



THE SOLUTION TO THE THREE-BODY PROBLEM AND SOME APPLICATIONS

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Abstract. Here we provide and explain the coordinate transformation according to which every weighted quadratic form of the absolute Cartesian coordinates or velocities of three particles is separable into quadratic terms of the relative and centre-of-mass coordinates or velocities. This solution is applied to define a new set of weighted colour coordinates YJK in the colour space, and also to solve the dynamical system Sun-Earth-Moon. The weighted Laplacian and hence the quantum Hamiltonian operator for a system of three particles are also given in relative coordinates, and applied to calculate the vibrational energy levels of carbon dioxide and the electronic energy of the ground state of the hydrogen-molecule-ion and two-electron atomic systems like the helium atom.

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