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ORTHOGONAL SPHERES, LIGHT CONES AND CAUSALITY IN MINKOWSKI SPACE

ROBERT J. LOW

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Abstract. We describe a curious relationship between orthogonal spheres and causal relationships in Minkowski space.

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1. Introduction

We are going to consider causal relationships in Minkowski space $\mathbb{M}^4$, equipped with the usual Minkowskian coordinates (t, x, y, z) and inner product $\langle , \rangle$ with signature $(+, -, -, -)$. In particular, we are going to consider the relationship between how two points are causally related with how their light cones intersect in a surface of constant time, and see how orthogonality of the resulting spheres is involved.

First, we recall that two spheres are orthogonal if the vectors from the centre of each to a (and hence every) point of intersection are orthogonal. This is clearly equivalent to the sum of the squares of the radii of the spheres being the square of the distance between their centres.

Now, let us consider the growth of two spheres each of whose radii grows in time, where one starts as a point and the other initially has radius R.

Then, for $p \in \mathbb{E}^3$, where $\mathbb{E}^3$ is the usual three dimensional Euclidean space, we denote by $S_p(r)$ the sphere of radius r with centre p. Let $p, q \in \mathbb{E}^3$, let the distance between p and q be d, and let $R > 0$. Then we have the fundamental geometric result

Lemma 1. *The spheres $S_p(|t|)$ and $S_q(|t + R|)$ intersect orthogonally for some t iff $d \geq R/\sqrt{2}$.*

Proof: The spheres intersect orthogonally when t satisfies

$$d^2 = t^2 + (t + R)^2$$