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FREDHOLM ANALYTIC OPERATOR FAMILIES AND PERTURBATION OF RESONANCES

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Abstract. The purpose of this contribution is to display an outlook of a certain approach that enables one to study various spectral perturbation phenomena such as perturbation of eigenvalues and resonances (scattering poles) from the general viewpoint. Some applications of this elaborated technique are presented as well.

1. Introduction

Let us consider a Hilbert space $\mathcal{H}$ and an operator function $K(\lambda, \alpha)$ which is analytic in two variables and takes its values in the class of compact operators. The variable λ stands for the "spectral parameter", while the second variable α is treated as a "perturbation parameter". For a fixed $\alpha = \alpha_0$ the point $\lambda = \lambda_0$ is called singular for the operator family $K(\lambda, \alpha_0)$ if $\ker(K(\lambda_0, \alpha_0) - I) \neq \{0\}$. The problem in question is to study the analytic properties of the **singular points** $\lambda(\alpha)$ as functions of the parameter α.

Below the notation $K(\lambda) = K(\lambda, \alpha_0)$ will be used. Let $n = \dim\ker(K(\lambda_0) - I)$ and $\{\varphi_1^{(0)}, \ldots, \varphi_n^{(0)}\}$ be a basis of the subspace $\ker(K(\lambda_0) - I)$. It may happens that for a given eigenvector $\varphi_j^{(0)}$ there exists a chain of adjoint vectors $\{\varphi_j^{(1)}, \ldots, \varphi_j^{(m_j-1)}\}$ of maximal length $(m_j - 1)$ such that

$$\begin{gathered}
K(\lambda_0)\varphi_j^{(1)} + K'(\lambda_0)\varphi_j^{(0)} = \varphi_j^{(1)} \\
K(\lambda_0)\varphi_j^{(2)} + K'(\lambda_0)\varphi_j^{(1)} + \frac{1}{2}K''(\lambda_0)\varphi_j^{(0)} = \varphi_j^{(2)} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
K(\lambda_0)\varphi_j^{(m_j-1)} + \ldots + \frac{1}{(m_j-1)!}K^{(m_j-1)}(\lambda_0)\varphi_j^{(0)} = \varphi_j^{(m_j-1)}.
\end{gathered}$$