



NEW PROPERTIES OF EUCLIDEAN KILLING TENSORS OF RANK TWO

MIRCEA CRASMAREANU

Communicated by Vladimir Kisil

Abstract. Due to the importance of Killing tensors of rank two in providing quadratic first integrals we point out several algebraic and geometrical features of this class of Killing tensor fields for the two-dimensional Euclidean metric.

MSC: 53C20, 53C30, 53C80

Keywords: Euclidean plane, killing tensor of rank two, quadratic first integral

A symmetric tensor field on a Riemannian manifold is called a *Killing tensor field* if the symmetric part of its covariant derivative is equal to zero. There exists a well-known bijection between Killing tensor fields and conserved quantities of the geodesic flow which depend polynomially on the momentum variables. In particular, Killing tensors of rank (or valence) two yields quadratic first integrals and we discuss some aspects of this process in Crasmareanu [7] from a dynamical point of view. Some classes of physical examples associated with the Euclidean 2D metric are provided in Crasmareanu and Baleanu [8].

The present paper returns to the Euclidean plane geometry \mathbb{E}^2 and its purpose is to derive other algebraic and geometrical properties of the generators of real vector space $\mathcal{K}^2(\mathbb{E}^2)$ of Killing tensors of rank two.

In Boccaletti and Pucacco [2, p. 195] is given the general expression of an element $A^{(2)} \in \mathcal{K}^2(\mathbb{E}^2)$

$$A^{(2)}(x, y) = aM + bL_1 + cL_2 + eE_1 + dE_2 + gE_3$$

with a, b, c, d, e, g arbitrary real numbers and

$$\begin{aligned} M(x, y) &= \frac{1}{2} \begin{pmatrix} y^2 & -xy \\ -xy & x^2 \end{pmatrix}, & L_1(x, y) &= \frac{1}{2} \begin{pmatrix} 0 & -y \\ -y & 2x \end{pmatrix}, & E_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ L_2(x, y) &= \frac{1}{2} \begin{pmatrix} 2y & -x \\ -x & 0 \end{pmatrix}, & E_2 &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, & E_3 &= \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \end{aligned} \quad (1)$$

So, the dimension of $\mathcal{K}^2(\mathbb{E}^2)$ is six and a general formula for this dimension appears in Chanu, Degiovanni and McLenaghan [5].