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A RIGOROUS FRAMEWORK FOR THE LANDAU AND LIFSHITZ APPROACH TO THOMSON ELECTROSTATICS

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Abstract. Landau and Lifshitz [7] proposed a novel formulation of the famous Thomson theorem, also known as the Thomson variational principle. In an attempt to explain, rather than postulate, the distribution of electrical charge exclusively on the surface of the conductor, Landau and Lifshitz allow the admissible variations in the electrical charge to penetrate the interior of the conductor. This is a valuable generalization of their predecessors' work, as well as a step towards basing more of the analysis on first principles.

Landau and Lifshitz' approach has not received the attention it deserves because it was not formulated as a rigorous technique, but rather as a slight of hand to arrive at a known result. In this paper, we construct a rigorous mathematical framework based on the Landau and Lifshitz idea. In particular, we prove that surface distribution of charges corresponds to the absolute minimum of electrostatic energy.

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1. The Thomson Principle

The Thomson principle, also known as the variational principle of electrostatics of conductors, has played in important role in a broad range of disciplines from electromagnetism to applied mathematics [1, 2, 7–9]. There are many different forms of the Thomson principle. It was formulated for conductors and, in its original form, states that the equilibrium distribution of electric charges on the surface of an electric conductor minimizes the total electrostatic energy. The principle has since been generalized in a number of ways. For instance, according to one of the alternative formulations [2], at equilibrium, the conductor's boundary is an equipotential surface.

One extension of the Thomson principle was suggested by Landau and Lifshitz [7]. Until Landau and Lifshitz, only admissible variations in the surface charge density were considered. In other words, only those variations that kept all electrical