



## SEIBERG–WITTEN EQUATIONS AND PSEUDOHOLOMORPHIC CURVES

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**Abstract.** We consider the Taubes correspondence between solutions of Seiberg–Witten equations on a compact four-dimensional symplectic manifold and pseudoholomorphic curves. We start from Kähler surfaces, in which case there is a direct correspondence between solutions of Seiberg–Witten equations and holomorphic curves. The general Taubes correspondence for symplectic four-manifolds involves, in contrast with the Kähler case, a limiting procedure, called the scaling limit. Under this scaling limit solutions of Seiberg–Witten equations reduce to families of solutions of certain vortex equations in the normal bundle of the limiting pseudoholomorphic curve.

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### 1. Introduction

One of the most remarkable results, related to the Seiberg–Witten equations (proposed by N. Seiberg and E. Witten in 1994), is the so called “Taubes equation”

$$SW = Gr.$$

It is a mnemonic formula, encoding a simple relation between two important invariants of a compact symplectic four-manifold, namely, its Seiberg–Witten invariant, produced from the moduli space of solutions of Seiberg–Witten equations, and the Gromov invariant of this manifold, counting the number of pseudoholomorphic curves in a given homology class.

This “equation” is based on a remarkable construction, proposed by C. Taubes in [9], which associates pseudoholomorphic curves with solutions of Seiberg–Witten equations. In the first part of the paper we explain how the Taubes correspondence is established in the simpler case of compact Kähler surfaces. In the Kähler case the moduli space of solutions of (perturbed) Seiberg–Witten equations can be identified with the space of effective divisors (i.e., holomorphic curves with multiplicities). This is analogous to the Bradlow’s description of the moduli space