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ON THE DISTANCE BETWEEN TWO ELLIPSES IN $\mathbb{R}^3$

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Abstract. We prove that in the general case the number of critical points of the distance function between two ellipses in $\mathbb{R}^3$ equals to 4, 6, 8, 10, 12, 14 or 16. As an example, the distance between the nowadays orbits of Neptune and Pluto has six critical points: one maximum two minima and three saddle points. The global minimum is 2.503 au (astronomical units), while the global maximum is 79.111 au. If we ignore the perturbations, then in year 21103 AD the distance between Neptune and Pluto would be 2.527 au.

MSC: 70F05, 51N10

Keywords: Critical points, distance function, elliptic orbits

1. Introduction

The distance problem between two ellipses in $\mathbb{R}^3$ can be found in widely disparate fields as celestial mechanics [1, 3], computer vision [5], computational geometry [7], robotics [12], CAD [15] and so on.

Several authors [2, 3, 6, 11, 14] has studied the special case of two ellipses with common focus. In [11] and [3] the proposed algorithms for numerically solving the problem were mainly affected by the difficulty in dealing with a non-linear one dimensional equation appearing when a component of the squared distance function is sought for and the dependency from suitable starting value.

In [6] the problem was algebraically solved in the case of two Keplerian orbits by finding all the critical points of trigonometric polynomial of degree eight, obtained with Gröbner basis theory. It is also proven that a trigonometric polynomial of any lesser degree does not exist. Later in [1] the authors extend their results for all types of conic sections.

In [7] the problem of finding the distance between two given ellipses in three dimensional space is considered. By using a geometric transformation of a standard ellipse in the XOY plane, the problem is reduced to the distance problem between one standard ellipse and the other ellipse. An iterative algorithm, which is mainly

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