



## MATHEMATICS IN CAGING OF ROBOTICS

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Communicated by Jan J. Slawianowski

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**Abstract.** It is a crucial problem in robotics field to cage an object using robots like multifingered hand. However the problem what is the caging for general geometrical objects and robots has not been well-described in mathematics though there were many rigorous studies on the methods how to cage an object by certain robots. In this article, we investigate the caging problem more mathematically and describe the problem in terms of recursion of the simple euclidean moves. Using this description, we show that the caging has the degree of difficulty which is closely related to a combinatorial problem and a wire puzzle. It implies that in order to capture an object by caging, from a practical viewpoint the difficulty plays an important role.

MSC: 51M04, 57N15, 57N35, 70B15, 70E60

Keywords: Caging, euclidean move, robotics, wire puzzle

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### 1. Introduction

In robotics fields, caging is a type of grasping where robots capture an object by surrounding or hooking it. Thus the caging problem based on the shape of the robots and the object is addressed via geometrical representation. Its mathematical description has been partially studied with a focus on methodology how to cage an object by certain robots. Though it is rigorous, it is not suitable for arbitrary target objects and robots. In this article, we propose an essential of caging to describe arbitrary target objects and robots from a mathematical viewpoint, and then it naturally leads us to a degree of difficulty of escaping and caging. It is a novel concept of the caging which is connected with practical approaches.

Caging or holding an object has been discussed in mathematics field in [3, 21], and has been applied to robotic manipulation in parallel. Rimon and Blake [16] raised a caging by two circular robots driven by one parameter in two dimensional planar space, and formulated its conditions. Wang and Kumar [19] proposed caging by multi-robot cooperation with mathematical abstract formulas. More than three dimensional caging problem is formulated by [14], although only circular and spherical robots are referred. There studies discuss existence of object's free movable