purpose of which was to understand the classification of cyclic linear Weingarten surfaces from [6]. In particular, we obtained a very simple proof for Corollary 3.6 of [6], using explicit parametrizations in terms of Jacobi elliptic functions based on [11]. This method will also be applicable to the results of [7]. Moreover, in our attempt to derive a more conceptual proof for the key step [6, Theorem 2.1] in the classification of cyclic linear Weingarten surfaces of [6, Theorem 1.1], we derived the classification of channel linear Weingarten surfaces Theorem 3 below – that, in fact, had already been obtained in [4], using different methods based on [3]. As a consequence of our Theorem 3 we obtain a partial but rather explicit classification, of channel linear Weingarten surfaces as special cyclic linear Weingarten surfaces, in Section 3 of this note.

This paper does not contain substantial new results, but merely employs various well known methods and results in order to elucidate the main classification result of [6]. We feel, however, that its publication may serve the mathematical community by recalling these methods and by demonstrating how they beautifully serve to classify channel linear Weingarten surfaces. In fact, basic scholarly work suggests