

## ON GIBBS STATES OF MECHANICAL SYSTEMS WITH SYMMETRIES\*

CHARLES-MICHEL MARLE

Communicated by Ivaïlo M. Mladenov

**Abstract.** The French mathematician and physicist Jean-Marie Souriau studied Gibbs states for the Hamiltonian action of a Lie group on a symplectic manifold and considered their possible applications in Physics and Cosmology. These Gibbs states are presented here with detailed proofs of all the stated results. A companion paper to appear will present examples of Gibbs states on various symplectic manifolds on which a Lie group of symmetries acts by a Hamiltonian action, including the Poincaré disk and the Poincaré half-plane.

*MSC:* 53D05, 53D20, 53D17, 82B03, 82B30

*Keywords:* Gibbs states, Hamiltonian systems, Liouville measure, moment maps, symplectic and Poisson manifolds, thermodynamic equilibrium

### Contents

<b>1</b>	<b>Introduction</b>	<b>46</b>
<b>2</b>	<b>Some Concepts Used in Statistical Mechanics</b>	<b>47</b>
2.1	The Birth of Statistical Mechanics . . . . .	47
2.2	Statistical States and Entropy . . . . .	48
2.3	The Use of Hamiltonian Vector Fields in Classical Mechanics . . . . .	48
2.4	The Liouville Measure on a Symplectic Manifold . . . . .	50
2.5	Comments about the Use of Statistical States . . . . .	51
2.6	Examples . . . . .	51
2.7	Evolution with Time of a Statistical State . . . . .	52
2.8	Comments about Entropy . . . . .	53
2.9	Gibbs States for a Hamiltonian System . . . . .	59
2.10	Some Properties of Gibbs States . . . . .	61
2.11	Gibbs States, Temperatures and Thermodynamic Equilibria . . . . .	62
2.12	Evolution Towards a Thermodynamic Equilibrium . . . . .	64
<b>3</b>	<b>Gibbs States for Hamiltonian Actions of Lie Groups</b>	<b>64</b>
3.1	Symmetries and Statistical States . . . . .	65
3.2	The Manifold of Motions of a Hamiltonian System . . . . .	65

---

\*In memory of the French mathematician and physicist Jean-Marie Souriau (1922–2012).